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Mean and Variance of the Time to Recruitment in a Two Graded Manpower System with Two Thresholds for the Organization

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Abstract

In this paper, a two graded manpower system which is subject to exit of personnel due to the policy decisions taken in the system is considered. There is an associated loss of manpower if a person quits. As the exit of personnel is unpredictable, a new recruitment policy involving two thresholds – one is optional and the other one mandatory is suggested to enable the organization to plan its decision on recruitment. Based on shock model approach, two mathematical models are constructed using a univariate policy of recruitment. The analytical expression for the mean and variance of the time to recruitment are obtained when i) the loss of manpower processes in grades 1& 2 form separately a sequence of independent and identically distributed exponential random variables ii) the inter-decision times are independent and identically distributed exponential random variables and iii) the optional and mandatory thresholds in both the grades follow exponential, exponentiated exponential distribution and the distribution having SCBZ property respectively. The results are numerically illustrated for both the models and relevant conclusions are made.

Key Words: Manpower planning, Univariate recruitment policy, shock models, exponentiated exponential distribution, Mean and Variance of the time to recruitment
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Introduction

Exit of personnel which is in other words known as wastage is an important aspect in the study of manpower planning. Many models have been discussed using different kinds of wastages and different types of distributions. For a two graded system, Sathiyamoothi and Parthasarathy [8] have found out the expected time to recruitment by considering the distribution of the thresholds as exponential and the threshold level of the organization as the maximum of thresholds of the two grades. Kasturri [7] has extended the above result when the inter-decision times are exchangeable and constantly correlated exponential random variables. In [1] Akilandeswari studied this model when the thresholds distribution has SCBZ property. Kasturri and Sendhamizh Selvi [7], [9] have extended the result of Akilandeswari for correlated inter-decision times when the loss of manpower follows exponential and Poisson distribution respectively. In [8] Sathiyamoorthi and Parthasarathy have obtained the performance measures when the threshold level of the organization is the minimum of the thresholds of two grades. Kasturri [7] studied this model when the inter-decision times are correlated. Vidhya [12] has extended the result when the threshold distribution has SCBZ property. In [7] and [9] Kasturri and Sendhamizh Selvi have extended the model of Vidhya for correlated inter-decision times according as the distribution of loss of manpower is exponential or Poisson. Recently in [2] Srinivasan et al. have obtained the mean and variance of the time to recruitment for a two graded system by considering different combinations of the loss of manhours in the two grades and for different forms of the threshold for the organization . In all the above models, they have taken the threshold for the organization as maximum or minimum of the thresholds of two grades. Since the number of exits in a policy decision making epoch is unpredictable and the time at which the cumulative loss of manpower crossing a single threshold is probabilistic, the organization has left with no choice except making recruitment immediately upon the threshold crossing. To remove this limitation, first for a single graded system, Srinivasan and Esther Clara [3-6] have incorporated the concept of alertness in their new recruitment policy which involves two thresholds – one is optional and the other one mandatory and obtained performance measures on time to recruitment under different conditions. Vasudevan and Srinivasan [10] have also used this new recruitment policy for a two graded manpower system and found out the mean and variance of the time to recruitment when i) the loss of manpower process forms a sequence of independent and identically distributed exponential random variables ii) the inter-decision times are independent and identically distributed exponential random variables and iii) the optional and mandatory thresholds follow exponential distribution. Again in [11], they have extended the above result when the two thresholds follow extended exponential distribution with shape parameter 2 given by $D(x) = (1- e^{-\mu x})^{2}$ which is more general than the conventionally used exponential distribution. The objective of the present paper is to study the result of Vasudevan and Srinivasan [10], [11] when the loss of manpower for the organization is taken as the maximum of loss of manpower of two grades. This paper is organized as follows: In sections 2and 3, model description and analytical expressions for the mean and variance of the time to recruitment for Models-I and II are given. In section 4, the main results are numerically illustrated for both the models by assuming specific distributions and relevant conclusions are made.

Model description and main results for Model-I

Consider a two graded organization taking decisions at random epochs in (0, ∞) and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that X_{1_i} (X_{2_i}) - the loss of manpower for grade-1(grade-2) due to the i[th] decision epoch, i=1, 2 … is linear and cumulative, forming a sequence of independent and identically distributed exponential random variables with parameter λ_1 (λ_2). Let $\bar{X}_i = \max(X_{1_i}, X_{2_i})$ be the loss of manpower for the organization due to the i[th] decision epoch, i=1, 2 . . . Let S_k be the cumulative loss of manpower in the first k decisions. Let g (.) (G (.)) be the probability density function (distribution function) of $\bar{X}_i$. Let $g_k(.)$ ($G_k(.)$) be k-fold convolution of g (.) (G (.)). Let $\bar{g}$ (.) be the Laplace transform of g (.).It is assumed that the inter-decision times are independent and identically distributed exponential random variables with parameter θ having probability density (distribution) function f (.) (F (.)). Let $f_k(.)$ ($F_k(.)$) be k fold convolution of f (.) (F (.)). The loss of manpower process and the process of inter decision times are assumed to be statistically independent. Let Y_1, Y_2 (Z_1, Z_2) be a continuous random variables denoting the optional (mandatory) thresholds for grade-1 (grade-2). Let Y= max (Y_1, Y_2) and Z= max (Z_1, Z_2) be the optional and mandatory thresholds for the organization respectively. Assume that $Y_1<Z_1$ and $Y_2<Z_2$. Also Y, Z and $\bar{X}_i$, i=1, 2 … are assumed to be statistically independent. The univariate recruitment policy employed in this model is as follows:

Recruitment is optional when the cumulative loss of manpower crosses the optional threshold Y. However, recruitment is necessary when the cumulative loss of manpower crosses the mandatory threshold Z. Let p be the probability that the organization is not going for recruitment whenever the cumulative loss of manpower crosses Y. Let W be a continuous random variable denoting the time for recruitment in the organization with probability density function l (.) and cumulative distribution function L (.). From Renewal theory, the probability $V_k(t)$ that there are exactly k-decisions are taken in (0, t] is given by $V_k(t)$ = $F_k(t)$ - $F_{k+1}(t)$ with $F_0(t)$ =1. Let E (W) and V (W) be the mean and variance of the time to recruitment.

Case (i): $\boldsymbol{Y_1, Y_2, Z_1}$ **and** $\boldsymbol{Z_2}$ **follow exponential distribution with parameters** $\boldsymbol{\mu_1, \mu_2, \mu_3}$ **and** $\boldsymbol{\mu_4}$ **respectively**.

The survival function of W is given by

P (W > t) = $\sum_{k=0}^{\infty}$ {Probability that exactly k-decisions are taken in (0, t], $\times$ (probability that the total number of exits in these k-decisions does not cross the optional threshold Y or the total number of exits in these k-decisions crosses the optional level Y but lies below the mandatory level Z and the organization is not going for recruitment)}

i.e., P (W > t) = $\sum_{k-0}^{\infty} V_k(t)\ P(S_k < Y)$ +

$$\sum_{k=0}^{\infty} V_k(t) \times p \times P(S_k \geq Y) \times P(S_k < Z) \quad \textbf{(1)}$$

By using the law of total probability and on simplification it can be shown that

$$\left.\begin{array}{l} P(S_k < Y) = A_1^{\,k} + A_2^{\,k} - A_3^{\,k} \\ \text{and} \\ P(S_k < Z) = A_4^{\,k} + A_5^{\,k} - A_6^{\,k} \end{array}\right\} \quad \textbf{(2)}$$

where

$$A_1 = \bar{g}(\mu_1) = \frac{\lambda_1}{\lambda_1+\mu_1} + \frac{\lambda_2}{\lambda_2+\mu_1} - \frac{(\lambda_1+\lambda_2)}{\lambda_1+\lambda_2+\mu_1}$$

$$A_2 = \bar{g}(\mu_2) = \frac{\lambda_1}{\lambda_1+\mu_2} + \frac{\lambda_2}{\lambda_2+\mu_2} - \frac{(\lambda_1+\lambda_2)}{\lambda_1+\lambda_2+\mu_2}$$

$$A_3 = \bar{g}(\mu_1+\mu_2) = \frac{\lambda_1}{\lambda_1+\mu_1+\mu_2} + \frac{\lambda_2}{\lambda_2+\mu_1+\mu_2} - \frac{(\lambda_1+\lambda_2)}{\lambda_1+\lambda_2+\mu_1+\mu_2} \qquad (3)$$

$$A_4 = \bar{g}(\mu_3) = \frac{\lambda_1}{\lambda_1+\mu_3} + \frac{\lambda_2}{\lambda_2+\mu_3} - \frac{(\lambda_1+\lambda_2)}{\lambda_1+\lambda_2+\mu_3}$$

$$A_5 = \bar{g}(\mu_4) = \frac{\lambda_1}{\lambda_1+\mu_4} + \frac{\lambda_2}{\lambda_2+\mu_4} - \frac{(\lambda_1+\lambda_2)}{\lambda_1+\lambda_2+\mu_4}$$

$$A_6 = \bar{g}(\mu_3+\mu_4) = \frac{\lambda_1}{\lambda_1+\mu_3+\mu_4} + \frac{\lambda_2}{\lambda_2+\mu_3+\mu_4} - \frac{(\lambda_1+\lambda_2)}{\lambda_1+\lambda_2+\mu_3+\mu_4}$$

Using (2) in (1) and on simplification

$$\begin{aligned} P(W>t) = 1 &- E_1(t) - E_2(t) + E_3(t) - pE_4(t) \\ &- pE_5(t) + pE_6(t) + pE_{1,4}(t) \\ &+ pE_{1,5}(t) - pE_{1,6}(t) + pE_{2,4}(t) \\ &+ pE_{2,5}(t) - pE_{2,6}(t) - pE_{3,4}(t) \\ &- pE_{3,5} + pE_{3,6}(t) \end{aligned} \qquad \textbf{(4)}$$

where

$$E_i(t) = (1 - A_i)\sum_{k=1}^{\infty} F_k(t)\,(A_i)^{k-1}, \quad i=1, 2, 3, 4, 5, 6$$

and for j=4, 5, 6

$$E_{1,j}(t) = (1 - A_1 A_j)\sum_{k=1}^{\infty} F_k(t)\,(A_1A_j)^{k-1}$$

$$E_{2,j}(t) = (1 - A_2 A_j)\sum_{k=1}^{\infty} F_k(t)\,(A_2A_j)^{k-1} \qquad E_{3,j}(t) = (1 - A_3 A_j)\sum_{k=1}^{\infty} F_k(t)\,(A_3A_j)^{k-1} \qquad \textbf{(5)}$$

Since L (t) =1 – P (W > t) from (4)

$$\begin{aligned} L(t) = {} & E_1(t) + E_2(t) - E_3(t) + pE_4(t) + pE_5(t) \\ &- pE_6(t) - pE_{1,4}(t) - pE_{1,5}(t) + pE_{1,6}(t) \\ &- pE_{2,4}(t) - pE_{2,5}(t) + pE_{2,6}(t) \\ &+ pE_{3,4}(t) + pE_{3,5}(t) - pE_{3,6}(t) \end{aligned} \qquad \textbf{(6)}$$

$$\begin{aligned} \ell(t) = {} & e_1(t) + e_2(t) - e_3(t) + pe_4(t) + pe_5(t) - pe_6(t) \\ &- pe_{1,4}(t) - pe_{1,5}(t) + pe_{1,6}(t) - pe_{2,4}(t) \\ &- pe_{2,5}(t) + pe_{2,6}(t) + pe_{3,4}(t) + pe_{3,5}(t) \\ &- pe_{3,6}(t) \end{aligned} \qquad \textbf{(7)}$$

and

$$\begin{aligned} \bar{l}(s) = {} & \bar{e}_1(s) + \bar{e}_2(s) - \bar{e}_3(s) + p\bar{e}_4(s) + p\bar{e}_5(s) \\ &- p\bar{e}_6(s) - p\bar{e}_{1,4}(s) - p\bar{e}_{1,5}(s) + p\bar{e}_{1,6}(s) \\ &- p\bar{e}_{2,4}(s) - p\bar{e}_{2,5}(s) + p\bar{e}_{2,6}(s) + p\bar{e}_{3,4}(s) \\ &+ p\bar{e}_{3,5}(s) - p\bar{e}_{3,6}(s) \end{aligned} \qquad \textbf{(8)}$$

It is known that

$$E(W) = -\left[\frac{d}{ds}\bar{l}(s)\right]_{s=0} \qquad \textbf{(9)}$$

$$E(W^2) = \left[\frac{d^2}{ds^2}\bar{l}(s)\right]_{s=0} \qquad \textbf{(10)}$$

$$V(W) = E(W^2) - [E(W)]^2 \qquad \textbf{(11)}$$

Using (3), (5) & (8) in (9) and (10), we have

$$\begin{aligned} E(W) = {} & C_1 + C_2 - C_3 + p[C_4 + C_5 - C_6 \\ &- H_{1,4} - H_{1,5} + H_{1,6} - H_{2,4} \\ &- H_{2,5} + H_{2,6} + H_{3,4} + H_{3,5} \\ &- H_{3,6}] \end{aligned} \qquad \textbf{(12)}$$

and

$$\begin{aligned} E(W^2) = 2\{ & C_1^2 + C_2^2 - C_3^2 + p[C_4^2 + C_5^2 \\ &- C_6^2 - H_{1,4}^2 - H_{1,5}^2 + H_{1,6}^2 \\ &- H_{2,4}^2 - H_{2,5}^2 \\ &+ H_{2,6}^2 + H_{3,4}^2 + H_{3,5}^2 - H_{3,6}^2]\} \end{aligned} \qquad \textbf{(13)}$$

where $C_i = \dfrac{1}{\theta(1 - A_i)}$, i=1, 2, 3, 4, 5, 6

and j=4, 5, 6,

$$H_{1,j} = \frac{1}{\theta(1 - A_1A_j)}, \quad H_{2,j} = \frac{1}{\theta(1 - A_2A_j)}, \quad H_{3,j} = \frac{1}{\theta(1 - A_3A_j)} \qquad \textbf{(14)}$$

Equation (12) gives the mean time to recruitment and equations (12) & (13) together with (11) give the variance of the time to recruitment for case (i) of Model - I.

Case (ii): Y_1, Y_2, Z_1 **and** Z_2 **follow exponentiated exponential distribution with parameters** μ_1, μ_2, μ_3 **and** μ_4 **respectively with shape parameter α = 2.**

$$P(S_k < Y) = 2A_1^{\,k} + 2A_2^{\,k} + 2A_7^{\,k} + 2A_8^{\,k} - A_9^{\,k} - A_{10}^{\,k} - A_{11}^{\,k} - 4A_3^{\,k}$$

and

$$P(S_k < Z) = 2A_4^{\,k} + 2A_5^{\,k} + 2A_{12}^{\,k} + 2A_{13}^{\,k} - A_{14}^{\,k} - A_{15}^{\,k} - A_{16}^{\,k} - 4A_6^{\,k} \qquad \textbf{(15)}$$

where A_i, i = 1 to 6 are given in (3) and

$$A_7 = \bar{g}(2\mu_1 + \mu_2) = \frac{\lambda_1}{\lambda_1 + 2\mu_1 + \mu_2} + \frac{\lambda_2}{\lambda_2 + 2\mu_1 + \mu_2} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + 2\mu_1 + \mu_2}$$

$$A_8 = \bar{g}(\mu_1 + 2\mu_2) = \frac{\lambda_1}{\lambda_1 + \mu_1 + 2\mu_2} + \frac{\lambda_2}{\lambda_2 + \mu_1 + 2\mu_2} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \mu_1 + 2\mu_2}$$

$$A_9 = \bar{g}(2\mu_1) = \frac{\lambda_1}{\lambda_1 + 2\mu_1} + \frac{\lambda_2}{\lambda_2 + 2\mu_1} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + 2\mu_1}$$

$$A_{10} = \bar{g}(2\mu_2) = \frac{\lambda_1}{\lambda_1 + 2\mu_2} + \frac{\lambda_2}{\lambda_2 + 2\mu_2} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + 2\mu_2}$$

$$A_{11} = \bar{g}(2\mu_1 + 2\mu_2) = \frac{\lambda_1}{\lambda_1 + 2\mu_1 + 2\mu_2} + \frac{\lambda_2}{\lambda_2 + 2\mu_1 + 2\mu_2} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + 2\mu_1 + 2\mu_2}$$

$$A_{12} = \bar{g}(2\mu_3 + \mu_4) = \frac{\lambda_1}{\lambda_1 + 2\mu_3 + \mu_4} + \frac{\lambda_2}{\lambda_2 + 2\mu_3 + \mu_4} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + 2\mu_3 + \mu_4}$$

In this case(2) becomes

$$A_{13} = \bar{g}(\mu_3 + 2\mu_4) = \frac{\lambda_1}{\lambda_1 + \mu_3 + 2\mu_4} + \frac{\lambda_2}{\lambda_2 + \mu_3 + 2\mu_4} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \mu_3 + 2\mu_4}$$

$$A_{14} = \bar{g}(2\mu_3) = \frac{\lambda_1}{\lambda_1 + 2\mu_3} + \frac{\lambda_2}{\lambda_2 + 2\mu_3} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + 2\mu_3}$$

$$A_{15} = \bar{g}(2\mu_4) = \frac{\lambda_1}{\lambda_1 + 2\mu_4} + \frac{\lambda_2}{\lambda_2 + 2\mu_4} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + 2\mu_4}$$

$$A_{16} = \bar{g}(2\mu_3 + 2\mu_4) = \frac{\lambda_1}{\lambda_1 + 2\mu_3 + 2\mu_4} + \frac{\lambda_2}{\lambda_2 + 2\mu_3 + 2\mu_4} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + 2\mu_3 + 2\mu_4} \qquad \textbf{(16)}$$

Using (15) in (1) and proceeding as in case (i) we get

$$\begin{aligned}
E(W) = {} & 2C_1 + 2C_2 + 2C_7 + 2C_8 - C_9 \\
& - C_{10} - C_{11} - 4C_3 + p[2C_4 + 2C_5 + 2C_{12} \\
& + 2C_{13} - C_{14} - C_{15} - C_{16} - 4C_6 - 4H_{1,4} \\
& - 4H_{1,5} - 4H_{1,12} - 4H_{1,13} + 2H_{1,14} \\
& + 2H_{1,15} + 2H_{1,16} + 8H_{1,6} - 4H_{2,4} - 4H_{2,5} \\
& - 4H_{2,12} - 4H_{2,13} + 2H_{2,14} + 2H_{2,15} + 2H_{2,16} \\
& + 8H_{2,6} - 4H_{7,4} - 4H_{7,5} - 4H_{7,12} - 4H_{7,13} \\
& + 2H_{7,14} + 2H_{7,15} + 2H_{7,16} + 8H_{7,6} - 4H_{8,4} \\
& - 4H_{8,5} - 4H_{8,12} - 4H_{8,13} + 2H_{8,14} + 2H_{8,15} \\
& + 2H_{8,16} + 8H_{8,6} + 2H_{9,4} + 2H_{9,5} + 2H_{9,12} \\
& + 2H_{9,13} - H_{9,14} - H_{9,15} - H_{9,16} - 4H_{9,6} \\
& + 2H_{10,4} + 2H_{10,5} + 2H_{10,12} + 2H_{10,13} \\
& - H_{10,14} - H_{10,15} - H_{10,16} - 4H_{10,6} + 2H_{11,4} \\
& + 2H_{11,5} + 2H_{11,12} + 2H_{11,13} - H_{11,14} \\
& - H_{11,15} - H_{11,16} - 4H_{11,6} + 8H_{3,4} + 8H_{3,5} \\
& + 8H_{3,12} + 8H_{3,13} - 4H_{3,14} - 4H_{3,15} \\
& - 4H_{3,16} - 16H_{3,6}]
\end{aligned} \qquad \textbf{(17)}$$

$$
\begin{aligned}
E(W^2) = {} & 2\{2C_1^2 + 2C_2^2 + 2C_7^2 + 2C_8^2 - C_9^2 \\
& - C_{10}^2 - C_{11}^2 - 4C_3^2 + p[2C_4^2 + 2C_5^2 + 2C_{12}^2 \\
& + 2C_{13}^2 - C_{14}^2 - C_{15}^2 - C_{16}^2 - 4C_6^2 - 4H_{1,4}^2 \\
& - 4H_{1,5}^2 - 4H_{1,12}^2 - 4H_{1,13}^2 + 2H_{1,14}^2 \\
& + 2H_{1,15}^2 + 2H_{1,16}^2 + 8H_{1,6}^2 - 4H_{2,4}^2 - 4H_{2,5}^2 \\
& - 4H_{2,12}^2 - 4H_{2,13}^2 + 2H_{2,14}^2 + 2H_{2,15}^2 \\
& + 2H_{2,15}^2 + 8H_{2,6}^2 - 4H_{7,4}^2 - 4H_{7,5}^2 - 4H_{7,12}^2 \\
& - 4H_{7,13}^2 + 2H_{7,14}^2 + 2H_{7,15}^2 + 2H_{7,16}^2 \\
& + 8H_{7,6}^2 - 4H_{8,4}^2 - 4H_{8,5}^2 - 4H_{8,12}^2 - 4H_{8,13}^2 \\
& + 2H_{8,14}^2 + 2H_{8,15}^2 + 2H_{8,16}^2 + 8H_{3,6}^2 + 2H_{9,4}^2 \\
& + 2H_{9,5}^2 + 2H_{9,12}^2 + 2H_{9,13}^2 - H_{9,14}^2 - H_{9,15}^2 \\
& - H_{9,16}^2 - 4H_{9,6}^2 + 2H_{10,4}^2 + 2H_{10,5}^2 \\
& + 2H_{10,12}^2 + 2H_{10,13}^2 - H_{10,14}^2 - H_{10,15}^2 \\
& - H_{10,16}^2 - 4H_{10,6}^2 + 2H_{11,4}^2 + 2H_{11,5}^2 \\
& + 2H_{11,12}^2 + 2H_{11,13}^2 - H_{11,14}^2 - H_{11,15}^2 \\
& - H_{11,16}^2 - 4H_{11,6}^2 + 8H_{3,4}^2 + 8H_{3,5}^2 + 8H_{3,12}^2 \\
& + 8H_{3,13}^2 - 4H_{3,14}^2 - 4H_{3,15}^2 - 4H_{3,16}^2 \\
& - 16H_{3,6}^2]\} \qquad \textbf{(18)}
\end{aligned}
$$

where $C_i = \dfrac{1}{\theta(1 - A_i)}$, i=1 to 16 and

for j = 4, 5, 12, 13, 14, 15, 16, 6,

$$
H_{1,j} = \frac{1}{\theta(1 - A_1 A_j)}, \quad H_{2,j} = \frac{1}{\theta(1 - A_2 A_j)},
$$

$$
H_{7,j} = \frac{1}{\theta(1 - A_7 A_j)} \quad H_{8,j} = \frac{1}{\theta(1 - A_8 A_j)}
$$

$$
H_{9,j} = \frac{1}{\theta(1 - A_9 A_j)} \quad H_{10,j} = \frac{1}{\theta(1 - A_{10} A_j)}
$$

$$
H_{11,j} = \frac{1}{\theta(1 - A_{11} A_j)} \quad H_{3,j} = \frac{1}{\theta(1 - A_3 A_j)} \qquad \textbf{(19)}
$$

Equation (17) gives the mean time to recruitment and equations (17) & (18) together with (11) give the variance of the time to recruitment for case (ii) of Model - I.

Case (iii): Y_1, Y_2, Z_1 and Z_2 have a continuous distribution with SCBZ property with parameters (μ_1, β_1, β_2), (μ_2, β_3, β_4), (μ_3, β_5, β_6) and (μ_4, β_7, β_8) respectively.

In this case (2) becomes

$$
\begin{aligned}
P(S_k < Y) = {} & p_2 B_1^k + q_2 B_2^k + p_1 B_3^k - p_1 p_2 B_4^k \\
& - p_1 q_2 B_5^k + q_1 B_6^k - q_1 p_2 B_7^k \\
& - q_1 q_2 B_8^k
\end{aligned}
$$

$$
\begin{aligned}
\text{and } P(S_k < Z) = {} & p_4 B_9^k + q_4 B_{10}^k + p_3 B_{11}^k \\
& - p_3 p_4 B_{12}^k - p_3 q_4 B_{13}^k + q_3 B_{14}^k \\
& - q_3 p_4 B_{15}^k - q_3 q_4 B_{16}^k \qquad \textbf{(20)}
\end{aligned}
$$

where

$$
B_1 = \bar{g}(\mu_2 + \beta_3) = \frac{\lambda_1}{\lambda_1 + \mu_2 + \beta_3} + \frac{\lambda_2}{\lambda_2 + \mu_2 + \beta_3} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \mu_2 + \beta_3}
$$

$$
B_2 = \bar{g}(\beta_4) = \frac{\lambda_1}{\lambda_1 + \beta_4} + \frac{\lambda_2}{\lambda_2 + \beta_4} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \beta_4}
$$

$$
B_3 = \bar{g}(\mu_1 + \beta_1) = \frac{\lambda_1}{\lambda_1 + \mu_1 + \beta_1} + \frac{\lambda_2}{\lambda_2 + \mu_1 + \beta_1} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \mu_1 + \beta_1}
$$

$$
B_4 = \bar{g}(\mu_1 + \mu_2 + \beta_1 + \beta_3) = \frac{\lambda_1}{\lambda_1 + \mu_1 + \mu_2 + \beta_1 + \beta_3} + \frac{\lambda_2}{\lambda_2 + \mu_1 + \mu_2 + \beta_1 + \beta_3} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \beta_1 + \beta_3}
$$

$$
B_5 = \bar{g}(\mu_1 + \beta_1 + \beta_4) = \frac{\lambda_1}{\lambda_1 + \mu_1 + \beta_1 + \beta_4} + \frac{\lambda_2}{\lambda_2 + \mu_1 + \beta_1 + \beta_4} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \mu_1 + \beta_1 + \beta_4}
$$

$$
B_6 = \bar{g}(\beta_2) = \frac{\lambda_1}{\lambda_1 + \beta_2} + \frac{\lambda_2}{\lambda_2 + \beta_2} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \beta_2}
$$

$$
B_7 = \bar{g}(\mu_2 + \beta_2 + \beta_3) = \frac{\lambda_1}{\lambda_1 + \mu_2 + \beta_2 + \beta_3} + \frac{\lambda_2}{\lambda_2 + \mu_2 + \beta_2 + \beta_3} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \mu_2 + \beta_2 + \beta_3}
$$

$$
B_8 = \bar{g}(\beta_2 + \beta_4) = \frac{\lambda_1}{\lambda_1 + \beta_2 + \beta_4} + \frac{\lambda_2}{\lambda_2 + \beta_2 + \beta_4} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \beta_2 + \beta_4}
$$

$$
B_9 = \bar{g}(\mu_4 + \beta_7) = \frac{\lambda_1}{\lambda_1 + \mu_4 + \beta_7} + \frac{\lambda_2}{\lambda_2 + \mu_4 + \beta_7} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \mu_4 + \beta_7}
$$

$$B_{10} = \bar{g}(\beta_8) = \frac{\lambda_1}{\lambda_1 + \beta_8} + \frac{\lambda_2}{\lambda_2 + \beta_8} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \beta_8}$$

$$B_{11} = \bar{g}(\mu_3 + \beta_5) = \frac{\lambda_1}{\lambda_1 + \mu_3 + \beta_5} + \frac{\lambda_2}{\lambda_2 + \mu_3 + \beta_5} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \mu_3 + \beta_5}$$

$$B_{12} = \bar{g}(\mu_3 + \mu_4 + \beta_5 + \beta_7) = \frac{\lambda_1}{\lambda_1 + \mu_3 + \mu_4 + \beta_5 + \beta_7} + \frac{\lambda_2}{\lambda_2 + \mu_3 + \mu_4 + \beta_5 + \beta_7} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \mu_3 + \mu_4 + \beta_5 + \beta_7}$$

$$B_{13} = \bar{g}(\mu_3 + \beta_5 + \beta_8) = \frac{\lambda_1}{\lambda_1 + \mu_3 + \beta_5 + \beta_8} + \frac{\lambda_2}{\lambda_2 + \mu_3 + \beta_5 + \beta_8} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \mu_3 + \beta_5 + \beta_8}$$

$$B_{14} = \bar{g}(\beta_6) = \frac{\lambda_1}{\lambda_1 + \beta_6} + \frac{\lambda_2}{\lambda_2 + \beta_6} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \beta_6}$$

$$B_{15} = \bar{g}(\mu_4 + \beta_6 + \beta_7) = \frac{\lambda_1}{\lambda_1 + \mu_4 + \beta_6 + \beta_7} + \frac{\lambda_2}{\lambda_2 + \mu_4 + \beta_6 + \beta_7} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \mu_4 + \beta_6 + \beta_7}$$

$$B_{16} = \bar{g}(\beta_6 + \beta_8) = \frac{\lambda_1}{\lambda_1 + \beta_6 + \beta_8} + \frac{\lambda_2}{\lambda_2 + \beta_6 + \beta_8} - \frac{(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2 + \beta_6 + \beta_8} \tag{21}$$

Using (20) in (1) and proceeding as in case (i) we get

$$
\begin{aligned}
E(W) = {} & p_2 C_1 + q_2 C_2 + p_1 C_3 - p_1 p_2 C_4 - p_1 q_2 C_5 + q_1 C_6 - q_1 p_2 C_7 - q_1 q_2 C_8 \\
& + p\,[p_4 C_9 + q_4 C_{10} + p_3 C_{11} - p_3 p_4 C_{12} - p_3 q_4 C_{13} + q_3 C_{14} - q_3 p_4 C_{15} - q_3 q_4 C_{16} \\
& - p_2 p_4 H_{1,9} - p_2 q_4 H_{1,10} - p_2 p_3 H_{1,11} + p_2 p_3 p_4 H_{1,12} + p_2 p_3 q_4 H_{1,13} - p_2 q_3 H_{1,14} + p_2 q_3 p_4 H_{1,15} + p_2 q_3 q_4 H_{1,16} \\
& - q_2 p_4 H_{2,9} - q_2 q_4 H_{2,10} - q_2 p_3 H_{2,11} + q_2 p_3 p_4 H_{2,12} + q_3 p_3 q_4 H_{2,13} - q_2 q_3 H_{2,14} + q_2 q_3 p_4 H_{2,15} + q_2 q_3 q_4 H_{2,16} \\
& - p_1 p_4 H_{3,9} - p_1 q_4 H_{3,10} - p_1 p_3 H_{3,11} + p_1 p_3 p_4 H_{3,12} + p_1 p_3 q_4 H_{3,13} - p_1 q_3 H_{3,14} + p_1 q_3 p_4 H_{3,15} + p_1 q_3 q_4 H_{3,16} \\
& + p_1 p_2 p_4 H_{4,9} + p_1 p_2 q_4 H_{4,10} + p_1 p_2 p_3 H_{4,11} - p_1 p_2 p_3 p_4 H_{4,12} - p_1 p_2 p_3 q_4 H_{4,13} + p_1 p_2 q_3 H_{4,14} - p_1 p_2 q_3 p_4 H_{4,15} - p_1 p_2 q_3 q_4 H_{4,16} \\
& + p_1 q_2 p_4 H_{5,9} + p_1 q_2 q_4 H_{5,10} + p_1 q_2 p_3 H_{5,11} - p_1 q_2 p_3 p_4 H_{5,12} - p_1 q_2 p_3 q_4 H_{5,13} + p_1 q_2 q_3 H_{5,14} - p_1 q_2 q_3 p_4 H_{5,15} - p_1 q_2 q_3 q_4 H_{5,16} \\
& - q_1 p_4 H_{6,9} - q_1 q_4 H_{6,10} - q_1 p_3 H_{6,11} + q_1 p_3 p_4 H_{6,12} + q_1 p_3 q_4 H_{6,13} - q_1 q_3 H_{6,14} + q_1 q_3 p_4 H_{6,15} + q_1 q_3 q_4 H_{6,16} \\
& + q_1 p_2 p_4 H_{7,9} + q_1 p_2 q_4 H_{7,10} + q_1 p_2 p_3 H_{7,11} - q_1 p_2 p_3 p_4 H_{7,12} - q_1 p_2 p_3 q_4 H_{7,13} + q_1 p_2 q_3 H_{7,14} - q_1 p_2 q_3 p_4 H_{7,15} - q_1 p_2 q_3 q_4 H_{7,16} \\
& + q_1 q_2 p_4 H_{8,9} + q_1 q_2 q_4 H_{8,10} + q_1 q_2 p_3 H_{8,11} - q_1 q_2 p_3 p_4 H_{8,12} - q_1 q_2 p_3 q_4 H_{8,13} + q_1 q_2 q_3 H_{8,14} - q_1 q_2 q_3 p_4 H_{8,15} - q_1 q_2 q_3 q_4 H_{8,16}]
\end{aligned}
\tag{22}
$$

and

$$
\begin{aligned}
E(W^2) = {} & 2[p_2 C_1^2 + q_2 C_2^2 + p_1 C_3^2 - p_1 p_2 C_4^2 - p_1 q_2 C_5^2 + q_1 C_6^2 - q_1 p_2 C_7^2 - q_1 q_2 C_8^2 \\
& + p\,[p_4 C_9^2 + q_4 C_{10}^2 + p_3 C_{11}^2 - p_3 p_4 C_{12}^2 - p_3 q_4 C_{13}^2 + q_3 C_{14}^2 - q_3 p_4 C_{15}^2 - q_3 q_4 C_{16}^2 \\
& - p_2 p_4 H_{1,9}^2 - p_2 q_4 H_{1,10}^2 - p_2 p_3 H_{1,11}^2 + p_2 p_3 p_4 H_{1,12}^2 + p_2 p_3 q_4 H_{1,13}^2 - p_2 q_3 H_{1,14}^2 + p_2 q_3 p_4 H_{1,15}^2 + p_2 q_3 q_4 H_{1,16}^2 \\
& - q_2 p_4 H_{2,9}^2 - q_2 q_4 H_{2,10}^2 - q_2 p_3 H_{2,11}^2 + q_2 p_3 p_4 H_{2,12}^2 + q_3 p_3 q_4 H_{2,13}^2 - q_2 q_3 H_{2,14}^2 + q_2 q_3 p_4 H_{2,15}^2 + q_2 q_3 q_4 H_{2,16}^2 \\
& - p_1 p_4 H_{3,9}^2 - p_1 q_4 H_{3,10}^2 - p_1 p_3 H_{3,11}^2 + p_1 p_3 p_4 H_{3,12}^2 + p_1 p_3 q_4 H_{3,13}^2 - p_1 q_3 H_{3,14}^2 + p_1 q_3 p_4 H_{3,15}^2 + p_1 q_3 q_4 H_{3,16}^2 \\
& + p_1 p_2 p_4 H_{4,9}^2 + p_1 p_2 q_4 H_{4,10}^2 + p_1 p_2 p_3 H_{4,11}^2 - p_1 p_2 p_3 p_4 H_{4,12}^2 - p_1 p_2 p_3 q_4 H_{4,13}^2 + p_1 p_2 q_3 H_{4,14}^2 - p_1 p_2 q_3 p_4 H_{4,15}^2 \\
& - p_1 p_2 q_3 q_4 H_{4,16}^2 + p_1 q_2 p_4 H_{5,9}^2
\end{aligned}
$$

$$+ p_1 q_2 q_4 H_{5,10}{}^2 + p_1 q_2 p_3 H_{5,11}{}^2 - p_1 q_2 p_3 p_4 H_{5,12}{}^2 - p_1 q_2 p_3 q_4 H_{5,13}{}^2 + p_1 q_2 q_3 H_{5,14}{}^2 - p_1 q_2 q_3 p_4 H_{5,15}{}^2 - p_1 q_2 q_3 q_4 H_{5,16}{}^2 - q_1 p_4 H_{6,9}{}^2 - q_1 q_4 H_{6,10}{}^2 - q_1 p_3 H_{6,11}{}^2 + q_1 p_3 p_4 H_{6,12}{}^2 + q_1 p_3 q_4 H_{5,13}{}^2 - q_1 q_3 H_{6,14}{}^2 + q_1 q_3 p_4 H_{6,15}{}^2 + q_1 q_3 q_4 H_{6,16}{}^2 + q_1 p_2 p_4 H_{7,9}{}^2 + q_1 p_2 q_4 H_{7,10}{}^2 + q_1 p_2 p_3 H_{7,11}{}^2 - q_1 p_2 p_3 p_4 H_{7,12}{}^2 - q_1 p_2 p_3 q_4 H_{7,13}{}^2 + q_1 p_2 q_3 H_{7,14}{}^2 - q_1 p_2 q_3 p_4 H_{7,15}{}^2 - q_1 p_2 q_3 q_4 H_{7,16}{}^2 + q_1 q_2 p_4 H_{8,9}{}^2 + q_1 q_2 q_4 H_{8,10}{}^2 + q_1 q_2 p_3 H_{8,11}{}^2 - q_1 q_2 p_3 p_4 H_{8,12}{}^2 - q_1 q_2 p_3 q_4 H_{8,13}{}^2 + q_1 q_2 q_3 H_{8,14}{}^2 - q_1 q_2 q_3 p_4 H_{8,15}{}^2 - q_1 q_2 q_3 q_4 H_{8,16}{}^2]\} \quad \textbf{(23)}$$

where $C_i = \dfrac{1}{\theta(1-B_i)}$, i = 1 to 16 and

for j = 9, 10, 11, 12, 13, 14, 15, 16,

$$H_{1,j} = \frac{1}{\theta(1-B_1B_j)}, \quad H_{2,j} = \frac{1}{\theta(1-B_2B_j)},$$
$$H_{3,j} = \frac{1}{\theta(1-B_3B_j)} \quad H_{4,j} = \frac{1}{\theta(1-B_4B_j)}$$
$$H_{5,j} = \frac{1}{\theta(1-B_5B_j)} \quad H_{6,j} = \frac{1}{\theta(1-B_6\ B_j)}$$
$$H_{7,j} = \frac{1}{\theta(1-B_7B_j)} \quad H_{8,j} = \frac{1}{\theta(1-B_8B_j)} \quad \textbf{(24)}$$

Equation (22) gives the mean time to recruitment and equations (22) & (23) together with (11) give the variance of the time to recruitment for case (iii) of Model - I.

Model description and main results for Model - II

In this Model, the thresholds Y and Z for the organization are taken as Y= min (Y_1, Y_2) and Z= min (Z_1, Z_2). All other assumptions and notations are same as in Model – I.

Case (i): $\boldsymbol{Y_1}$, $\boldsymbol{Y_2}$, $\boldsymbol{Z_1}$ *and* $\boldsymbol{Z_2}$ *follow exponential distribution with parameters* $\boldsymbol{\mu_1}$, $\boldsymbol{\mu_2}$, $\boldsymbol{\mu_3}$ **and** $\boldsymbol{\mu_4}$ **respectively**

In this case equation (2) becomes

P ($S_k < Y$) = $A_3{}^k$

P ($S_k < Z$) = $A_6{}^k$ **(25)**

where A_3 *and* A_6 are given in (3).

Using (25) in (1) and proceeding as in Model – I, we get

E (W) = C_3 + p [C_6 -$H_{3,6}$] **(26)**

E (W^2) = 2 {$C_3{}^2$ + p [$C_6{}^2$ -$H_{3,6}{}^2$]} **(27)**

Where C_3, C_6 and $H_{3,6}$ are given in (14).

Equation (26) gives the mean time to recruitment and equations (26) & (27) together with (11) give the variance of the time to recruitment for case (i) of Model - II.

Case (ii): $\boldsymbol{Y_1}$, $\boldsymbol{Y_2}$, $\boldsymbol{Z_1}$ *and* $\boldsymbol{Z_2}$ *follow exponentiated exponential distribution with parameters* $\boldsymbol{\mu_1}$, $\boldsymbol{\mu_2}$, $\boldsymbol{\mu_3}$ **and** $\boldsymbol{\mu_4}$ **respectively with shape parameter =2**

In this case (2) becomes

P ($S_k < Y$) = 4$A_3{}^k$ - 2$A_8{}^k$ - 2$A_7{}^k$ + $A_{11}{}^k$

P ($S_k < Z$) = 4$A_6{}^k$ - 2$A_{13}{}^k$ - 2$A_{12}{}^k$ + $A_{16}{}^k$ **(28)**

where A_3, A_6, A_7, A_8, A_{11}, A_{12}, A_{13} and A_{16} are given in (16).

Using (28) in (1) and proceeding as in Model – I, we have

E (W) = 4C_3 - 2C_8 - 2C_7 + C_{11} + p [4C_6 - 2C_{13} - 2C_{12} + C_{16} - 16$H_{3,6}$ + 8$H_{3,13}$+ 8$H_{3,12}$ - 4$H_{3,16}$ + 8$H_{8,6}$ - 4$H_{8,13}$ - 4$H_{8,12}$ + 2$H_{8,16}$ + 8$H_{7,6}$ - 4$H_{7,13}$ - 4$H_{7,12}$ + 2$H_{7,16}$ - 4$H_{11,6}$ + 2$H_{11,13}$ + 2$H_{11,12}$ - $H_{11,16}$] **(29)**

E (W^2) = 2 {4$C_3{}^2$ - 2$C_8{}^2$ - 2$C_7{}^2$ + $C_{11}{}^2$ + p [4$C_6{}^2$ - 2$C_{13}{}^2$ - 2$C_{12}{}^2$ + $C_{16}{}^2$ - 16$H_{3,6}{}^2$ + 8$H_{3,13}{}^2$ + 8$H_{3,12}{}^2$ - 4$H_{3,16}{}^2$+ 8$H_{8,6}{}^2$ - 4$H_{8,13}{}^2$ - 4$H_{8,12}{}^2$ + 2$H_{8,16}{}^2$ + 8$H_{7,6}{}^2$ - 4$H_{7,13}{}^2$- 4$H_{7,12}{}^2$ + 2$H_{7,16}{}^2$ - 4$H_{11,6}{}^2$ + 2$H_{11,13}{}^2$ +2$H_{11,12}{}^2$-$H_{11,16}{}^2$]} **(30)**

where C_3, C_6, C_7, C_8, C_{11}, C_{12}, C_{13}, C_{16} and $H_{3,j}$, $H_{8,j}$, $H_{7,j}$, $H_{11,j}$, j = 6, 12, 13, 16 are given in equation (19).

Equation (29) gives the mean time to recruitment and equations (29) & (30) together with (11) give the variance of the time to recruitment for case (ii) of Model - II.

Case (iii): $\boldsymbol{Y_1}$, $\boldsymbol{Y_2}$, $\boldsymbol{Z_1}$ *and* $\boldsymbol{Z_2}$ *have a continuous distribution with SCBZ property with parameters* ($\boldsymbol{\mu_1,\beta_1,\beta_2}$), ($\boldsymbol{\mu_2,\beta_3,\beta_4}$), ($\boldsymbol{\mu_3,\beta_5,\beta_6}$) *and* ($\boldsymbol{\mu_4,\beta_7,\beta_8}$) **respectively.**

In this case (2) becomes

P ($S_k < Y$) = $p_1\, p_2\, B_4{}^k$ + $p_1\, q_2\, B_5{}^k$ + $q_1\, p_2\, B_7{}^k$ + $q_1\, q_2\, B_8{}^k$

P($S_k < Z$)= $p_3\, p_4\, B_{12}{}^k$ + $p_3\, q_4\, B_{13}{}^k$+ $q_3\, p_4\, B_{15}{}^k$ + $q_3\, q_4\, B_{16}{}^k$ **(31)**

where B_4, B_5, B_7, B_8, B_{12}, B_{13}, B_{15} and B_{16} are given in (21).

Using (31) in (1) and proceeding as in Model – I, we get

$$
\begin{aligned}
E(W) = {} & p_1 p_2 C_4 + p_1 q_2 C_5 + q_1 p_2 C_7 + q_1 q_2 C_8 \\
& + p\,[p_3 p_4 C_{12} + p_3 q_4 C_{13} + q_3 p_4 C_{15} + q_3 q_4 C_{16} \\
& - p_1 p_2 p_3 p_4 H_{4,12} - p_1 p_2 p_3 q_4 H_{4,13} \\
& - p_1 p_2 q_3 p_4 H_{4,15} - p_1 p_2 q_3 q_4 H_{4,16} \\
& - p_1 q_2 p_3 p_4 H_{5,12} - p_1 q_2 p_3 q_4 H_{5,13} \\
& - p_1 q_2 q_3 p_4 H_{5,15} - p_1 q_2 q_3 q_4 H_{5,16} \\
& - q_1 p_2 p_3 p_4 H_{7,12} - q_1 p_2 p_3 q_4 H_{7,13} \\
& - q_1 p_2 q_3 p_4 H_{7,15} - q_1 p_2 q_3 q_4 H_{7,16} \\
& - q_1 q_2 p_3 p_4 H_{8,12} - q_1 q_2 p_3 q_4 H_{8,13} \\
& - q_1 q_2 q_3 p_4 H_{8,15} - q_1 q_2 q_3 q_4 H_{8,15}] \qquad \textbf{(32)}
\end{aligned}
$$

$$
\begin{aligned}
E(W^2) = {} & 2\{p_1 p_2 C_4^2 + p_1 q_2 C_5^2 + q_1 p_2 C_7^2 \\
& + q_1 q_2 C_8^2 + p\,[p_3 p_4 C_{12}^2 + p_3 q_4 C_{13}^2 \\
& + q_3 p_4 C_{15}^2 + q_3 q_4 C_{16}^2 - p_1 p_2 p_3 p_4 H_{4,12}^2 \\
& - p_1 p_2 p_3 q_4 H_{4,13}^2 - p_1 p_2 q_3 p_4 H_{4,15}^2 \\
& - p_1 p_2 q_3 q_4 H_{4,16}^2 - p_1 q_2 p_3 p_4 H_{5,12}^2 \\
& - p_1 q_2 p_3 q_4 H_{5,13}^2 - p_1 q_2 q_3 p_4 H_{5,15}^2 \\
& - p_1 q_2 q_3 q_4 H_{5,16}^2 - q_1 p_2 p_3 p_4 H_{7,12}^2 \\
& - q_1 p_2 p_3 q_4 H_{7,13}^2 - q_1 p_2 q_3 p_4 H_{7,15}^2 \\
& - q_1 p_2 q_3 q_4 H_{7,16}^2 - q_1 q_2 p_3 p_4 H_{8,12}^2 \\
& - q_1 q_2 p_3 q_4 H_{8,13}^2 - q_1 q_2 q_3 p_4 H_{8,15}^2 \\
& - q_1 q_2 q_3 q_4 H_{8,16}^2]\} \qquad \textbf{(33)}
\end{aligned}
$$

where C_4, C_5, C_7, C_8, C_{12}, C_{13}, C_{15}, C_{16} and $H_{4,j}$, $H_{5,j}$, $H_{7,j}$, $H_{8,j}$, j = 12, 13, 15, 16 are given in equation (24).

Equation (32) gives the mean time to recruitment and equations (32) & (33) together with (11) give the variance of the time to recruitment for case (iii) of Model - II.

Numerical illustrations and Conclusions

In this section the analytical expressions for expectation and variance of the time to recruitment are analyzed for Models I and II. The influence of nodal parameters λ_1, λ_2, θ and p on the performance measures namely mean and variance of the time to recruitment for Model - I is shown in table-I by varying one parameter and keeping other parameters fixed. In table-II corresponding computation for Model - II is made and the results are tabulated.

Table 1: Effect of λ1, λ2, θ and p on performance measures.

($\mu_1 = 0.7$; $\mu_2 = 0.4$; $\mu_3 = 0.5$ $\mu_4 = 0.2$ $\beta_1 = 0.6$; $\beta_2 = 0.3$; $\beta_3 = 0.4$; $\beta_4 = 0.7$
$\beta_5 = 0.5$; $\beta_6 = 0.2$; $\beta_7 = 0.8$; $\beta_8 = 0.4$)

λ_1	λ_2	θ	p	Model – I					
				Case(i)		Case(ii)		Case(iii)	
				E(W)	V(W)	E(W)	V(W)	E(W)	V(W)
0.1	0.2	0.10	0.5	12.9510	163.0749	14.4662	192.6646	12.7794	159.1932
0.3	0.2	0.10	0.5	16.0885	241.0517	18.9214	299.3584	15.7811	232.6867
0.5	0.2	0.10	0.5	17.6114	282.7408	20.9604	354.4096	17.2561	272.2334
0.2	0.2	0.15	0.5	9.8844	92.5874	11.4429	113.4186	9.7120	89.6831
0.2	0.4	0.15	0.5	11.3147	117.7624	13.4115	147.1595	11.0902	113.4845
0.2	0.6	0.15	0.5	12.0580	131.6499	14.3817	165.2815	11.8130	126.6990
0.4	0.3	0.10	0.5	19.2401	332.6607	23.3129	423.0164	18.8154	318.6007
0.4	0.3	0.15	0.5	12.8267	147.8492	15.5419	188.0073	12.5436	141.6003
0.4	0.3	0.20	0.5	9.6201	83.1652	11.6565	105.7541	9.4077	79.6502
0.3	0.5	0.25	0.2	7.0123	46.4272	8.4049	58.9194	7.1954	48.2009
0.3	0.5	0.25	0.4	7.7391	54.6495	9.3855	69.8024	7.6782	53.3857
0.3	0.5	0.25	0.6	8.4659	61.8152	10.3662	78.7620	8.1610	58.1043

Table 2: Effect of λ1, λ2, θ and p on performance measures.

($\mu_1 = 0.7$; $\mu_2 = 0.4$; $\mu_3 = 0.5$ $\mu_4 = 0.2$ $\beta_1 = 0.6$; $\beta_2 = 0.3$; $\beta_3 = 0.4$; $\beta_4 = 0.7$
$\beta_5 = 0.5$; $\beta_6 = 0.2$; $\beta_7 = 0.8$; $\beta_8 = 0.4$)

λ_1	λ_2	θ	p	Model – II					
				Case(i)		Case(ii)		Case(iii)	
				E(W)	V(W)	E(W)	V(W)	E(W)	V(W)
0.1	0.2	0.10	0.5	10.4767	109.6622	11.0468	120.8837	10.4341	108.7980
0.3	0.2	0.10	0.5	11.1496	123.7540	12.4172	148.4414	11.0546	121.7852
0.5	0.2	0.10	0.5	11.5917	133.2997	13.2454	165.5040	11.4675	130.6851
0.2	0.2	0.15	0.5	7.2338	52.1906	7.8828	60.6424	7.1852	51.5260
0.2	0.4	0.15	0.5	7.5948	57.3174	8.5861	70.1802	7.5204	56.2810
0.2	0.6	0.15	0.5	7.8385	60.8630	9.0262	76.3078	7.7491	59.6015
0.4	0.3	0.10	0.5	11.9158	140.5061	13.8960	179.0610	11.7680	137.3453
0.4	0.3	0.15	0.5	7.9438	62.4472	9.2640	79.5827	7.8454	61.0424
0.4	0.3	0.20	0.5	5.9579	35.1265	6.9480	44.7653	5.8840	34.3363
0.3	0.5	0.25	0.2	4.6367	21.4557	5.3186	26.9613	4.6854	21.8974
0.3	0.5	0.25	0.4	4.8005	22.8506	5.6204	29.3460	4.7726	22.6055
0.3	0.5	0.25	0.6	4.9643	24.1918	5.9223	31.5485	4.8598	23.2984

From tables 1 and 2, we observe the following which agrees with reality:

1. When λ_1 increases and keeping other parameters fixed, the mean and variance of the time to recruitment increase.

2. When λ_2 increases and keeping other parameters fixed, the mean and variance of the time to recruitment increase.

3. When θ increases and keeping other parameters fixed, the mean and variance of the time to recruitment decrease.

4. When p increases and keeping other parameters fixed, the mean and variance of the time to recruitment increase.

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