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MATHEMATICS

A NOTE ON CONVERGENCE OF WILKINSON MATRIX WITH PROJECTION METHOD

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Abstract

An attempt has made in this paper to describe an efficient method for solving a system of linear equations with non-zero rows, whose coefficient matrix is almost singular or inconsistent. The equation systems containing 4 by 4 Wilkinson matrix as coefficient matrix has been experimented in order to estimate the potential of projection method .The work has been carried out in FORTRAN on new computational facilities. All calculations have been done in double precision. It is found that this method is very efficient, fast converging and less time consuming

Keywords: 4 by 4 Wilkinson matrix, Projection method

Introduction

In order to obtain the solution of the system of linear equation

$BX = U$ (1.1)

where B is 4 by 4 Wilkinson Matrix, X and U are column matrices of 4 by 1, the various methods for solving system of linear equations get failed or give solutions which are not good. It is investigated that various factorization methods are failed to solve this problem. And, when this problem is solved by Iterative methods, they do not give good solutions[1-5].

The projection method devised by *Kaczmarz* can be proved to converge for any system of linear equations with non-zero rows, even when it is singular and inconsistent and the arithmetic operations required in an iteration of the method are comparatively low. This method works well for both singular and non-singular systems and it determines the affine space formed by the solutions, if they exist. The method also provides an iterative procedure for computing a generalized inverse of matrix[1-5]. On application of the projection method, the solutions of the problem (1.1) is found very good.

Method and Algorithm

A finite form of the method of Kaczmarz is as follows-

We consider the solution of

$BX = U$ (2.1)

under the assumption that B is non-singular. The procedure consists in first converting the system of BX = U into an equivalent system

$AX = V$ (2.2)

$A^* = A^{-1}$

such that (2.1) and (2.2) have the same solution vector "X". Then, using an arbitrary initial vector r_0, we define-

$r_j = r_{j-1} - [<\alpha_j , r_{j-1}> - V_j]\ \alpha_j$, $1 \le j \le n$ (2.3)

where $A = (a_{ij})$

$\alpha_j = [a'_{j1}, a'_{j2}, \ldots , a'_{jn}]^T$

Then, $Ar_n = V$ and $Br_n = U$ (2.4)

Here, r_n is the required solution vector and the sequences $\{\alpha_j\}$, $\{V_j\}$ and $\{r_j\}$ can progress together, so that method can also be called as"n-step method".

Construction of [A:V] by using given matrix [B:U]:

First row of [A:V] = first row of [B:U] / Norm

$= (\beta_{11},\beta_{12},\ldots,\beta_{1n},\beta_{1n+1})/\sqrt{(\beta_{11}{}^2+ \beta_{12}{}^2+\ldots+\beta_{1n}{}^2)}$

Prior to normalizing, the second row of [A:V] = second row of [B:U] - $<\alpha_1 , \beta_2>$ first row of [A:V]

$= (\beta'_{21},\beta'_{22},\ldots,\beta'_{2n},\beta'_{2n+1})$ (say)

where, $\alpha_i = [a_{i1}, a_{i2}, \ldots , a_{in}]^T$

$\beta_i = [b_{i1}, b_{i2}, \ldots , b_{in}]^T$

Second row of [A :V] = $(\beta'_{21},\beta'_{22},\ldots,\beta'_{2n},\beta'_{2n+1})/\sqrt{(\beta'_{21}{}^2+ \beta'_{22}{}^2+\ldots+\beta'_{2n}{}^2)}$

Prior to normalizing, the third row of [A:V] = third row of [B:U] - $<\alpha_1 , \beta_3>$ first row of [A:V] - $<\alpha_2 , \beta_3>$ second row of [A:V]

$= (\beta'_{31},\beta'_{32},\ldots,\beta'_{3n},\beta'_{3n+1})$ (say)

Third row of [A :V] = $(\beta'_{31},\beta'_{32},\ldots,\beta'_{3n},\beta'_{3n+1})/\sqrt{(\beta'_{31}{}^2+ \beta'_{32}{}^2+\ldots+\beta'_{3n}{}^2)}$

Continuing this procedure, we can construct [A:V].

Algorithm

Given a non-singular matrix B, the following algorithm solves the linear system BX = U.

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(i) set: initial solution vector r(0) = 0.0
(ii) construct [A:V] by using [B:U]
(iii) for j = 1, n
r(j) = r(j –1) – [<a(j), r(j –1)> – V(j)] a(j –1)
(end of j loop)
(iv) r(n) will be required solution vector.

Results and Discussion

The following 4 by 4 Wilkinson matrix is deceptively simple, as it is already in lower triangular form. However, it posses severe computational problems.

Wilkinson matrix B =

0.9143E-4	0	0	0
0.8762	0.7156E-4	0	0
0.7943	0.8143	0.9504E-4	0
0.8017	0.6123	0.7165	0.7123E-4

Taking U = [0.00009143 0.87627156 1.60869504 2.13057123 $]^T$

This makes true solution of BX = U, the vector $[1,1,1,1]^T$. When this problem is solved by Direct methods, it is observe that factorization methods do not give a good result. This is due to, in the time of triangulization the diagonal element of last row tends to zero. Therefore one of the factor goes to singular. Thus, perhaps all factorization methods are failed to solve this problem. On the other hand, when we try to solve this problem by Iterative methods, it has been observed that most of them converge. But the solutions are not good.

For this matrix, we have-

$$||B|| = 2.13$$

$$||B^{-1}|| = 1.15 \times 10^{16}$$

$$||r|| = 1.46 \times 10^{-14}$$

$$||X - X'|| = 0.00177$$

$$(||X - X'||)/||X|| = 0.00177$$

Conclusion

Projection method is a fast converging, less time consuming and robust for solving a system of linear equations with non-zero rows, whose coefficient matrix is almost singular or inconsistent. This method gives very good result and advantageous in terms of guaranteed convergence, i.e., if given system is n by n, then at n^{th} step, it converges to the result.

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