Recent Research in Science and Technology 2011, 3(10): 55-58
ISSN: 2076-5061
www.scholarjournals.org

www.recent-science.com

RRST-Mathematics

Stochastic Models on Time to Recruitment in a Two Grade Manpower System using Different Policies of Recruitment

A. Srinivasan* and S. Mohanalakshmi

P.G. and Research Department of Mathematics, Bishop Heber College, Trichy-620017, Tamilnadu, India

Article Info

Article History

Received : 27-06-2011
Revised : 25-08-2011
Accepted : 01-09-2011

***Corresponding Author**

Tel : +91-4312351696

Email: mathsrinivas@yahoo.com

Abstract

In this paper a two grade organization in which depletion of manpower occurs due to its policy decisions is considered. Two mathematical models are constructed employing two different univariate recruitment policies, based on shock model approach. The mean and variance of the time to recruitment are obtained for both the models under different conditions. The analytical results are numerically illustrated and relevant conclusions are presented.

Key Words: Two grade system, Shock models, Univariate policies of recruitment, Mean and variance of the time to recruitment.
AMS MSC 2010: 91D35, 91B40, 90B70

Introduction

Consider an organization having two grades in which depletion of manpower occurs at every decision epoch. In the univariate policy of recruitment based on shock model approach recruitment is made as and when the cumulative loss of manpower crosses a threshold. Employing this recruitment policy, expected time to recruitment is obtained under different conditions for several models in [1], [3], [4], [6], [8], [9], [10] and [11]. Recently in [2], Dhivya and Srinivasan have constructed two mathematical models and obtained the performance measures on the time to recruitment under different conditions. More specifically, in model 1 they have obtained these analytical results when the loss of manhours in the organization is the maximum of the loss of manhours for the two grades and the threshold for the organization is either the maximum or minimum of the thresholds for the loss of manhours in the two grades, assuming different distribution for these two thresholds for the grades. This work is extended in the present paper when the threshold for the loss of manhours in the organization is the sum of the corresponding thresholds of the two grades according as the two thresholds are exponential or extended exponential thresholds. This is done in section 2.1. In section 2.2, the performance measures are obtained when the threshold for the organization is minimum of the thresholds for the loss of manhours in the two grades where the loss of manhours in the two grades are put together to get the loss of manhours for the organization. The analytical results are numerically illustrated and the influence of nodal parameters on these results is studied and relevant conclusions are given.

Model description and analysis for model –I

Consider an organization having two grades A and B, taking decisions at random epochs in $[0, \infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of man power if the person quits. This loss of manhours linear and Cumulative and $X_i=\max(X_{iA}, X_{iB})$. Let X_i, i=1,2,3…… be the loss of man hours due to the i^{th} decision epoch in the organization and G(.) [g(.)] be its distribution function[density function].Let $g_k(.)$ $[G_k(.)]$ be the k-fold convolution of g(.) [G(.)]. Let $\bar{g}(.)$ be the Laplace transform of g(.). Let S_k be the total loss of man power in first k decisions. It is assumed that the inter decision times are independent and identically distributed exponential random variables and f(.) [F(.)] be its density function [distribution function] with parameter θ. Let $f_k(.)$ $[F_k(.)]$ be the k-fold convolution of f(.) [F(.)]. Let $V_k(t)$ be the probability that there are exactly 'k' decisions in [0,t]. Since the number of decision taken form a renewal process, from renewal theory it is known that $V_k(t) = F_k(t) - F_{k+1}(t)$ where $F_0(t)=1$. The loss of man power process and process of inter decision times are assumed to be statistically independent. Let $Y= Y_A+Y_B$ be the threshold level for the organization. For all i=1,2,3…, it is assumed that X_i and Y are independent. **Recruitment is done whenever the cumulative loss of man hours in the organization crosses Y**. Let W be the time for recruitment in the organization and l (.) [L(.)] be its density function[distribution function]. Let E(W) and V(W) be the mean and variance of the time for recruitment.

Main result

The survival function of W is given by,

$P(W > t) = \sum_{k=0}^{\infty}$ {Probability that there are exactly k decisions in [0,t) and cumulative loss of manhours does not crosses the threshold level Y in these 'k' decisions}

By the law of total probability

$P(W > t) = \sum_{k=0}^{\infty} V_k(t) P(S_k<Y=Y_A+Y_B)$

$= \sum_{k=0}^{\infty} V_k(t) \int_0^{\infty} P(Y_A + Y_B > x) g_k(x)dx$

$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \int_0^{\infty} P(Y_A + Y_B > x) g_k(x)dx$ ....(1)

Since X_{iA} and X_{iB} follow exponential distribution with parameters $\lambda_A \& \lambda_B$ respectively we find that

$g(x)= \lambda_A e^{-\lambda_A x} + \lambda_B e^{-\lambda_B x} - (\lambda_A + \lambda_B)e^{-(\lambda_A+\lambda_B)x}$ ..(2)

Case(i) :

Suppose Y_A and Y_B follow exponential distribution with parameters μ_A and μ_B respectively. In this case it is found that

$\int_0^{\infty} P(Y_A + Y_B > x) g_k(x)dx = \frac{\mu_A}{\mu_A-\mu_B}[A_1]^k - \frac{\mu_B}{\mu_A-\mu_B}[A_2]^k$ ……(3)

where

$A_1 = \frac{\lambda_A}{\mu_B+\lambda_A} + \frac{\lambda_B}{\mu_B+\lambda_A} - \frac{\lambda_A}{\mu_B+\lambda_A+\lambda_B}$

$A_2 = \frac{\lambda_A}{\mu_A+\lambda_A} + \frac{\lambda_B}{\mu_A+\lambda_A} - \frac{\lambda_A}{\mu_A+\lambda_A+\lambda_B}$ …….(4)

From (1) ,(3) and on simplification we get

$L(t) = 1- P(W > t) = \frac{\mu_A}{\mu_A-\mu_B} D_1 + \frac{\mu_B}{\mu_A-\mu_B} D_2$ …..(5)

and

$\bar{l}(s) = \frac{\mu_A}{\mu_A-\mu_B}\overline{a_1} + \frac{\mu_B}{\mu_A-\mu_B}\overline{a_2}$ …….(6)

where

for j=1,2 , $D_j = D_j(t) = (1-A_j) \sum_{k=0}^{\infty} F_k(t)[A_j]^{k-1}$

and

$\bar{a}_j = \bar{a}_j(s) = (1-A_j) \sum_{k=0}^{\infty} \{\bar{f}(s)\}^k [A_j]^{k-1}$ ……(7)

Since $\bar{f}(s) = \frac{\theta}{\theta+s}$, E(W) = $-\left[\frac{d}{ds}\bar{l}(s)\right]_{s=0}$ and

$E(W^2) = -\left[\frac{d^2}{ds^2}\bar{l}(s)\right]_{s=0}$ …….(8)

from (5),(6),(8) and on simplification we get

$E(W) = \frac{1}{\theta}\left\{\frac{\mu_A}{\mu_A-\mu_B}C_1 - \frac{\mu_B}{\mu_A-\mu_B}C_2\right\}$ & ……(9)

$E(W^2) = \frac{2}{\theta^2}\left\{\frac{\mu_A}{\mu_A-\mu_B}C_1^2 - \frac{\mu_B}{\mu_A-\mu_B}C_2^2\right\}$ …..(10)

where $C_j = \frac{1}{[1-A_j]}$, j=1,2, and A_j's are given by(4).

When Y_A and Y_B are exponential random variables (9) gives the mean time to recruitment. From (9) and (10) the variance of the time to recruitment can be computed for this case.

Case(ii) :

Suppose Y_A and Y_B follow extended exponential distribution with scale parameters μ_A and μ_B respectively and shape parameter 2. In this case it is found that

$\int_0^{\infty} P(Y_A + Y_B > x) g_k(x)dx = B_1 [I_1]^k - B_2 [I_2]^k + B_3 [I_3]^k - B_4 [I_4]^k$ ……(11)

where

$I_1 = \frac{\lambda_A}{\mu_B+\lambda_A} + \frac{\lambda_B}{\mu_B+\lambda_A} - \frac{\lambda_A}{\mu_B+\lambda_A+\lambda_B}$

$I_2 = \frac{\lambda_A}{2\mu_A+\lambda_A} + \frac{\lambda_B}{2\mu_A+\lambda_A} - \frac{\lambda_A}{2\mu_A+\lambda_A+\lambda_B}$

$I_3 = \frac{\lambda_A}{\mu_A+\lambda_A} + \frac{\lambda_B}{\mu_A+\lambda_A} - \frac{\lambda_A}{\mu_A+\lambda_A+\lambda_B}$

$I_4 = \frac{\lambda_A}{2\mu_B+\lambda_A} + \frac{\lambda_B}{2\mu_B+\lambda_A} - \frac{\lambda_A}{2\mu_B+\lambda_A+\lambda_B}$ ……..(12)

$B_1 = \frac{4\mu_A{}^2}{(2\mu_A-\mu_B)(\mu_A-\mu_B)}$

$B_2 = \frac{\mu_B{}^2}{(2\mu_A-\mu_B)(\mu_A-\mu_B)}$ ……..(13)

$B_3 = \frac{4\mu_B{}^2}{(\mu_A-\mu_B)(\mu_A-2\mu_B)}$

$B_1 = \frac{\mu_A{}^2}{(\mu_A-\mu_B)(\mu_A-2\mu_B)}$

From (1) , (11) and on simplification we get

$L(t) = 1- P(W > t) = B_1 D_1 - B_2 D_2 + B_3 D_3 - B_4 D_4$….(14)

and

$\bar{l}(s) = B_2\overline{a_1} - B_2\overline{a_2} + B_3\overline{a_3} - B_4\overline{a_4}$ …….(15)

Where, for m=1,2,3,4,

$D_m = D_m(t) = (1-I_m) \sum_{k=0}^{\infty} F_k(t)[I_m]^{k-1}$

and

$\overline{a_m} = \overline{a_m}(s) = (1-I_m)[I_m]^{k-1}$ ………..(16)

Since $\bar{f}(s) = \frac{\theta}{\theta+s}$, E(W) = $-\left[\frac{d}{ds}\bar{l}(s)\right]_{s=0}$ and

$E(W^2) = -\left[\frac{d^2}{ds^2}\bar{l}(s)\right]_{s=0}$ …….(17)

from (14),(15),(17) and on simplification we get

$E(W) = \frac{1}{\theta}\{B_1E_1 - B_2E_2 + B_3E_3 - B_4E_4\}$ ...(18)

and

$E(W^2) = \frac{2}{\theta^2}\{B_1E_1^2 - B_2E_2^2 + B_3E_3^2 - B_4E_4^2\}$ (19)

where $E_m = \frac{1}{[1-I_m]}$, m=1,2,3,4, I_m's are given by(12) and B_m's are given by(13).

When Y_A and Y_B are extended exponential random variables (18) gives the mean time to recruitment. From (18) and (19) the variance of the time to recruitment can be computed for this case.

Numerical illustration for model 1

The influence of nodal parameters on the performance measures namely mean and variance of the time to recruitment are studied numerically and the results are tabulated in Table-1. As per the Table-1, the researcher made the following observations.

(i) As λ_1 increases, the mean loss of man hours decreases and hence the mean of the time to recruitment increases. The variance of the time to recruitment also increases with λ_1.

(ii) As λ_2 increases, the mean loss of man hours decreases and hence the mean of the time to recruitment increases. The variance of the time to recruitment also increases with λ_2.

(iii) As θ increases, the mean inter-decision time decreases and hence the mean of the time to recruitment decreases. In this case the variance of the time to recruitment also decreases.

Model description and analysis for model –2

All the assumptions and notations are same as in model 1 except the loss of man hours and the threshold for the organization. In this model we take the X_i as the loss of manhours for the organization and Y = min(Y_A , Y_B) as the threshold for the organization.

As in model-1, it can be shown that

By the law of total probability,

$$P(W > t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \int_0^{\infty} [1 - H(x)] g_k(x)dx \quad \text{..................(20)}$$

Since X_i follows exponential distribution with parameter λ, we find that $g(x) = \lambda e^{-\lambda x}$ ....(21)

Suppose Y_A and Y_B follow extended exponential distribution with scale parameters μ_A and μ_B respectively and shape parameter 2. In this case it is found that

$$\int_0^{\infty} [1 - H(x)] g_k(x)dx = [R_1]^k - 2[R_2]^k - 2[R_3]^k + 4[R_4]^k \quad \text{......(22)}$$

where

$$R_1 = \bar{g}(2\mu_A + 2\mu_B) = \frac{\lambda}{2\mu_A + 2\mu_B + \lambda}$$

$$R_2 = \bar{g}(\mu_A + 2\mu_B) = \frac{\lambda}{\mu_A + 2\mu_B + \lambda}$$

$$R_3 = \bar{g}(2\mu_A + \mu_B) = \frac{\lambda}{2\mu_A + \mu_B + \lambda}$$

$$R_4 = \bar{g}(\mu_A + \mu_B) = \frac{\lambda}{\mu_A + \mu_B + \lambda} \quad \text{..................(23)}$$

From (20) , (22) and on simplification we get

$L(t) = 1 - P(W > t) = D_1 - 2D_2 - 2D_3 + 4D_4$ ....(24)

and

$$\bar{l}(s) = \overline{a_1} - 2\overline{a_2} - 2\overline{a_3} + 4\overline{a_4} \quad \text{.............(25)}$$

where for m=1,2,3,4 ,

$D_m = D_m(t) = (1-R_m) \sum_{k=0}^{\infty} F_k(t)[R_m]^{k-1}$

and

$\overline{a_m} = \overline{a_m}(s) = (1-R_m)[R_m]^{k-1}$ ....(26)

Since $\bar{f}(s) = \frac{\theta}{\theta+s}$, $E(W) = -\left[\frac{d}{ds}\bar{l}(s)\right]_{s=0}$ and

$E(W^2) = -\left[\frac{d^2}{ds^2}\bar{l}(s)\right]_{s=0}$ ....(27)

from (24),(25),(27) and on simplification we get

$$E(W) = \frac{1}{\theta}\{T_1 - 2T_2 - 2T_3 + 4T_4\} \quad \text{............(28)}$$

and

$$E(W^2) = \frac{2}{\theta^2}\{T_1^2 - 2T_2^2 - 2T_3^2 + 4T_4^2\} \quad \text{......(29)}$$

where $T_m = \frac{1}{[1-R_m]}$, m=1,2,3,4. R_m's are given by(23).

When Y_A and Y_B are extended exponential random variables (28) gives the mean time to recruitment. From (28) and (29) the variance of the time to recruitment can be computed for this model.

Numerical illustration for model-2

The influence of nodal parameters on the performance measures namely mean and variance of the three to recruitment are studied numerically and the results are tabulated in Table-2. As per the Table-2, the researcher made the following observations.

(i) As λ increases, the mean loss of man hours decreases and hence the mean of the time to recruitment increases. The variance of the time to recruitment also increases with λ.

(ii) As θ increases, the mean inter-decision time decreases and hence the mean of the time to recruitment decreases. In this case the variance of the time to recruitment also decreases.

Table 1: Effect of λ_1, λ_2 and θ on performance measures

(μ_1 = 0.6, μ_2 = 0.4)

λ_1	λ_2	μ_1	μ_2	θ	case-1		case-2	
					E(W)	V(W)	E(W)	V(W)
0.1	0.2	0.6	0.4	0.10	12.2448	146.4712	13.7960	176.1273
0.3	0.2	0.6	0.4	0.10	14.8435	205.2293	17.9318	263.7550
0.5	0.2	0.6	0.4	0.10	16.1519	237.8067	19.8385	309.5899
0.3	0.2	0.6	0.4	0.10	14.8435	205.2293	17.9318	263.7550
0.3	0.4	0.6	0.4	0.10	17.5446	272.8019	22.1191	360.4827
0.3	0.6	0.6	0.4	0.10	19.1117	316.1597	24.4110	420.9452
0.3	0.2	0.6	0.4	0.10	14.8435	205.2293	17.9318	263.7550
0.3	0.2	0.6	0.4	0.15	9.8957	91.2130	11.9545	117.2245
0.3	0.2	0.6	0.4	0.20	7.4217	51.3073	8.9659	65.9388

Table 2: Effect of λ and θ on performance measures

(μ_A= 0.6 , μ_B = 0.4)

λ	θ	μ_A	μ_B	E(W)	V(W)
0.1	0.10	0.6	0.4	11.8214	138.0077
0.3	0.10	0.6	0.4	15.4643	223.4974
0.5	0.10	0.6	0.4	19.1071	321.6199
0.3	0.10	0.6	0.4	15.4643	223.4974
0.3	0.15	0.6	0.4	10.3095	99.3322
0.3	0.20	0.6	0.4	7.7321	55.8744

References

[1] Akilandeswari, M. and Srinivasan, A., 2007, "Mean time to recruitment for a two graded manpower system when thresholds distribution has SCBZ property", Acta Ciencia Indica, XXXIII M(3):1113-1118.

[2] Dhivya, S., Srinivasan, A., and Mariappan, P., 2011, "Stochastic models on time to recruitment in a two grade manpower system using different policies of recruitment", Recent Research in Science and Technology, 3(4): 162-168.

[3] Esther Clara,J.B., 2008, "Stochastic model for the mean and variance of the time to recruitment in a two graded manpower system", M.Phil., Dissertation, Bharathidasan University.

[4] Kasturri, K., 2009,"Mean time for recruitment and cost analysis on some univariate policies of recruitment in manpower model", Ph.D., Thesis, Bharathidasan University.

[5] Mercy Alice, B., 2009, "Some stochastic models on the variance of the time to recruitment for a two graded manpower system associated with a univariate policy of recruitment involving combined thresholds", M.Phil., Dissertation, Bharathidasan University.

[6] Parthasarathy, S., Ravichandran, M.K. & Vinoth, R., 2010, "An application of stochastic models-grading system in manpower planning", International Business Research,3(2): 79-86.

[7] Sathiyamoorthi, R., and Parthasarathy, S., 2002, "On the expected time to recruitment in a two graded marketing organization", Indian Association for Productivity Quality and Reliability,27(1):77-81.

[8] Sendhamizh Selvi, S., 2009, "A Study on expected time to recruitment in manpower models for a multi graded system associated with an univariate policy of recruitment", Ph.D., Thesis, Bharathidasan University.

[9] Sheldon M. Ross, 2004, "Stochastic processes", Second Edition, John Wiley & Sons, INC.

[10] Uma, G., Uma, K.P., and Srinivasan, A., 2008, "Mean and variance of the time to recruitment in a two graded manpower system using a univariate policy of recruitment involving geometric threshold", Acta Ciencia Indica. XXXIV M (4):1643-1648.

[11] Vidhya, S. 2007,"A Stochastic model for mean and variance of the time to recruitment in a two graded manpower system", M.Phil., Dissertation, Bharathidasan University.