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A stochastic model on the mean time to recruitment for a two graded manpower system associated with a univariate policy of recruitment involving combined thresholds using same geometric process for inter-decesion times

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Abstract
In this paper, an organization with two grades subjected to loss of manpower due to the policy decisions taken by the organization is considered. A mathematical model is constructed and an appropriate univariate recruitment policy, based on shock model approach involving combined optional thresholds and combined mandatory thresholds for the loss of manpower in the organization is suggested. The expected time for recruitment is obtained for different cases on the distribution of the thresholds when (i) the loss of manpower forms a sequence of independent and identically distributed exponential random variables and (ii) the inter-decision times for the two grades form the same geometric process. The analytical results are substantiated by numerical illustrations and relevant conclusions are presented.

Keywords: Manpower planning, Two grades, Shock models, Univariate recruitment policy, Geometric process, Mean time to recruitment.

INTRODUCTION

Exodus of personnel is a common phenomenon in any marketing organization whenever the organization announces revised policies regarding sales target, revision of wages, incentives and perquisites. This in turn produces loss in manpower, which adversely affects the sales turnover of the organization. Frequent recruitment is not advisable as it will be expensive due to the cost of recruitment. As the loss of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. One univariate recruitment policy based on shock model approach in reliability theory is to make recruitment when the total loss of manpower crosses a threshold.

Many models have been discussed using different kinds of wastages and different types of distributions for the threshold. Such models could be seen in [2], [3], [4] and [10]. The problem of time to recruitment is studied by several authors both for a single and multi-graded systems for different types of thresholds according as the inter-decision times are independent and identically distributed random variables or correlated random variables. In a multi-graded system, transfer of personnel from one grade to another may or may not be permitted. Most of these authors have used univariate CUM policy of recruitment by which recruitment is done whenever the cumulative loss of manpower crosses a threshold. In [16] the author has obtained the performance measures namely mean and variance of the time to recruitment for a two graded system when (i) the loss of manpower and the threshold for the loss of manpower in each grade are exponential random variables (ii) the inter-decision times are independent and identically distributed exponential random variables forming the same renewal process for both grades and (iii) threshold for the organization is the max (min) of the thresholds for the two grades (max (min) model) using the above cited univariate cumulative policy of recruitment. In [1] the authors have studied the maximum model discussed in [16] when both the distributions of the thresholds have SCBZ property. In [28] the authors have obtained the performance measures when the loss of manpower follows Poisson distribution and the threshold for the loss of manpower in the two grades are geometric random variables. Assuming that the inter-decision times are exchangeable and constantly correlated random variables, the performance measures of time to recruitment are obtained in [12] according as the loss of manpower and thresholds are discrete or continuous random variables. In [19] the author has extended the results in [12] for geometric thresholds when the inter-decision times for the two grades form two different renewal processes. In [29] the author has studied the results in [12] and [16] using a bivariate policy of recruitment. In [31] these performance measures are obtained when the inter-decision times are exchangeable and constantly correlated exponential random variables and the distributions of the thresholds have SCBZ property. In [27] the authors have studied the results in [16] when the threshold for the organization is the sum of the thresholds for the grades. This paper has been extended in [19] when threshold distributions have SCBZ property. In [13] the work in [27] is studied when the loss of manpower and thresholds are geometric random variables according as the inter-decision times for the two grades are correlated random variables or forming two different renewal processes. This author has also obtained the mean time for recruitment for constant combined thresholds using a univariate max policy of recruitment. In [17] the authors have studied the work in [16] when the thresholds for the loss of manpower in the two grades

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follow an extended exponential distribution with shape parameter 2. In [6], [7] and [8] the authors have considered a new univariate recruitment policy involving two thresholds in which one is optional and the other a mandatory and obtained the mean time to recruitment under different conditions on the nature of the thresholds according as the inter-decision times are independent and identically distributed random variables or the inter-decision times are exchangeable and constantly correlated exponential random variables. In [9] the authors have also obtained the mean time to recruitment when the optional and mandatory thresholds are geometric random variables. In [20] the authors have studied the problem of time to recruitment for a two graded manpower system when (i) the loss of manpower in the organization due to i^{th} decision is maximum of the loss of manpower in this decision in grades A and B (ii) the threshold for the organization is max(min) of the thresholds for the loss of manpower in the two grades under different conditions using a univariate CUM policy of recruitment. They have also studied this problem using max policy of recruitment by assuming constant threshold. In [21] the authors have extended the work of [20] when the threshold for the loss of manhours in the organization is the sum of the corresponding thresholds of the two grades according as the two thresholds are exponential or extended exponential thresholds. In [22], [23], [24] and [25] the authors have extended the results in [6] for a two grade system according as the thresholds are exponential random variables or extended exponential random variables or SCBZ property possessing random variables or geometric random variables. In [26] the authors have extended the result in [6] for a two graded system according as the optional thresholds are exponential random variable and the distributions of the mandatory thresholds have SCBZ property. For a single graded manpower system, in [18] the authors have obtained the mean and variance of time to recruitment when (i) the loss of manpower form a sequence of independent and identically distributed Poisson random variables (ii) the threshold for the loss of manpower follow geometric distribution and the number of policy decisions announced by the organization is governed by a renewal process with independent and identically distributed exponential inter-decision times. In all the earlier research works the monotonicity of inter-decision times which do exists in reality, is not taken into account. In [15] the above limitation is removed and the authors have obtained the mean time to recruitment for a single grade manpower system by assuming that (i) the inter-decision times form a geometric process in which the monotonicity is inbuilt in the process itself (ii) the loss of manpower is a sequence of independent and identically distributed exponential random variables and (iii) the distribution of the threshold for the loss of manpower in the organization is exponential. In [5] the authors have studied the results of [15] for a two graded system when the threshold for the loss of manpower in the two grades are exponential thresholds or SCBZ property possessing thresholds or extended exponential thresholds or geometric thresholds. They have also studied this work in [30] by considering optional and mandatory thresholds for the loss of manpower in the two grades. In all the above cited research work involving optional and mandatory thresholds, the allowable loss of manpower to the system is not maximum. Recently in [14] the authors have extended the work of [[23],[26]] when the loss of manpower for the organization is the maximum of the loss of manpower in the two grades by assuming exponential, extended exponential and SCBZ property possessing thresholds for the loss of manpower. In order to provide a maximum allowable loss in the organization, an attempt has been made in this paper by considering combined optional thresholds and combined mandatory thresholds.

The objective of the present paper is to study the problem of time to recruitment for a two graded manpower system and to obtain the mean time for recruitment using CUM univariate recruitment policy for different cases of the threshold distributions by assuming that the inter-decision times for the two grades form the same geometric process. The analytical results are numerically illustrated and the influence of nodal parameters on the mean time to recruitment is studied.

Model Description And Analysis

Consider an organization having two grade A and B in which decisions are taken at random epochs $[0,\infty)$. At every decision making epoch a random number of persons quit. It is assumed that the loss manpower is linear and cumulative. Let $V_k(t)$ be the probability that there are exactly k-decisions in $[0,t)$, $k = 1,2,3,\ldots$. From renewal theory [11], $V_k(t) = F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$. Let X_k be the loss of manpower in the organization in the k^{th} decision epoch, $k = 1,2,3,\ldots$ forming a sequence of independent and identically distributed random variables. For $k = 1,2,3,\ldots$, let S_k be the cumulative loss of manpower in the first k-decisions. It is assumed that the inter-decision times U_k, $k = 1,2,3,\ldots$ form a geometric process with parameter a (a>0). This means $\{a^{k-1} U_k\}_{k=1}^{\infty}$ is a renewal process. It is assumed that $U_1, U_2, \ldots$ are independent random variables. Let $f(.)(F(.))$ be density (distribution) function for the renewal process with parameter θ. Let $f^*(.)$ be the Laplace transform of $f(.)$. It is assumed that loss of manpower process and the process of inter-decision times are statistically independent. Let $w_k(.)$, $(W_k(.))$ be the density (distribution) of the k^{th} term of geometric process $\{U_k\}_{k=1}^{\infty}$. Let $f_k(.)(F_k(.))$ be probability density (distribution) function of $\sum_{i=1}^{k} U_i$. Let Y_A and Y_B be the random variables denoting optional thresholds for the loss of manpower in grade A and grade B respectively. Let Z_A and Z_B be the random variables denoting mandatory thresholds for the loss of manpower in grade A and grade B respectively. It is assumed that $Y_A < Z_A$ and $Y_B < Z_B$. The optional and mandatory thresholds Y and Z for the loss of manpower in the organization are defined as $Y = Y_A + Y_B$ and $Z = Z_A + Z_B$. The recruitment policy employed in this paper is as follows: *If the total loss of manpower crosses the optional threshold level Y, the organization may or may not go for recruitment, but if the total loss of manpower crosses the mandatory threshold Z, recruitment is necessary.* Let p be the probability that the organization is not going for recruitment whenever the total loss of manpower crosses optional level Y. Let W be a continuous random variable denoting the time for recruitment in the organization with probability density function $\ell(.)$, cumulative distribution function $L(.)$. Let $E(W)$ be the mean time to recruitment.

RESULTS

In this section, we obtain an explicit analytic expression for mean time to recruitment by considering four cases on the distributions of the thresholds. As in [6] the survival function of W is given by

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\, P(S_k < Y) + p \sum_{k=0}^{\infty} V_k(t)\, P(S_k \geq Y) \times P(S_k < Z) \quad (1)$$

Case (i)

$Y_A \sim \exp(\lambda_A)$, $Y_B \sim \exp(\lambda_B)$, $Z_A \sim \exp(\mu_A)$, $Z_B \sim \exp(\mu_B)$ and $X_i \sim \exp(\alpha)$

In this case, by using the law of total probability it can be shown that

$$P[S_k < Y] = C_1 (D_2)^k - C_2 (D_1)^k \quad (2)$$

and

$$P[S_k < Z] = C_3 (D_4)^k - C_4 (D_3)^k \quad (3)$$

where

$$C_1 = \frac{\lambda_A}{(\lambda_A - \lambda_B)},\ C_2 = \frac{\lambda_B}{(\lambda_A - \lambda_B)},\ C_3 = \frac{\mu_A}{(\mu_A - \mu_B)},\ C_4 = \frac{\mu_B}{(\mu_A - \mu_B)},$$

$D_1 = g^*(\lambda_A)$, $D_2 = g^*(\lambda_B)$, $D_3 = g^*(\mu_A)$ and $D_4 = g^*(\mu_B)$, where $g^*(\cdot)$ is the Laplace transform of $g(\cdot)$, the density of function of X_i, $i = 1,2,3,\ldots$.

For $m = 1,2,3,4$, define $E_m(t) = [1 - D_m] \sum_{k=1}^{\infty} F_k(t)\, (D_m)^{k-1}$ (4)

For $r = 1,2$, $n = 3,4$, define

$$E_{r,n}(t) = [1 - D_r D_n] \sum_{k=1}^{\infty} F_k(t)\, (D_r D_n)^{k-1} \quad (5)$$

From (1), (2), (3), (4) and (5) and on simplification we get,

$$P(W>t) = 1 - C_1E_2(t) + C_2E_1(t) + P[- C_3E_4(t) + C_4E_3(t) + C_1C_3E_{2,4}(t) - C_1C_4E_{2,3}(t) - C_2C_3E_{1,4}(t) + C_2C_4E_{1,3}(t)]$$
(6)

Since $\ell(t) = \frac{d}{dt}[1 - P(W > t)]$, from (6) it can be shown that

$$\ell^*(s) = C_1 e_2^*(s) - C_2 e_1^*(s) + p\,[C_3 e_4^*(s) - C_4 e_3^*(s) - C_1C_3 e_{2,4}^*(s) + C_1C_4 e_{2,3}^*(s) + C_2C_3 e_{1,4}^*(s) - C_2C_4 e_{1,3}^*(s)] \quad (7)$$

$$\left.\begin{aligned} &\text{where } e_m^*(s) = L[E'_m(t)] = [1 - D_m] \sum_{k=1}^{\infty} f_k(s)\, (D_m)^{k-1},\ m = 1,2,3,4 \\ &\text{and} \\ &e_{r,n}^*(s) = L[E'_{r,n}(t)] = [1 - D_rD_n] \sum_{k=1}^{\infty} f_k(s)\, (D_rD_n)^{k-1},\ r = 1,2,\ n = 3,4 \end{aligned}\right\} \quad (8)$$

Since $\{U_k\}_{k=1}^{\infty}$ is a geometric process, we find that

$$f_k^*(s) = \prod_{j=1}^{k} f^*\left(\frac{s}{a^{j-1}}\right) \quad (9)$$

Using (9) in (7), we get

$$\ell^*(s) = C_1 A_2^*(s) - C_2 A_1^*(s) + p\,[C_3 A_4^*(s) - C_4 A_3^*(s) - C_1C_3 A_{2,4}^*(s) + C_1C_4 A_{2,3}^*(s) + C_2C_3 A_{1,4}^*(s) - C_2C_4 A_{1,3}^*(s)] \quad (10)$$

$$\left.\begin{aligned} &\text{where } A_m^*(s) = [1 - D_m] \sum_{k=1}^{\infty} F_k(t)\, (D_m)^{k-1} \prod_{j=1}^{k} f^*\left(\frac{s}{a^{j-1}}\right),\ m = 1,2,3,4 \\ &\text{and} \\ &A_{r,n}^*(s) = [1 - D_rD_n] \sum_{k=1}^{\infty} F_k(t)\, (D_r D_n)^{k-1} \prod_{j=1}^{k} f^*\left(\frac{s}{a^{j-1}}\right),\ r = 1,2,\ n = 3,4 \end{aligned}\right\} \quad (11)$$

By hypothesis $g^*(s) = \frac{\alpha}{\alpha + s}$, $E(U_1) = -f^{*\prime}(0) = \frac{1}{\theta}$ (12)

On simplification, it is found that

$$\frac{d}{ds}\left\{\prod_{j=1}^{k} f^*\left(\frac{s}{a^{j-1}}\right)\right\}_{s=0} = \frac{(a^k - 1) f^{*\prime}(0)}{a^{k-1}(a-1)} \quad (13)$$

Since $E(W) = -\left\{\frac{d}{ds}\ell^*(s)\right\}_{s=0}$ (14)

from (10), (11), (12), (13) and (14), on simplification one can show that.

$$E(W) = C_1a_2 - C_2a_1 + p[C_3a_4 - C_4a_3 - C_1C_3a_{2,4} + C_1C_4a_{2,3} + C_2C_3a_{1,4} - C_2C_4a_{1,3}] \quad (15)$$

where $a_m = \frac{a}{\theta[a - D_m]}$, $m = 1,2,3,4$ and $a_{r,n} = \frac{a}{\theta[a - D_rD_n]}$, $r = 1,2$, $n = 3,4$,

(15) gives the mean time to recruitment for case (i).

Case (ii)

Y_A, Y_B, Z_A and Z_B follow extended exponential distribution with scale parameters λ_A, λ_B, μ_A and μ_B respectively and shape parameter 2 and $X_i \sim \exp(\alpha)$.

If V follows an extended exponential distribution with scale parameter λ and shape parameter 2, then

$$P(V \leq x) = (1 - e^{-\lambda x})^2,\ \lambda > 0$$

As in case (i), we find on simplification that

$$P(S_k < Y) = C_5 (D_5)^k - C_6 (D_6)^k + C_7 (D_7)^k - C_8 (D_8)^k \quad (16)$$

and $P(S_k < Z) = C_9 (D_9)^k - C_{10}(D_{10})^k + C_{11}(D_{11})^k - C_{12}(D_{12})^k$ (17)

where

$$C_5 = \frac{4\lambda_B^2}{(\lambda_A - \lambda_B)(\lambda_A - 2\lambda_B)},\ C_6 = \frac{2\lambda_B^2}{(2\lambda_A - \lambda_B)(2\lambda_A - 2\lambda_B)},$$

$$C_7 = \frac{4\lambda_A^2}{(\lambda_A - \lambda_B)(2\lambda_A - \lambda_B)},\ C_8 = \frac{2\lambda_A^2}{(\lambda_A - 2\lambda_B)(2\lambda_A - 2\lambda_B)},$$

$$C_9 = \frac{2\mu_A^2}{(\mu_A - \mu_B)(\mu_A - 2\mu_B)},\ C_{10} = \frac{2\mu_B^2}{(2\mu_A - \mu_B)(2\mu_A - 2\mu_B)},$$

$$C_{11} = \frac{4\mu_A^2}{(\mu_A - \mu_B)(2\mu_A - \mu_B)},\ C_{12} = \frac{2\mu_A^2}{(\mu_A - 2\mu_B)(2\mu_A - 2\mu_B)},$$

$D_5 = g^*(\lambda_A)$, $D_6 = g^*(2\lambda_A)$, $D_7 = g^*(\lambda_B)$, $D_8 = g^*(2\lambda_B)$, $D_9 = g^*(\mu_A)$, $D_{10} = g^*(2\mu_A)$, $D_{11} = g^*(\mu_B)$, $D_{12} = g^*(2\mu_B)$

where $f^{*\prime}(\cdot)$ and $g^*(\cdot)$ are given by (12)

Proceeding as in case (i), we get

$$E(W) = C_5a_5 - C_6a_6 + C_7a_7 - C_8a_8 + p[C_9a_9 - C_{10}a_{10} + C_{11}a_{11} - C_{12}a_{12} - C_5C_9a_{5,9} + C_5C_{10}a_{5,10} - C_5C_{11}a_{5,11} + C_5C_{12}a_{5,12} + C_6C_9a_{6,9} - C_6C_{10}a_{6,10} + C_6C_{11}a_{6,11} - C_6C_{12}a_{6,12} - C_7C_9a_{7,9} + C_7C_{10}a_{7,10} - C_7C_{11}a_{7,11} + C_7C_{12}a_{7,12} + C_8C_9a_{8,9} - C_8C_{10}a_{8,10} + C_8C_{11}a_{8,11} - C_8C_{12}a_{8,12}] \quad (18)$$

where $a_m = \frac{a}{\theta[a - D_m]}$, $m = 5,6,\ldots,12$ and $a_{r,n} = \frac{a}{\theta[a - D_rD_n]}$,

$r = 5,\ldots,8$, $n = 9,\ldots,12$,

(18) gives the mean time to recruitment for case (ii).

Case (iii)
The distributions of Y_A, Y_B, Z_A and Z_B have SCBZ property and $X_i \sim \exp(\alpha)$.

In this case distribution of Y_A, Y_B, Z_A and Z_B are respectively given by

$$P(Y_A \le x) = 1 - p_1 e^{-(\lambda_{A_1}+\lambda_A)x} - q_1 e^{-\lambda_{A_2}x};$$

$$P(Y_B \le x) = 1 - p_2 e^{-(\lambda_{B_1}+\lambda_B)x} - q_2 e^{-\lambda_{B_2}x};$$

$$P(Z_A \le x) = 1 - p_3 e^{-(\mu_{A_1}+\mu_A)x} - q_3 e^{-\mu_{A_2}x};$$

$$P(Z_A \le x) = 1 - p_4 e^{-(\mu_{B_1}+\mu_B)x} - q_4 e^{-\mu_{B_2}x}$$

where $p_1 = \dfrac{(\lambda_{A_1} - \lambda_{A_2})}{(\lambda_{A_1} - \lambda_{A_2} + \lambda_A)}$, $q_1 = 1-p_1$; $p_2 = \dfrac{(\lambda_{B_1} - \lambda_{B_2})}{(\lambda_{B_1} - \lambda_{B_2} + \lambda_B)}$, $q_2 = 1-p_2$

$p_3 = \dfrac{(\mu_{A_1} - \mu_{A_2})}{(\mu_{A_1} - \mu_{A_2} + \mu_A)}$, $q_3 = 1-p_3$; $p_4 = \dfrac{(\mu_{B_1} - \mu_{B_2})}{(\mu_{B_1} - \mu_{B_2} + \mu_B)}$, $q_4 = 1-p_4$

Therefore on simplification as in case (i), we find that

$$P(S_k < Y) = C_{13}(D_{13})^k + C_{14}(D_{14})^k + C_{15}(D_{15})^k + C_{16}(D_{16})^k \qquad (19)$$

and

$$P(S_k < Z) = C_{17}D_{17}{}^k + C_{18}D_{18}{}^k + C_{19}D_{19}{}^k + C_{20}D_{20}{}^k \qquad (20)$$

where $C_{13} = \left\{ P_1 - \dfrac{p_1 p_2(\lambda_{A_1}+\lambda_A)}{(\lambda_{A_1}+\lambda_A-\lambda_{B_1}-\lambda_B)} - \dfrac{p_1 q_2(\lambda_{A_1}+\lambda_A)}{(\lambda_{A_1}+\lambda_A-\lambda_{B_2})} \right\}$,

$C_{14} = \left\{ \dfrac{p_1 p_2(\lambda_{A_1}+\lambda_A)}{(\lambda_{A_1}+\lambda_A-\lambda_{B_1}-\lambda_B)} + \dfrac{p_2 q_1 \lambda_{A_2}}{(\lambda_{A_2}+\lambda_{B_1}-\lambda_B)} \right\}$,

$C_{15} = \left\{ q_1 - \dfrac{p_2 q_1 \lambda_{A_2}}{(\lambda_{A_2}-\lambda_{B_1}-\lambda_B)} - \dfrac{q_1 q_2 \lambda_{A_2}}{(\lambda_{A_2}-\lambda_{B_2})} \right\}$,

$C_{16} = \left\{ \dfrac{p_1 q_1(\lambda_{A_1}+\lambda_A)}{(\lambda_{A_1}+\lambda_A-\lambda_{B_2})} + \dfrac{q_1 q_2 \lambda_{A_2}}{(\lambda_{A_2}-\lambda_{B_2})} \right\}$,

$C_{17} = \left\{ p_3 - \dfrac{p_3 p_4(\mu_{A_1}+\mu_A)}{(\mu_{A_1}+\mu_A-\mu_{B_1}-\mu_B)} - \dfrac{p_3 q_4(\mu_{A_1}+\mu_A)}{(\mu_{A_1}+\mu_A-\mu_{B_2})} \right\}$,

$C_{18} = \left\{ \dfrac{p_3 p_4(\mu_{A_1}+\mu_A)}{(\mu_{A_1}+\mu_A-\mu_{B_1}-\mu_B)} + \dfrac{p_4 q_3 \mu_{A_2}}{(\mu_{A2}-\mu_{B_1}-\mu_B)} \right\}$,

$C_{19} = \left\{ q_3 - \dfrac{p_4 q_3 \mu_{A_2}}{(\mu_{A_2}-\mu_{B_1}-\mu_B)} - \dfrac{q_3 q_4 \mu_{A_2}}{(\mu_{A_2}-\mu_{B_2})} \right\}$,

$C_{20} = \left\{ \dfrac{p_3 q_3(\mu_{A_1}+\mu_A)}{(\mu_{A_1}+\mu_A-\mu_{B_2})} + \dfrac{q_3 q_4 \mu_{A_2}}{(\mu_{A_2}-\mu_{B_2})} \right\}$

$D_{13} = g^*(\lambda_{A_1}+\lambda_A)$, $D_{14} = g^*(\lambda_{B_1}+\lambda_B)$, $D_{15} = g^*(\lambda_{A_2})$, $D_{16} = g^*(\lambda_{B_2})$

$D_{17} = g^*(\mu_{A_1}+\mu_A)$, $D_{18} = g^*(\mu_{B_1}+\mu_B)$, $D_{19} = g^*(\mu_{A_2})$, $D_{20} = g^*(\mu_{B_2})$

where $f^{*'}(\cdot)$ and $g^*(\cdot)$ are given by (12)

Proceeding as in case (i), we get

$$E(W) = C_{13}a_{13} + C_{14}a_{14} + C_{15}a_{15} + C_{16}a_{16} + p\,\{C_{17}a_{17} + C_{18}a_{18} + C_{19}a_{19} + C_{20}a_{20} - C_{13}C_{17}a_{13,17} - C_{13}C_{18}a_{13,18} - C_{13}C_{19}a_{13,19} - C_{13}C_{20}a_{13,20} - C_{14}C_{17}a_{14,17} - C_{14}C_{18}a_{14,18} - C_{14}C_{19}a_{14,19} - C_{14}C_{20}a_{14,20} - C_{15}C_{17}a_{15,17} - C_{15}C_{18}a_{15,18} - C_{15}C_{19}a_{15,19} - C_{15}C_{20}a_{15,20} - C_{16}C_{17}a_{16,17} - C_{16}C_{18}a_{16,18} - C_{16}C_{19}a_{16,19} - C_{16}C_{20}a_{16,20}\} \qquad (21)$$

where $a_m = \dfrac{a}{\theta[a - D_m]}$, $m = 13,\ldots,20$, and $a_{r,n} = \dfrac{a}{\theta[a - D_r D_n]}$, $r = 13,\ldots,16$, $n = 17,\ldots,20$,

(21) gives mean time to recruitment for case (iii).

Case (iv)
$Y_A \sim$ Geo (λ_A), $Y_B \sim$ Geo (λ_B), $Z_A \sim$ Geo (μ_A), $Z_B \sim$ Geo (μ_B) and $X_i \sim$ Poiss(β)

where $\phi(.)$ is the probability generating function of X_i, $i = 1,2,3,\ldots$
As in case(i), we can derive that

$$P(S_k < Y) = C_{21}(D_{21})^k - C_{22}(D_{22})^k \qquad (22)$$

and

$$P(S_k < Z) = C_{23}(D_{23})^k - C_{24}(D_{24})^k \qquad (23)$$

where $C_{21} = \dfrac{\lambda_B}{(\bar{\lambda}_A - \bar{\lambda}_B)}$, $C_{22} = \dfrac{\lambda_A}{(\bar{\lambda}_A - \bar{\lambda}_B)}$,

$C_{23} = \dfrac{\mu_B}{(\bar{\mu}_A - \bar{\mu}_B)}$, $C_{24} = \dfrac{\mu_A}{(\bar{\mu}_A - \bar{\mu}_B)}$,

$D_{21} = \varphi(\bar{\lambda}_A)$, $D_{22} = \varphi(\bar{\lambda}_B)$, $D_{23} = \varphi(\bar{\mu}_A)$, and $D_{24} = \varphi(\bar{\mu}_B)$,

By hypothesis $f^*(s) = \dfrac{\theta}{\theta + s}$ and $\phi(\lambda) = \dfrac{\alpha}{(\alpha+\lambda)}$

Proceeding as in case (i), we get,

$$E(W) = C_{21}a_{21} - C_{22}a_{22} + p\,\{C_{23}a_{23} - C_{24}a_{24} - C_{21}C_{23}a_{21,23} + C_{21}C_{24}a_{21,24} + C_{22}C_{23}a_{22,23} - C_{22}C_{24}a_{22,24}\} \qquad (24)$$

where $a_m = \dfrac{a}{\theta[a - D_m]}$, $m = 21,22,23,24$, and $a_{r,n} = \dfrac{a}{\theta[a - D_r D_n]}$, $r = 21,22$, $n = 23,24$

(24) gives the mean time to recruitment for case (iv).

Numerical Illustrations And Conclusions

The analytical expressions for the mean time to recruitment is analyzed for both the models. The influence of nodal parameter 'a' on the performance measures namely the mean time for recruitment for both the models is shown in the following tables when $0 < a < 1$ and $a > 1$, by fixing $\lambda_A = 0.8$, $\lambda_B = 0.55$, $\mu_A = 0.45$, $\mu_B = 0.35$, $p = 0.8$, $\theta = 0.6$, $\alpha = 0.4$, $\lambda_{A_1} = 0.65$, $\lambda_{A_2} = 0.3$, $\lambda_{B_1} = 0.6$, $\lambda_{B_2} = 0.5$, $\mu_{A_1} = 0.7$, $\mu_{A_2} = 0.5$, $\mu_{B_1} = 0.66$, $\mu_{B_2} = 0.77$, $\beta = 2$.

In order to provide a feasible solution, the denominator in the expression for E(W) in each case of the two models is assumed to be positive. This aspect is taken into account in the construction of the following tables.

Table 1. Effect of 'a' on performance measures

0 < a < 1 E(W)					a > 1 E(W)				
a	Case(i)	Case(ii)	Case(iii)	Case(iv)	a	Case(i)	Case(ii)	Case(iii)	Case(iv)
0.6	38.47	87.44	56.94	23.52	1.2	4.21	5.06	3.86	4.10
0.7	14.37	24.96	13.90	11.68	1.3	3.84	4.50	3.52	3.76
0.8	9.00	13.53	8.33	7.98	1.4	3.56	4.10	3.27	3.51
0.9	6.73	9.24	6.16	6.22	1.5	3.35	3.79	3.08	3.31

CONCLUSIONS

Since $\{U_k\}$ is a geometric process with parameter 'a', the average inter-decision times $E(U_k)$ is given by

$$E(U_k) = \frac{1}{\theta a^{k-1}}, \ k = 1,2\ldots$$

Therefore, we observe the following:

(i) As 'a' increases (decreases), $E(U_k)$ decreases (increases) and hence the mean time to recruitment has to decrease (increase). This realistic observation is indicated in Tables 1 and 2.

(ii) If a > 1, then U_i's, (i = 1,2,…) form a decreasing sequence. Hence the inter-decision times will decrease. Consequently the mean time to recruitment will also decrease, since the loss of manpower would be more frequent.

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