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PHYSICS

SIMPLE 4D – HYPERCHAOTIC CANONICAL VAN DER POL DUFFING OSCILLATOR USING CURRENT FEEDBACK OP-AMP

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Abstract

In this paper, in order to show some interesting phenomena of fourth-order hyperchaotic Canonical Van der Pol Duffing oscillator circuit with a smooth cubic nonlinearity, different kinds of attractors, time waveforms and corresponding Lyapunov exponent spectra of systems are presented, respectively. The perturbation transforms an unpredictable hyperchaotic behavior into a predictable hyperchaotic or periodic motion via stabilization of unstable, aperiodic, or periodic orbits of the strange hyperchaotic attractor. One advantage of the method is its robustness against noise. A theoretical analysis of the circuit equations is presented, along with experimental simulation and numerical results.

Keywords: Simple 4D – hyperchaotic canonical Van der Pol Duffing oscillator; Hyperchaos; Time waveforms; Lyapunov exponents

Introduction

In original canonical Chua's circuit, a nonlinear resistor is called Chua's diode is the unique nonlinear electric element. It plays an important role in the circuit. Due to the existance of this nonlinear element Chua's circuit exhibits a variety of nonlinear phenomena, such as chaos, bifurcation and so on [1-5]. The characteristic of Chua's diode is described by a continuous piecewise - linear function with three segments and two nondifferential break points [6-9]. However, the characteristics of nonlinear devices in practical circuits are always smooth and the implementation of piecewise-linear function requires a large amount of circuitry compared with smooth cubic function. Therefore, it is significant to investigate Chua's circuit with a smooth cubic nonlinearity from practical view point [10]. Hartley (1989) proposed to replace the piecewise-linear nonlinearity in Chua's circuit with a smooth cubic nonlinearity.

In the present report the behavior of a fourth-order autonomous hyperchaotic Canonical Van der Pol Duffing oscillator circuit has been studied. This circuit consists of two active elements, one linear negative conductance and one smooth cubic nonlinearity exhibiting a symmetrical piecewise-linear *v-i* characteristic. Two inductances (L_1, L_2) two capacitances (C_1, C_2) and one locally active resistor (R) is also included in the circuit, serve as the control parameters.

Hyperchaos is defined as a chaotic attractor with more than one positive Lyapunov exponents, i.e., its dynamics expand in more than one direction [5]. In otherwords, the dynamics expand not only small line segments, but also small area elements, there giving rise to a 'thick' chaotic attractor. Most hyperchaotic and bifurcation effects cited in the literature have been observed in electric circuits. They include the period-doubling route to chaos, the intermittency route to chaos, the quasiperiodicity route to chaos and of course the crisis [7, 11-13]. This popularity is attributed to the advantages which electric circuits offer to experimental hyperchaos studies, such complicated hyperchaotic wave forms are expected to be utilized for realization of several hyperchaotic applications such a chaos communication system with robustness against various interferences including multi-user access [9-16]. The plan of the paper is as follows. In section 2, we present the details of realization of the proposed autonomous circuit. The results of the observations from the laboratory experimental simulation and the conformation through analytical calculation and numerical simulation on the dynamics of the circuit are presented in section 3. Finally, in section 4, we summarize and conclude the results and indicate further direction.

Circuit description and Simulation results

The fourth-order hyperchaotic Canonical Van der Pol Duffing oscillator circuit we have studied is presented in Fig. 1. It consists of two active elements, one linear negative conductance (G_1) using current feedback op-amp and one smooth cubic nonlinearity with an odd symmetric piecewise-linear *v-i*

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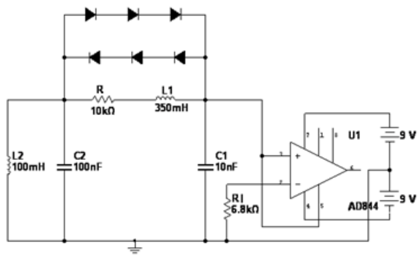
characteristic [10]. This fourth-order circuit is based on a third-order autonomous piecewise-linear circuit introduced by Chua and Lin, capable to realize every member of the canonical Chua's circuit family [9]. Applying Kirchoff's laws, the set of four first-order coupled autonomous differential equations as given below:

Fig. 1 Circuit realization of the fourth-order hyperchaotic Canonical Van der Pol Duffing oscillator circuit



$$C_1 \frac{dV_1}{dt} = G_1 V_1 - i_{L_1} - f(V_1 - V_2)$$
$$C_2 \frac{dV_2}{dt} = i_{L_1} - i_{L_2} + f(V_1 - V_2)$$
$$L_1 \frac{di_{L_1}}{dt} = V_1 - V_2 - i_{L_1} R$$
$$L_2 \frac{di_{L_2}}{dt} = V_2 \qquad (1)$$

While V_1 and V_2 are the voltages across the Capacitors C_1 and C_2, i_{L1} and i_{L2} denotes the currents through the inductances L_1 and L_2 respectively, the term $f(V_1\text{-}V_2)$ representing the characteristic of the smooth cubic nonlinearity can be expressed mathematically:

$$f(V_1 - V_2) = a(V_1 - V_2) + b(V_1 - V_2)^3 \qquad (2)$$

For our present experimental study we have chosen the following typical values of the circuit in Fig. 1. Were L_1 = 350*mH*, L_2 = 100*mH*, C_1 = 10*nF*, C_2 = 100*nF* and G_1 = -0.14706*mS*. Here the variable resistor '*R*' is assumed to be the control parameter. By decreasing the value of '*R*' from 14,000Ω ≥ 9,000Ω, the circuit behavior of Fig. 1 is found to transit from a period-doubling route to chaos and then to hyperchaotic attractor through border collision bifurcation behavior followed by period-doubling windows and boundary crisis [14-17], etc. The hyperchaotic attractors of fourth-order autonomous circuit with the smooth cubic nonlinearity projected onto different planes are shown in Fig. 2. Experimental time series were registered using a simulation storage oscilloscope for discrete values of C_1 and C_2 are shown if Fig. 3.

Fig. 2 Simulation results of the projections of hyperchaotic attractor onto different planes

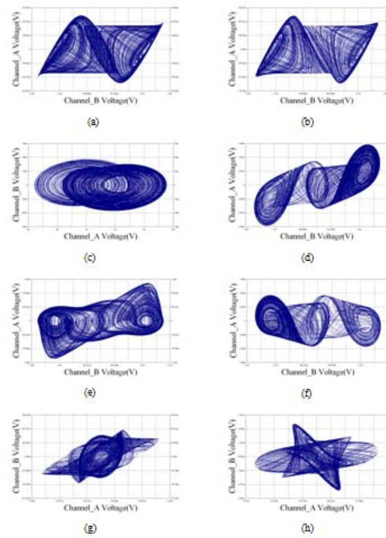


Fig. 3 Simulation results of the hyperchaotic time series

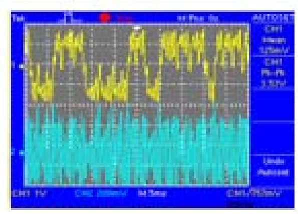
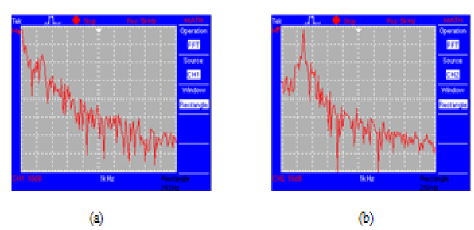


Fig. 4 Simulation results of the projections of hyperchaotic Lyapunov exponent spectra



The distribution of power in a signal *x(t)* is the most commonly quantified by means of the power density spectrum or simply power spectrum. It is the magnitude-square of the Fourier transforms of the signal *x(t)*. It can detect the presence of hyperchaos when the spectrum is broad-banded. The power spectrum corresponding to the voltages $V_1(t)$ and $V_2(t)$ waveforms across the capacitors C_1 and C_2 for the hyperchaotic regimes are shown in Fig. 4 which resembles broad-band spectrum noise.

Numerical confirmation

The hyperchaotic dynamics of circuit as shown in Fig. 1 is studied by numerical integration of the normalized differential equations [15]. For a convenient numerical analysis of the experimental system given by Eq. (1), we rescale the parameters as

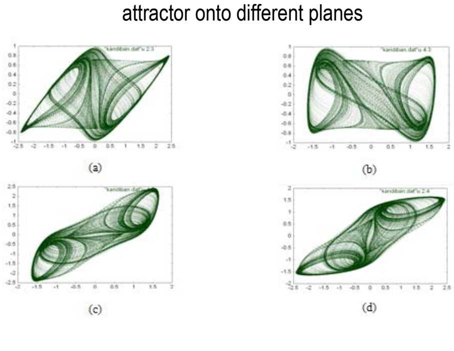
$$x_1 = \sqrt{bR}\,V_1, x_2 = \sqrt{bR}\,V_2, x_3 = \sqrt{bR^3}\,i_{L_1}, x_4 = \sqrt{bR^3}\,i_{L_2}, \tau = \frac{t}{RC_2}, \beta_1 = \frac{C_2R^2}{L_1}, \beta_2 = \frac{C_2R^2}{L_2}, \alpha = aR, \gamma = G_1R, \upsilon_1 = \frac{C_2}{C_1}, \upsilon_2 = \frac{C_2}{C_2}$$

and then redefine τ as t. Eqs. (1) and (2) reduce to the following set of normalized equations of the fourth-order Canonical Van der Pol Duffing oscillator circuit as given below:

$$\dot{x}_1 = \upsilon_1(\gamma x_1 - x_3 - \alpha(x_1 - x_2) - (x_1 - x_2)^3)$$
$$\dot{x}_2 = \upsilon_2(x_3 - x_4 + \alpha(x_1 - x_2) + (x_1 - x_2)^3)$$
$$\dot{x}_3 = \beta_1(x_1 - x_2 - x_3)$$
$$\dot{x}_4 = \beta_2(x_2) \qquad (3)$$

The dynamics of Eq. (3) now depends upon the parameters υ_1, υ_2, γ, α, β_1 and β_2. The experimental results have been verified by numerical simulation of the normalized Eq. (3) using the standard fourth-order Runge-Kutta method for a specific choice of system parameters employed in the experimental simulation results. That is, in the actual experimental set up the resistor '*R*' is varied from R = 14,000Ω ≥ 9,000Ω. Therefore in the numerical simulation, we study the corresponding Eq. (3) for in the range R = 14,000Ω ≥ 9,000Ω. From our numerical investigations, we find that for the value of '*R*' above 14,000Ω periodic limit cycle motion is obtained. When the value of '*R*' is decreased to lower than 14,000Ω particularly in the range R = (14,000Ω ≥ 9,000Ω) the system displays a period-doubling route to chaos and then to hyperchaos through boundary condition [8]. These numerical results of the hyperchaotic attractor of fourth-order autonomous circuit with the smooth cubic nonlinearity projected onto different planes are shown in Fig. 5. It is gratifying to note that the numerical results agree qualitatively very well with that of the experimental simulation results.

Fig. 5 Numerical results of the projections of hyperchaotic attractor onto different planes



Conclusions

We have presented a simple-4D hyperchaotic Canonical Van der Pol Duffing oscillator circuit which has symmetrical piecewise-linear elements. We can confirm hyperchaotic attractor on computer simulation or circuit experiments. The attractive feature of this circuit is the presence of hyperchaotic attractor over a range of parameter values, which might be useful for applications in controlling of hyperchaos, synchronization and in secure communication system.

References

1. Kyprianidis I.M., Stouboulos I.N., Haralabidis P. and Bountis T., *Int J Bifurc Chaos*., 8,1903-1915, (2000).
2. Chua L.O. and Lin G.N., *IEEE Trans Circ Syst.,* CAS-38, 510-520, (1991).
3. Kennedy M.P., *IEEE Trans Circ Syst I*., 40, 640-656, (1993).
4. Murali K. and Lakshmanan M., *Int J Bifurc Chaos.,* 1, 369-384, (1991).
5. Motsumoto T., Chua L.O. and Kobaiashi K., *IEEE Trans Circ Syst.,* CAS-33, 1143-1147, (1986).
6. Tamasevicius A., Namajunas A. and Cenys A., *Electron Lett.,* 32, 957-958, (1996).
7. Thamilmaran K., Lakshmanan M. and Venkatesan A., *Int J Bifurc Chaos.,* 14, 221-243, (2004).
8. Barboza R., *Int J Bifurc Chaos.,*18, 1151-1159, (2008).
9. Chua L.O. and Lin G.N., *IEEE Trans Circ Syst.,* 37, 885-902, (1990).
10. Balachandran V. and Kandiban G., *Indian J Pure & Appl Phys.,* 47, 823-827, (2009).
11. Tamasevicius A., Cenys A., Mykolaitis G., Namajunas A., and Lindberg E.,*Electron Lett.,* 33, 542-544, (1997).
12. Tamasevicius A., and Cenys A., *Chaos Solit Fract.,* 9,115-119, (1998).
13. Koliopanos C.L., Kyprianidis I.M., Stouboulos I.N., Anagnostopoulos A.N., and Magafas L., *Chaos Solit Fract*., 16,173-182, (2003).
14. Liu X., Wang J., and Huang L., *Int J Bifurc Chaos.,* 17, 2705-2722, (2007).
15. Barboza R., and Chua L.O., *Int J Bifurc Chaos*., 18, 943-955, (2008).
16. Kapitaniak T., Chua L.O., *Int J Bifurc Chaos*., 4, 477-482, (1994).
17. King G.P., Gaito S.T., *Phys Rev A.,* 46, 3093-3099, (1992).