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Kaehlerian manifolds admitting a metric semi-symmetric F-connection-II

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Abstract
In Kaehlerian space of real dimension n a semi-symmetric F-connection was introduce by S.S.Pujar[2] about one decade ago. Using this affine F-connection we obtain some global results. The main purpose of the paper is to establish that a conformally flat real Kaehlerian manifold is W-flat if and only if the curvature tensor of the metric semi-symmetric F-connection vanishes.

Keywords: Real Kaehlerian manifolds, affine connection, Concircular curvature tensor.

INTRODUCTION

Let M be the Kaehlerian manifold of real dimension n ≥4 (n=2m, m ≥1), with structure tensors (g, F), where g is Riemannian metric and F is a skew symmetric tensor field of type (1,1), called structure tensor on M, satisfying

G(F(X),F(Y)) = g(X,Y), ($\nabla_X F$)(Y) = 0,
F^2(X) = - X and F(X,Y) = - F(Y,X).

Previous author [2] define an affine connection on M induced by a smooth function ρ on M by

$$\overset{*}{\nabla}_X Y = \nabla_X Y + F(Y)\omega(X) \tag{1.1}$$

where X and Y are any vector fields on M, $\omega = d\rho$ is the 1-form associated with the vector field $D\rho$ and we call such an affine connection as a metric semi-symmetric F-connection.
The author [2] proved the following Theorem

Theorem A[2]. In order for an n-dimensional real Kaehlerian manifold M, n ≥ 4 to be conformally flat, it is necessary and sufficient that curvature tensor of the metric semi-symmetric F-connection vanishes.

Theorem B[1]. If a real Kaehlerian manifold is conformally flat, it is of zero curvature.
The purpose of the paper is to prove the following theorems:

Theorem1.1. If, in a Kaehlerian manifold M of dimension n ≥ 4 (n=2m, m ≥1) there exists a smooth function ρ such that the metric semi-symmetric F-connection(1.1) is of zero curvature, then manifold M is of zero curvature and hence the W-curvature tensor of the manifold vanishes.
Theorem1.2. If, in a Kaehlerian manifold M of dimension n ≥ 4 (n=2m, m ≥ 1) there exists a smooth function ρ such that the metric semi-symmetric F-connection(1.1) is of zero curvature, then manifold M is of zero curvature and hence the concircular curvature tensor of the manifold vanishes.
Theorem1.3. If, in a Kaehlerian manifold M of dimension n ≥ 4 (n=2m, m ≥ 1) there exists a smooth function ρ such that the metric semi-symmetric F-connection(1.1) is of zero curvature, then manifold M is of zero curvature and hence the U-curvature tensor of the manifold vanishes.

Theorem1.4. If, in a Kaehlerian manifold M of dimension n ≥ 4 (n=2m, m ≥ 1) there exists a smooth function ρ such that the metric semi-symmetric F-connection(1.1) is of zero curvature, then manifold M is of zero curvature and hence the holomorphically projective(H-projective) curvature tensor of the manifold vanishes.

Theorem 1.5. A conformally flat real Kaehlerian manifold is W-flat if and only if the curvature tensor of the metric semi-symmetric F-connection vanishes.

Notations:

Let x^i, i=1,2,3,….n be the local coordinates in the neighborhood of the point x of M. Let F_j^i, g_{ji} be the components of the complex structure tensor F and the covariant component of the metric tensor g respectively. Let $\left\{ {h \atop j\ i} \right\}$ denote the Christoffel symbols formed by g which are the components of the connection ∇. Then g and F are related by

$$F_j^t F_i^s g_{ts} = g_{ji} \text{ or } F_{js} F_i^s = g_{ji}, \nabla_j F_i^h = 0 \tag{2.1}$$
$$F_{ji} = -F_{ij} \text{ and } F_t^h F_j^t = -\delta_j^h \tag{2.2}$$

where $F_{ji} = g_{ti} F_j^t$ and the indices h,j,i,… etc taking on the values over the range 1,2,3,…n and in this paper the Einstein summation conventions are used over the repeated suffixes to avoid numerous summation signs. The lowering and raising of a tensor are done using g_{ji} and g^{ji} which are covariant and contravariant components of g, respectively. Let ρ be a smooth function on M. Let $D\rho$ be

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the vector field on M associated with the closed 1-form $\omega = d\rho$. The local components of $d\rho$ are denoted by $\rho_i = \nabla_i \rho = \frac{\partial \rho}{\partial x^i}$ and that of $D\rho$ by $\rho^i = g^{ji}\rho_j$.

We now introduce an affine connection on M. An affine connection $\overset{*}{\nabla}$ on M whose components are denoted by $\overset{*}{\Gamma}{}^h_{ji}$, is defined by

$$\overset{*}{\nabla}_X Y = \nabla_X Y + F(Y)\omega(X)$$

or local coordinates

$$\overset{*}{\Gamma}{}^h_{ji} = \left\{ {h \atop j \;\; i} \right\} + F_i^{\ h}\rho_j \tag{2.3}$$

We proceed here after with the tensor notations for the sake of convenience. Since the affine connection (2.3) satisfies torsion T with local components given by

$$T^h_{ji} = \delta^h_i \rho_j - \delta^h_j \rho_i \tag{2.4}$$

Further,

$$\overset{*}{\nabla}_k\, g_{ji} = 0 \tag{2.5}$$

$$\overset{*}{\nabla}_k\, F^i_j = 0 \tag{2.6}$$

the affine connection (2.3) is semi-symmetric by (2.4), metric by (2.5) and F is invariant by (2.6) also. We call such a connection a metric semi-symmetric F-connection.

Let, $K_{kji}{}^h$, K_{ji} and K respectively denote the components of curvature tensor, Ricci tensor R, and the scalar curvature of M with respective to ∇ respectively. If $\overset{*}{K}_{kji}{}^h$, $\overset{*}{K}_{ji}$ and $\overset{*}{K}$ denote the components of curvature tensor, Ricci tensor $\overset{*}{R}$, and the scalar curvature of M with respective to $\overset{*}{\nabla}$ respectively, then using (2.1) and (2.2), we have

$$\overset{*}{K}_{kji}{}^h = K_{kji}{}^h \tag{2.7}$$

since

$$\partial_j \rho_i - \partial_i \rho_j = 0.$$

On M we define curvature tensor [9], [10] of type (1,3) with components $K_{kji}{}^h$ by

$$K_{kji}{}^h = \frac{K}{4n(n+1)}(\delta^h_k g_{ji} - \delta^h_j g_{ki} + F^h_k F_{ji} - F_j^{\ h} F_{ki} - F_{kj} F_i^{\ h}) \tag{2.8}$$

deviation tensor G[3],[4],[10] of type (0,2) with components G_{ji} by

$$G_{ji} = K_{ji} - \frac{K}{2n} g_{ji}, \tag{2.9}$$

concircular tensor Z [5], [9], [10] of type (1,3) with components $Z_{kji}{}^h$ by

$$Z_{kji}{}^h = K_{kji}{}^h - \frac{K}{4n(n+1)}(\delta^h_k g_{ji} - \delta^h_j g_{ki} + F_k^{\ h} F_{ji} - F_j^{\ h} F_{ki} - 2F_{kj} F_i^{\ h}), \tag{2.10}$$

W- tensor [3] of type (1,3) with components $W_{kji}{}^h$ by

$$W_{kji}{}^h = aZ_{kji}{}^h + b_1(\delta^h_k G_{ji} - \delta^h_i G_{kj}) + b_2(g_{ji} G^h_k - g_{ki} G^h_j), \tag{2.11}$$

H-projective tensor P [3],[5],[9] of type (1,3) with components $P_{kji}{}^h$ by

$$P_{kji}{}^h = K_{kji}{}^h - \frac{1}{2(n+1)}(\delta^h_k K_{ji} - \delta^h_j K_{ki} - F^h_k K_{jt} F^t_i + F^h_j K_{kt} F^t_i + K_{kt} F^t_j F^h_i - K_{jt} F^t_k F^h_i), \tag{2.12}$$

U-tensor [4],[6],[7] of type (1,3) with components $U_{kji}{}^h$ by

$$U_{kji}{}^h = cP_{kji}{}^h + d_1(\delta^h_k G_{ji} - \delta^h_i G_{kj}) + d_2(g_{ji} G^h_k - g_{ki} G^h_j), \tag{2.13}$$

and the Weyl Conformal curvature tensor C [8] of type (1,3) with components $C_{kji}{}^h$ by

$$C_{kji}{}^h = K_{kji}{}^h + \delta^h_k C_{ji} - \delta^h_j C_{ki} + \delta^h_k g_{ji} - \delta^h_j g_{ki} \tag{2.14}$$

where

$$C_{ji} = -\frac{1}{n-2} K_{ji} + \frac{1}{2(n-1)(n-2)} K g_{ji}.$$

If $C_{kji}{}^h = 0$, then M is conformally flat.

If $W_{kji}{}^h = 0$, then M is W-flat.

Lemma 2.1: If, in a Kaehlerian manifold M of dimension n (n=2m, m ≥ 1), there exists a smooth function ρ such that it induces the metric semi-symmetric F-connection(1.1),then the curvature tensor of the manifold is identically to the curvature tensor of metric semi-symmetric F-connection.

Proof: Follows from (2.7).

Proof of Theorems:

From Lemma 2.1 and the condition stated in the Theorem 1.1 to Theorem 1.4, it follows that

$$\overset{*}{K}_{kji}{}^h = 0, \ \overset{*}{K}_{ji} = 0, \ \overset{*}{K} = 0 \tag{2.15}$$

so that

$$K_{kji}{}^h = 0, \ K_{ji} = 0, \ K = 0 \tag{2.16}$$

Proof of Theorem1.1: Follows from (2.16) and (2.11).

Proof of Theorem1.2: Follows from (2.16) and (2.10).

Proof of Theorem1.3: Follows from (2.16) and (2.13).

Proof of Theorem1.4: Follows from (2.16) and (2.12).

Proof of Theorem1.5: From Lemma2.1, (2.10), (2.11) and the condition stated in the Theorem 1.5, it follows that

$$W_{kji}{}^{h} = 0$$

which shows that M is W-flat.

On the other hand, if M is conformally and W-flat, then from Theorem B, it follows that

$$K_{kji}{}^{h} = 0$$

which, in view of (2.7), shows that $\overset{*}{K}_{kji}{}^{h} = 0$.

Thus the proof of Theorem1.5 completes.

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