Recent Research in Science and Technology 2012, 4(5): 10-14
ISSN: 2076-5061
Available Online: http://recent-science.com/

# Different strategies to solve fuzzy linear programming problems

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**Abstract**

Fuzzy linear programming problems have an essential role in fuzzy modeling, which can formulate uncertainty in actual environment In this paper we present methods to solve (i) the fuzzy linear programming problem in which the coefficients of objective function are trapezoidal fuzzy numbers, the coefficients of the constraints, right hand side of the constraints are triangular fuzzy numbers, and (ii) the fuzzy linear programming problem in which the variables are trapezoidal fuzzy variables, the coefficients of objective function and right hand side of the constraints are trapezoidal fuzzy numbers, (iii) the fuzzy linear programming problem in which the coefficients of objective function, the coefficients of the constraints, right hand side of the constraints are triangular fuzzy numbers. Here we use α –cut and ranking functions for ordering the triangular fuzzy numbers and trapezoidal fuzzy numbers. Finally numerical examples are provided to illustrate the various methods of the fuzzy linear programming problem and compared with the solution of the problem obtained after defuzzyfing in the beginning using ranking functions.

**Keywords:** Fuzzy sets, α - cut, Fuzzy numbers, trapezoidal fuzzy number, triangular fuzzy number , Ranking function, fuzzy linear programming problem.

## INTRODUCTION

Fuzzy set theory has been applied to many fields of Operations Research. The concept of fuzzy linear programming (FLP) was first formulated by Zimmermann[12]. Fuzzy Linear Programming Problems have an essential role in fuzzy modeling which can formulate uncertainty in actual environment. Afterwards, many authors ([2],[3],[4],[7],[ 8],[9],[10],[11]) considered various types of the Fuzzy Linear Programming Problems and proposed several approaches for solving these problems.

Ranking fuzzy numbers is one of the fundamental problems of fuzzy arithmetic and fuzzy decision making. Fuzzy numbers must be ranked before an action is taken by a decision maker. Real numbers can be linearly ordered by the relation ≤ or ≥ , however this type of inequality does not exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than other. An efficient method for ordering the fuzzy numbers is by the use of a ranking function, which maps each fuzzy number into the real line, where a natural order exists. The concept of ranking function for comparing normal fuzzy numbers is compared in [6].

In this paper we present methods to solve (i) the fuzzy LPP in which the coefficients of objective function are trapezoidal fuzzy numbers, the coefficients of the constraints, right hand side of the constraints are triangular fuzzy numbers, and (ii) the fuzzy LPP in which the variables are trapezoidal fuzzy variables, the coefficients of objective function and right hand side of the constraints are trapezoidal fuzzy numbers, (iii) the fuzzy LPP in which the coefficients of objective function, the coefficients of the constraints, right hand side of the constraints are triangular fuzzy numbers using α –cut and ranking functions. Numerical examples are provided to illustrate the various methods of the fuzzy linear programming problem and compared with the solution of the problem obtained after defuzzyfing in the beginning using ranking functions.

Received: March 05, 2012; Revised: April 18, 2012; Accepted: May 24, 2012.

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**Preliminaries**[1][8][9][10]

Definition : Let X be a classical set of objects called the universe whose generic elements are denoted by x. The membership in a crisp subset of X is often viewed as characteristic function $\mu_A(x)$ from X to {0, 1} such that
$\mu_A(x) = 1$ , if x Є A
= 0 , otherwise.where {0, 1} is called valuation set.

If the valuation set is allowed to be the real interval [0, 1], A is called a fuzzy set. $\mu_A(x)$ is the degree of membership of x in A. The closer the value of $\mu_A(x)$ is to 1, the more x belong to A. Therefore, A is completely characterized by the set of ordered pairs:
A = { ( x, $\mu_A(x)$) / x Є X }

Definition :The support of a fuzzy set A is the crisp subset of X and is presented as :
Supp A = { x Є X / $\mu_A(x) > 0$ }

Definition :The α level (α – cut ) set of a fuzzy set A is a crisp subset of X and is denoted by Aα = { (x Є X / $\mu_A(x) > \alpha$) }

Definition : A fuzzy set A in X is convex if $\mu_A(\lambda x + (1-\lambda)y) \geq \min\{\mu_A(x), \mu_A(y)\}$ x,y Є X and λ Є [0, 1]. Alternatively, a fuzzy set is convex if all α level sets are convex .
Note that in this paper we suppose that X = R.

Definition : A fuzzy number Ã is a convex normalized fuzzy set on

the real line R such that
1) it exists atleast one $x_0 \in R$ with $\mu_A(x_0) = 1$.
2) $\mu_A(x)$ is piecewise continuous.

Among the various types of fuzzy numbers, triangular and trapezoidal fuzzy numbers are of the most important.

Definition : We can define trapezoidal fuzzy number $\tilde{A}$ as $\tilde{A} = (a_1, a_2, a_3, a_4)$ the membership function of this fuzzy number will be interpreted as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4, \\ 0, & x > a_4 \end{cases}$$

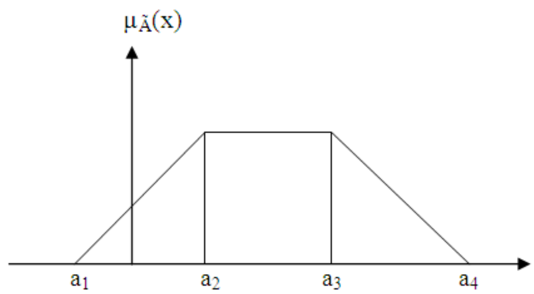
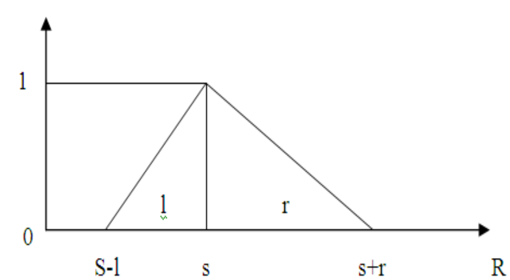



Fig. 1. Trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$

α – cut interval for this shape is written below
For all $\alpha \in [0, 1]$ $\tilde{A}_\alpha = [(a_2 - a_1)\alpha + a_1, \ -(a_4 - a_3)\alpha + a_4]$
Let F( R) denote the set of all trapezoidal fuzzy numbers.

Definition :Triangular fuzzy number $\tilde{A}$ can be represented by three real numbers, s, l, r whose meanings are defined as




Using this representation we write $\tilde{A} = (s, l, r)$
Let $F_1(R)$ be the set of all triangular fuzzy numbers.

**Operations of Trapezoidal Fuzzy Number**

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers.
Addition : $\tilde{A} (+) \tilde{B} = (a_1, a_2, a_3, a_4) (+) (b_1, b_2, b_3, b_4)$
$= (a_1 + b_1, \ a_2 + b_2, \ a_3 + b_3, \ a_4 + b_4)$
Subtraction: $\tilde{A} (-) \tilde{B} = (a_1, a_2, a_3, a_4) (-) (b_1, b_2, b_3, b_4)$
$= (a_1 - b_4, \ a_2 - b_3, \ a_3 - b_2, \ a_4 - b_1)$
Scalar Multiplication : $x > 0$, $x\tilde{A} = (xa_1, xa_2, xa_3, xa_4)$
$x < 0$, $x\tilde{A} = (xa_4, xa_3, xa_2, xa_1)$

Multiplication:
Multiplication of two trapezoidal fuzzy numbers need not be a trapezoidal fuzzy number. But in many cases, the operation results from multiplication is approximated trapezoidal shape. Multiplication operations should be done by using membership functions. For the approximation of operation results, we use α – cut interval.

**Operations of Triangular Fuzzy Number**

Let $\tilde{A} = (s_1, l_1, r_1)$ and $\tilde{B} = (s_2, l_2, r_2)$ be two triangular fuzzy numbers
Addition : $\tilde{A} (+) \tilde{B} = (s_1, l_1, r_1) (+) (s_2, l_2, r_2)$
$= (s_1 + s_2, l_1 + l_2, r_1 + r_2)$
Subtraction : $\tilde{A} (-) \tilde{B} = (s_1, l_1, r_1) (-) (s_2, l_2, r_2)$
$= (s_1 - s_2, l_1 - l_2, r_1 - r_2)$
Scalar Multiplication:
$x > 0$, $\tilde{A}x = (s_1x, l_1x, r_1x)$
$x < 0$, $\tilde{A}x = (r_1x, l_1x, s_1x)$
The partial order ≤ is defined by A ≤ B iff MAX (A , B) = B.

For any two triangular fuzzy numbers $\tilde{A} = (s_1, l_1, r_1)$ and $\tilde{B} = (s_2, l_2, r_2)$, $\tilde{A} \leq \tilde{B}$ iff $s_1 \leq s_2$, $s_1 - l_1 \leq s_2 - l_2$, $s_1 + r_1 \leq s_2 + r_2$.

**Ranking functions**

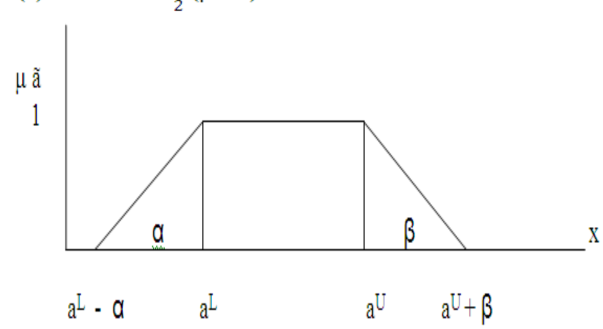
A convenient method for comparing of the fuzzy numbers is by use of ranking functions. A ranking function is a map from F(R) into the real line. Now , we define orders on F(R) as following:

$\tilde{a} \geq \tilde{b}$ if and only if $R(\tilde{a}) \underset{R}{\geq} R(\tilde{b})$
$\tilde{a} > \tilde{b}$ if and only if $R(\tilde{a}) \underset{R}{>} R(\tilde{b})$
$\tilde{a} = \tilde{b}$ if and only if $R(\tilde{a}) \underset{R}{=} R(\tilde{b})$ where $\tilde{a}$ and $\tilde{b}$ are in F(R).
It is obvious that we may write $\tilde{a} \underset{R}{\leq} \tilde{b}$ if and only if $\tilde{b} \underset{R}{\geq} \tilde{a}$

Since there are many ranking function for comparing fuzzy numbers we only apply linear ranking functions. So, it is obvious that if we suppose that R be any linear ranking function, then

i) $\tilde{a} \underset{R}{\geq} \tilde{b}$ if and only if $\tilde{a} - \tilde{b} \underset{R}{\geq} 0$ if and only if $-\tilde{b} \underset{R}{\geq} -0$
ii) $\tilde{a} \underset{R}{\geq} \tilde{b}$ and $\tilde{c} \underset{R}{\geq} \tilde{d}$, then $\tilde{a} + \tilde{c} \underset{R}{\geq} \tilde{b} + \tilde{d}$

One suggestion for a linear ranking function is following:
$$R(\tilde{a}) = a^L + a^U + \frac{1}{2}(\beta - \alpha)$$



where $\tilde{a} = (a^L, a^U, \alpha, \beta)$ For any arbitrary fuzzy number $\tilde{A} = (\underline{A}(r), \overline{A}(r))$, we use ranking function as follows :

$$R(\tilde{A}) = \frac{1}{2}\int_{[0,1]}(\underline{A}(r)) + \int_{[0,1]}(\overline{A}(r))$$

For triangular fuzzy number this reduces to

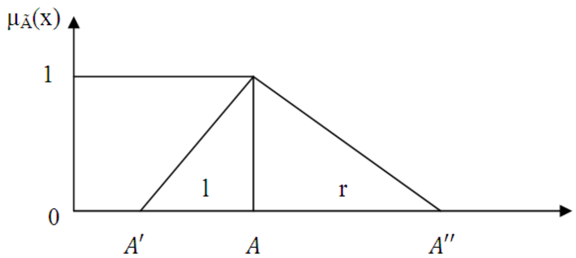
$$R(\tilde{A}) = A + \frac{1}{4}(A'' - A')$$

Where

$\tilde{A} = (A, A', A'')$.

Then for triangular fuzzy number $\tilde{A}$ and $\tilde{B}$, we have $\tilde{A} \geq \tilde{B}$ if and only if

$$A + \frac{1}{4}(A'' - A') \geq B + \frac{1}{4}(B'' - B')$$




**Fuzzy Linear Programming Problem**

(i) Consider the Fuzzy linear programming problem in which the coefficients of objective function are trapezoidal fuzzy numbers, the coefficients of the constraints, right hand side of the constraints are triangular fuzzy number

$$\text{Max } \tilde{z} \underset{R}{=} \sum_{j=1}^{n} \tilde{c}_j \, x_j$$
$$\text{Sub to } \sum_{j=1}^{n} A_{ij}\, x_j \leq B_i \quad (i \in N_m)$$
$$x_j \geq 0 \quad (j \in N_n)$$

According to [5] the problem can be rewritten as

$$\text{Max } \tilde{z} \underset{R}{=} \sum_{j=1}^{n} \tilde{c}_j \; x_j$$
$$\text{Sub to } \sum_{j=1}^{n}(s_{ij}, l_{ij}, r_{ij})\, x_{ij} \leq (t_i, u_i, v_i)(i \in N_m)$$
$$x_j \geq 0 \; (j \in N_n)$$

where $A_{ij} = (s_{ij}, l_{ij}, r_{ij})$ and $B_i = (t_i, u_i, v_i)$ are triangular fuzzy numbers.

$\tilde{c}_j \in F(R)$ Using operations on fuzzy numbers, the problem can be rewritten as

$$\text{Max } \tilde{z} \underset{R}{=} \sum_{j=1}^{n} \tilde{c}_j \; x_j$$
$$\text{Sub to } \sum_{j=1}^{n} s_{ij}\, x_j \leq t_i$$
$$\sum_{j=1}^{n}(s_{ij} - l_{ij})\, x_j \leq t_i - u_i$$
$$\sum_{j=1}^{n}(s_{ij} + r_{ij})\, x_j \leq t_i + v_i \quad (i \in N_m)$$
$$x_j \geq 0 \quad (j \in N_n)$$

Now the Fuzzy linear programming problem becomes

$$\text{Max } \tilde{z} \underset{R}{=} \tilde{c}\, x$$
$$\text{Sub to } A\, x \leq b$$
$$x \geq 0$$

$\tilde{c}^T \in (F(R))^n$, $b \in R^m$, $x \in R^n$, R is a linear ranking function. Then the problem can be solved by using simplex method. While solving the FLPP we use ranking function.

(ii) Consider the Fuzzy linear programming problem in which the variables are trapezoidal fuzzy variables, the coefficients of objective function and right hand side of the constraints are trapezoidal fuzzy numbers,

$$\text{Max } \tilde{z} \underset{R}{=} \tilde{c}\, \tilde{x}$$
$$\text{Sub to } A\, \tilde{x} \underset{R}{\leq} \tilde{b}$$
$$\tilde{x} \underset{R}{\geq} \tilde{0}$$

where $\tilde{b} \in (F(R))^m$, $\tilde{x} \in (F(R))^n$, $A \in R^{m\times n}$, $\tilde{c} \in (F(R))^n$

Then the problem can be solved by using simplex method. While solving the FLPP we use α- cut interval and ranking function.

(iii) Consider the Fuzzy linear programming problem in which the coefficients of objective function, the coefficients of the constraints, right hand side of the constraints are triangular fuzzy numbers.

$$\text{Max } \tilde{z} \underset{R}{=} \sum_{j=1}^{n} \tilde{c}_j \, x_j$$
$$\text{Sub to } \sum_{j=1}^{n} A_{ij}\, x_j \leq B_i \quad (i \in N_m)$$
$$x_j \geq 0 \quad (j \in N_n)$$

According to [5] the problem can be rewritten as

$$\text{Max } \tilde{z} \underset{R}{=} \sum_{j=1}^{n} \tilde{c}_j \; x_j$$
$$\text{Sub to } \sum_{j=1}^{n}(s_{ij}, l_{ij}, r_{ij})\, x_{ij} \leq (t_i, u_i, v_i)(i \in N_m)$$
$$x_j \geq 0 \; (j \in N_n)$$

where $A_{ij} = (s_{ij}, l_{ij}, r_{ij})$ and $B_i = (t_i, u_i, v_i)$ are triangular fuzzy numbers.

$\tilde{c}_j \in F_1(R)$ Using operations on fuzzy numbers, the problem can be rewritten as

$$\text{Max } \tilde{z} \underset{R}{=} \sum_{j=1}^{n} \tilde{c}_j \; x_j$$
$$\text{Sub to } \sum_{j=1}^{n} s_{ij}\, x_j \leq t_i$$
$$\sum_{j=1}^{n}(s_{ij} - l_{ij})\, x_j \leq t_i - u_i$$
$$\sum_{j=1}^{n}(s_{ij} + r_{ij})\, x_j \leq t_i + v_i \quad (i \in N_m)$$
$$x_j \geq 0 \quad (j \in N_n)$$

Now the Fuzzy linear programming problem becomes

$$\text{Max } \tilde{z} \underset{R}{=} \tilde{c}\, x$$
$$\text{Sub to } A\, x \leq b$$
$$x \geq 0$$

$\tilde{c}^T \in (F_1(R))^n$, $b \in R^m$, $x \in R^n$, R is a linear ranking function. Then the problem can be solved by using simplex method. While solving the FLPP we use ranking function.

**Numerical examples**

**Example 1**

For an illustration of the above method (i) consider a FLP problem

Max $\tilde{z}$ = ( 3, 5, 8, 13 )$x_1$ + ( 4, 6, 10, 16) $x_2$
Sub to ( 4, 2, 1) $x_1$ + ( 5, 3, 1) $x_2$ $\leq$ ( 24, 5, 8)
( 4, 1, 2) $x_1$ + ( 1, 0.5,1) $x_2$ $\leq$ ( 12, 6, 3)
$x_1$ , $x_2 \geq 0$

we can rewrite it as Max $\tilde{z}$ = ( 3, 5, 8, 13 )$x_1$ + ( 4, 6, 10, 16) $x_2$

Sub to
$4x_1 + 5x_2 \leq 24$
$4x_1 + x_2 \leq 12$
$2x_1 + 2x_2 \leq 19$
$3x_1 + 0.5x_2 \leq 6$
$5x_1 + 6x_2 \leq 32$
$6x_1 + 2x_2 \leq 15$
$x_1, x_2 \geq 0$

Adding the slack variables, constraints of the problem can be rewritten as :

$4x_1 + 5x_2 + x_3 = 24$
$4x_1 + x_2 + x_4 = 12$
$2x_1 + 2x_2 + x_5 = 19$
$3x_1 + 0.5x_2 + x_6 = 6$
$5x_1 + 6x_2 + x_7 = 32$
$6x_1 + 2x_2 + x_8 = 15$
$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0$

Now we rewrite the above problem as the FLPP simplex tableau (Table 1)

Table 1. The first simplex tableau of the FLPP

| Basis | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | RHS |
|---|---|---|---|---|---|---|---|---|---|
| $\tilde{z}$ | (-13,-8,-5,-3) | (-16,-10,-6,-4) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $x_3$ | 4 | 5 | 1 | 0 | 0 | 0 | 0 | 0 | 24 |
| $x_4$ | 4 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 12 |
| $x_5$ | 2 | 2 | 0 | 0 | 1 | 0 | 0 | 0 | 19 |
| $x_6$ | 3 | ½ | 0 | 0 | 0 | 1 | 0 | 0 | 6 |
| $x_7$ | 5 | 6 | 0 | 0 | 0 | 0 | 1 | 0 | 32 |
| $x_8$ | 6 | 2 | 0 | 0 | 0 | 0 | 0 | 1 | 15 |

Since min{ R (-13,-8,-5,-3) , R (-16,-10,-6,-4) } = -18, $x_2$ enters the basis and since minimum ratio {$x_B$/ $x_i$}= 4.8, $x_3$ leaves the basis. Then pivoting on $y_{12}$ = 5, we obtain the next tableau given as Table 2.

Table 2. The second simplex tableau of the FLPP

| Basis | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | RHS |
|---|---|---|---|---|---|---|---|---|---|
| $\tilde{z}$ | (-49/5,-16/5,3,49/5) | (-12,-4,4,12) | (4/5,6/5,2,16/5) | 0 | 0 | 0 | 0 | 0 | (96/5,144/5,48,384/5) |
| $x_2$ | 4/5 | 1 | 1/5 | 0 | 0 | 0 | 0 | 0 | 24/5 |
| $x_4$ | 16/5 | 0 | -1/5 | 1 | 0 | 0 | 0 | 0 | 36/5 |
| $x_5$ | 2/5 | 0 | -2/5 | 0 | 1 | 0 | 0 | 0 | 47/5 |
| $x_6$ | 13/5 | 0 | -1/10 | 0 | 0 | 1 | 0 | 0 | 18/5 |
| $x_7$ | 1/5 | 0 | -6/5 | 0 | 0 | 0 | 1 | 0 | 16/5 |
| $x_8$ | 22/5 | 0 | -2/5 | 0 | 0 | 0 | 0 | 1 | 27/5 |

Since R (-49/5,-16/5,3,49/5)= -0.1 is minimum, $x_1$ enters the basis and since minimum ratio {$x_B$/ $x_i$}= 1.23, $x_8$ leaves the basis. Then pivoting on $y_{61}$ = 22/5, we obtain the next tableau given as Table 3.

Table 3. The optimal simplex tableau of the FLPP

| Basis | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | RHS |
|---|---|---|---|---|---|---|---|---|---|
| $\tilde{z}$ | (-10,-3,3,10) | (-12,-4,4,12) | (-1/11,10/11,25/11,45/11) | 0 | 0 | 0 | 0 | (-49/22,-15/22,8/11,48/22) | (417/22,639/22,48,1695/22) |
| $x_2$ | 0 | 1 | 3/11 | 0 | 0 | 0 | 0 | -2/11 | 42/11 |
| $x_4$ | 0 | 0 | 1/11 | 1 | 0 | 0 | 0 | -8/11 | 36/11 |
| $x_5$ | 0 | 0 | -4/11 | 0 | 1 | 0 | 0 | -1/11 | 98/11 |
| $x_6$ | 0 | 0 | 3/22 | 0 | 0 | 1 | 0 | -13/22 | 9/22 |
| $x_7$ | 0 | 0 | -13/11 | 0 | 0 | 0 | 1 | -1/22 | 65/22 |
| $x_1$ | 1 | 0 | -1/11 | 0 | 0 | 0 | 0 | 5/22 | 27/22 |

Since R(-10,-3,3,10), R(-12,-4,4,12)R (-1/11, 10/11, 25/11, 45/11) , R (-49/22,-15/22,8/11,48/22) are non-negative, the basis is optimal and the optimal solution is
$x_1$ = 1.23, $x_2$ = 3.82 , max $\tilde{z}$ = (417/22,639/22,48,1695/22) ,
R($\tilde{z}$) = 86.515. After defuzzfing using ranking functions in the beginning and then solving we get maxz = 86.52.

**Example 2**

For an illustration of the above method (ii) consider a FLPP

Max $\tilde{z}$ = ( 3, 5, 8, 13 ) $\tilde{x}_1$ + ( 4, 6, 10, 16) $\tilde{x}_2$
R

Sub to $3\tilde{x}_1 + \tilde{x}_2 \leq (1, 2, 4, 7)$
R

$2\tilde{x}_1 + \tilde{x}_2 \leq ( 0, 3, \frac{9}{2}, 5)$
R

$\tilde{x}_1$ , $\tilde{x}_2 \geq \tilde{0}$
R

Adding the slack variables, we rewrite the constraints of the problem as

$3\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3 = (1, 2, 4, 7)$
R
$2\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_4 = ( 0, 3, 9/2 , 5)$
R
$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq \tilde{0}$
R

Now we rewrite the above problem as the FLPP simplex tableau (Table 4)

Table 4. The first simplex tableau of the FLPP

| Basis | $\tilde{x}_1$ | $\tilde{x}_2$ | $\tilde{x}_3$ | $\tilde{x}_4$ | RHS |
|---|---|---|---|---|---|
| $\tilde{z}$ | (-13,-8,-5,-3) | (-16,-10,-6,-4) | 0 | 0 | (0,0,0,0) |
| $\tilde{x}_3$ | 3 | 1 | 1 | 0 | (1,2,4,7) |
| $\tilde{x}_4$ | 2 | 1 | 0 | 1 | (0,3,9/2,5) |

Since min{ R (-13,-8,-5,-3) , R (-16,-10,-6,-4) } = -18, $\tilde{x}_2$ enters the basis and since min { R (1,2,4,7) , R ( 0, 3, 9/2 , 5) }= 6.25, $\tilde{x}_4$ leaves the basis. Then pivoting on $y_{22}$ = 1, we obtain the next tableau given as Table 5.

Table 5. The optimal simplex tableau of the FLPP

| Basis | $\tilde{x}_1$ | $\tilde{x}_2$ | $\tilde{x}_3$ | $\tilde{x}_4$ | RHS |
|---|---|---|---|---|---|
| $\tilde{z}$ | (-5,4,15,29) | (-12,-4,4,12) | 0 | (4,6,10,16) | (0,18,45,80) |
| $\tilde{x}_3$ | 1 | 0 | 1 | -1 | (-4,-5/2,1,7) |
| $\tilde{x}_2$ | 2 | 1 | 0 | 1 | (0,3,9/2,5) |

Here for the multiplication of (4,6,10,16) and (0, 3,9/2, 5) we use the α cut interval $(6\alpha^2+12\alpha, 3\alpha^2-8\alpha-30\alpha+80)$. Since R(-5,4,15,29) , R(-12,-4,4,12) , R(4,6,10,16) are non-negative, the basis is optimal and the optimal solution is $\tilde{x}_1=0$, $\tilde{x}_2=(0,3,9/2,5)$
Max $\tilde{z}$ = (0,18,45,80).

**Example 3**

For an illustration of the above method (iii) consider a FLPP

$$\begin{aligned} &\text{Max } \tilde{z} = (8, 5, 2)x_1 + (10, 6, 2)x_2 \\ &\text{Sub to } (4, 2, 1)x_1 + (5, 3, 1)x_2 \le (24, 5, 8) \\ &(4, 1, 2)x_1 + (1, 0.5, 1)x_2 \le (12, 6, 3) \\ &x_1, \quad x_2 \ge 0 \end{aligned}$$

we can rewrite it as

$$\begin{aligned} &\text{Max } \tilde{z} = (8, 5, 2)x_1 + (10, 6, 2)x_2 \\ &\text{Sub to } \quad 4x_1 + 5x_2 \le 24 \\ &4x_1 + x_2 \le 12 \\ &2x_1 + 2x_2 \le 19 \\ &3x_1 + 0.5x_2 \le 6 \\ &5x_1 + 6x_2 \le 32 \\ &6x_1 + 2x_2 \le 15 \\ &x_1, x_2 \ge 0 \end{aligned}$$

Adding the slack variables, constraints of the problem can be rewritten as :

$$\begin{aligned} &4x_1 + 5x_2 + x_3 = 24 \\ &4x_1 + x_2 + x_4 = 12 \\ &2x_1 + 2x_2 + x_5 = 19 \\ &3x_1 + 0.5x_2 + x_6 = 6 \\ &5x_1 + 6x_2 + x_7 = 32 \\ &6x_1 + 2x_2 + x_8 = 15 \\ &x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \ge 0 \end{aligned}$$

Solving this problem as in the example 1 , we obtain the optimal solution $x_1 = 0$, $x_2 = 4.8$ , max $\tilde{z}$ = ( 48, 144/5, 48/5), R($\tilde{z}$) =57.6. After defuzzfing using ranking functions in the beginning and then solving we get maxz = 57.78.

**CONCLUSION**

In this paper we proposed different methods for solving the FLP problems such as the fuzzy LPP in which the coefficients of objective function are trapezoidal fuzzy numbers, the coefficients of the constraints, right hand side of the constraints are triangular fuzzy numbers and the fuzzy LPP in which the variables are trapezoidal fuzzy variables, the coefficients of objective function and right hand side of the constraints are trapezoidal fuzzy numbers and the fuzzy LPP in which the coefficients of objective function, the coefficients of the constraints, right hand side of the constraints are triangular fuzzy numbers. The significance of this paper is to use α cut interval and ranking function in solving various fuzzy linear programming problems and compared with the solution of the problem obtained after defuzzyfing in the beginning using ranking functions.

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