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MATHEMATICS

VARIANCE OF THE TIME TO RECRUITMENT IN AN ORGANIZATION WITH TWO GRADES

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Abstract

In this paper, a two grade organization subjected to random exit of personal due to policy decisions taken by the organization is considered. There is an associated loss of manpower if a person quits. As the exit of personnel is unpredictable, a new recruitment policy involving two thresholds for each grade-one is optional and the other mandatory is suggested to enable the organization to plan its decision on recruitment. Based on shock model approach two mathematical models are constructed using an appropriate univariate policy of recruitment. Performance measures namely mean and variance of the time to recruitment are obtained for both the models when (i) the loss of manhours process forms a sequence of independent and identically distributed exponential random variables (ii) the inter-decision times are independent and identically distributed exponential random variables and (iii) the optional and mandatory thresholds are exponential random variables. The analytical results are substantiated by numerical illustrations and the influence of nodal parameters on the performance measures is also analyzed.

Keywords: Manpower planning, Shock models, Univariate recruitment policy, Mean and variance of the time to recruitment
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Introduction

Consider an organization having two grades in which depletion of manpower occurs at every decision epoch. In the univariate policy of recruitment, based on shock model approach, recruitment is made as and when the cumulative loss of manpower crosses a threshold. Employing this recruitment policy, expected time to recruit is obtained under different conditions for several models in [[1], [5], [6], [7], [8], [9], [10], [11]]. Recently in [[2], [3], [4]] for a single grade system, a new univariate recruitment policy involving two thresholds in which one is optional and the other mandatory is suggested and the mean time to recruitment is obtained under different conditions on the nature of the thresholds for the two cases (i) the inter decision times are independent and identically distributed random variables and (ii) the inter decision times are exchangeable and constantly correlated exponential random variables. The objective of the present paper is to obtain the variance of time to recruitment for a two graded system using the univariate recruitment policy considering optional and mandatory thresholds for both the grades. The present paper extends the result in [2] for a two graded system.

Model description and analysis for model - I

Consider an organization having two grades in which decisions are taken at random epochs in $(0, \infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manhours to the organization if a person quits. It is assumed that the loss of manhours are linear and cumulative.

Let X_i, be the loss of manhours due to the i^{th} decision epoch, i = 1, 2, 3 … …, forming a sequence of independent and identically distributed exponential random variables with parameter α $(\alpha > 0)$.

Let $S_k = \sum_{i=0}^{k} X_i$ be the cumulative loss of man hours in the first k decisions(k=1,2,…).Let g(.) be the probability density function of X_i, i = 1, 2, 3 … … It is assumed that the inter-decision times are independent and identically distributed exponential random variables with probability density function (distribution function) f(.) (F(.)) and parameter θ (θ > 0). Let f_k (.) (F_k (.)) be the k fold convolution of f(.) (F(.)). Let f*(.) (g*(.)) be the Laplace transform of f(.) (g(.)). It is assumed that loss of manhours process and the process of inter-decision times are statistically independent. Let Y_A, Y_B (Z_A, Z_B) be exponential random variables denoting the optional (mandatory) thresholds for grades A and B with parameters λ_A and λ_B (μ_A and μ_B)respectively .Assume that $Y_A<Z_A$ and $Y_B<Z_B$.
Let Y = max (Y_A, Y_B) and Z = max (Z_A, Z_B) be the optional and mandatory thresholds for the organization. The recruitment policy employed in this paper is as follows: *If the total loss of manhours crosses the optional threshold level Y, the organization may or may not go for recruitment, but if the total loss of manhours crosses the mandatory threshold Z, recruitment is necessary.* Let p be the probability that

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the organization is not going for recruitment whenever the total loss of manhours crosses optional level Y. Let W be a continuous random variable denoting the time for recruitment in the organization with probability density function ℓ (.), cumulative distribution function L(.) and ℓ *(.) be the Laplace transform of ℓ (.). Let V_k (t) be the probability that there are exactly k–decision epochs in (0, t].Since the number of decisions made in (0,t] form a renewal process we note that $V_k(t) = F_k(t) - F_{k+1}(t)$ where $F_0(t) = 1$. Let E(W) and V(W) be the mean and variance of time to recruitment respectively.

As in [2] the survival function of W is given by

$$P(W > t) = \sum_{k=0}^{\infty} V_k(t)\, P(S_k < Y) + \sum_{k=0}^{\infty} V_k(t) \times p \times P(S_k \ge Y) \times P(S_k < Z) \qquad (1)$$

Invoking the law of total probability it can be shown that

$$P(S_k < Y) = (D_1)^k + (D_2)^k - (D_3)^k \qquad (2)$$

and

$$P(S_k < Z) = (D_4)^k + (D_5)^k - (D_6)^k \qquad (3)$$

where $D_1 = g^*(\lambda_A), D_2 = g^*(\lambda_B), D_3 = g^*(\lambda_A + \lambda_B),$ $D_4 = g^*(\mu_A), D_5 = g^*(\mu_B), D_6 = g^*(\mu_A + \mu_B)$ (4)

For i=1,2,….6 ,write

$$E_i(t) = [1 - D_i] \sum_{k=1}^{\infty} F_k(t)\, (D_i)^{k-1}$$

For j=4,5,6,write

$$\left.\begin{aligned} E_{1,j}(t) &= [1 - D_1 D_j] \sum_{k=1}^{\infty} F_k(t)\, (D_1 D_j)^{k-1}, \\ E_{2,j}(t) &= \sum_{k=1}^{\infty} F_k(t)\, (D_2 D_j)^{k-1} \\ E_{3,j}(t) &= [1 - D_3 D_j] \sum_{k=1}^{\infty} F_k(t)\, (D_3 D_j)^{k-1} \end{aligned}\right\} \qquad (5)$$

From (1), (2), (3) (4)and (5) and on simplification we get

$$\begin{aligned} P(W > t) = 1 &- E_1(t) - E_2(t) + E_3(t) + p[- E_4(t) + E_{1,4}(t) + E_{2,4}(t) - E_{3,4}(t) - E_5(t) + E_{1,5}(t) + E_{2,5}(t) \\ &- E_{3,5}(t) + E_6(t) - E_{1,6}(t) - E_{2,6}(t) + E_{3,6}(t)] \end{aligned} \qquad (6)$$

Since L(t) = 1 – P (W > t) from (6)

$$\begin{aligned} L(t) = E_1(t) &+ E_2(t) - E_3(t) + p[E_4(t) - E_{1,4}(t) - E_{2,4}(t) \\ &+ E_{3,4}(t) + E_5(t) - E_{1,5}(t) - E_{2,5}(t) + E_{3,5}(t) - E_6(t) + \\ &E_{1,6}(t) + E_{2,6}(t) - E_{3,6}(t)] \end{aligned} \qquad (7)$$

$$\begin{aligned} \therefore \ell(t) = e_1(t) &+ e_2(t) - e_3(t) + p[e_4(t) - e_{1,4}(t) - e_{2,4}(t) \\ &+ e_{3,4}(t) + e_5(t) - e_{1,5}(t) - e_{2,5}(t) + e_{3,5}(t) - e_6(t) \\ &+ e_{1,6}(t) + e_{2,6}(t) - e_{3,6}(t)] \end{aligned} \qquad (8)$$

where for i=1,2,…6, $e_i(t) = \left\{\frac{d}{dt} E_i(t)\right\}$ and j=4,5,6 ,

$e_{1,j}(t) = \left\{\frac{d}{dt} E_{1,j}(t)\right\}$, $e_{2,j}(t) = \left\{\frac{d}{dt} E_{2,j}(t)\right\}$,

$e_{3,j}(t) = \left\{\frac{d}{dt} E_{3,j}(t)\right\}$

From (8)

$$\begin{aligned} \ell^*(s) = e_1^*(s) &+ e_2^*(s) - e_3^*(s) + p[e_4^*(s) - e_{1,4}^*(s) \\ &- e_{2,4}^*(s) + e_{3,4}^*(s) + e_5^*(s) - e_{1,5}^*(s) - e_{2,5}^*(s) \\ &+ e_{3,5}^*(s) - e_6^*(s) + e_{1,6}^*(s) + e_{2,6}^*(s) - e_{3,6}^*(s)] \end{aligned} \qquad (9)$$

For i=1,2,..6,note that $e_i^*(s) = \frac{[1 - D_i] f^*(s)}{[1 - f^*(s) D_i]}$. For j=4,5,6,

$$e_{1,j}^*(s) = \frac{[1 - D_1 D_j] f^*(s)}{[1 - f^*(s) D_1 D_j]}, \quad e_{2,j}^*(s) = \frac{[1 - D_2 D_j] f^*(s)}{[1 - f^*(s) D_2 D_j]},$$

$$e_{3,j}^*(s) = \frac{[1 - D_3 D_j] f^*(s)}{[1 - f^*(s) D_3 D_j]} \qquad (10)$$

By hypothesis

$$f^*(s) = \theta/(\theta + s), \quad g^*(s) = \alpha/(\alpha + s) \qquad (11)$$

Since

$$E(W) = -\left\{\frac{d}{ds} \ell^*(s)\right\}_{s=0} \qquad (12)$$

$$E(W^2) = \left\{\frac{d^2}{ds^2} \ell^*(s)\right\}_{s=0} \qquad (13)$$

using (4),(9),(10), (11) in(12) and (13) and on simplification one can show that

$$E(W) = C_1 + C_2 - C_3 + p[C_4 + C_5 - C_6 - H_{14} - H_{15} + H_{16} - H_{24} - H_{25} + H_{26} + H_{34} + H_{35} - H_{36}] \qquad (14)$$

$$E(W^2) = 2\{C_1^2 + C_2^2 - C_3^2 + p[C_4^2 + C_5^2 - C_6^2 - H_{14}^2 - H_{15}^2 + H_{16}^2 - H_{24}^2 - H_{25}^2 + H_{26}^2 + H_{34}^2 + H_{35}^2 - H_{36}^2]\}$$

where for i=1,2,…6 $C_i = 1/[\theta(1-D_i)]$ and for m=1,2,3 $H_{m4} = 1/[\theta(1-D_m D_4)]$,

$$H_{m5} = 1/[\theta(1-D_m D_5)], H_{m6} = 1/[\theta(1-D_m D_6)]. \qquad (15)$$

Since $V(W) = E(W^2) - [E(W)]^2$ (16)

while (14) gives the mean time to recruitment,(14) and (15) together with (16) gives variance of the time for recruitment for the present model.

Model description and analysis for model - II

For model II, the optional and mandatory thresholds for the organization are given by Y = min(Y_A, Y_B) and Z = min(Z_A, Z_B). All other assumptions and notations are as in Model I.

For this model it can be shown that

$P(S_k < Y) = (D_3)^k$ and $P(S_k < Z) = (D_6)^k$

$L(t) = E_3(t) + pE_6(t) - pE_{3,6}(t)$

$\ell^*(s) = e_3^*(s) + pe_6^*(s) - pe_{3,6}^*(s)$

$E(W) = C_3 + pC_6 - pH_{36}$ and (17)

$$E(W^2) = 2[C_3^{\,2} + pC_6^{\,2} - pH_{36}^{\,2}] \quad (18)$$

(17) gives the mean time to recruitment and (17) and (18) together with (16) gives variance of the time for recruitment for the present model.

Numerical illustration

The analytical expression for expectation and variance of time to recruitment are analyzed numerically by varing parameters. The values of the mean and variance of time to recruitment are calculated for both the models and presented in table I by varing the mean 1/θ of the inter decision time, keeping the other parameters fixed. In table II the corresponding results are tabulated when the mean 1/α of the loss of manhours varies, keeping the other parameters-fixed.

Table – I: Effect of θ and α on performance measures.
(λ_A = .5; λ_B = .8; μ_A = .4; μ_B = .6, p = .8)

θ	α	E(W)		V(W)	
		Model I	Model II	Model I	Model II
.3	.7	11.64	4.49	161.13	76.491
.6	.7	5.82	2.25	34.43	17.30
.9	.7	3.88	1.50	12.70	6.88

Table – II: Effect of θ and α on performance measures.
(λ_A = .5; λ_B = .8; μ_A = .3; μ_B = .4, p = .6)

θ	α	E(W)		V(W)	
		Model I	Model II	Model I	Model II
.7	.2	2.54	0.99	7.49	6.69
.7	.4	3.58	1.40	14.18	9.02
.7	.6	4.61	1.80	22.89	11.59

(i) From table – I, the expected time and the variance of time to recruitment decrease, with the mean of the inter-decision time.
(ii) From table – II, mean and variance of time to recruitment increase when the mean loss of manhours decreases for both the models.

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