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REFORMULATION ALGORITHM A CONVEX/CONCAVE ENVELOPS FOR LOGIC BASED OPTIMIZATION

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Abstract

This paper treats convex/concave nonlinear envelops for odd power terms of the form x^{2k+1}, $(k \in N)$ where $x \in [a,b]$ and $a<0<b$. These envelopes are continuous and differentiable everywhere in $[a,b]$. A novel automatic reformulation method for non convex NLPs that include linear constraints and bilinear terms we also drive convex relaxations we compare both of these relaxations with relaxations for the same terms derived using other methods. We shall with a slight about of notation, Speak about "envelop" to mean the region enclosed between the convex and concave envelopes.

Keywords: Global optimization, Odd degree, Monomial

Introduction

Our work has considered the derivation of convex relaxations for monomial terms of odd degree when the variables range includes zero. The main innovation of the $\alpha\beta\beta$ algorithm is in the underestimations of a general non convex function.

One of the most effective techniques for the solution of nonlinear Programming problems (NLPs) to global optimality is the spatial Branch-and-Bound (SBB) method. This requires the computation of a lower bound to the solution, usually obtained by solving a convex relaxation of the original NLP. The formation and tightness of such a convex relaxation are critical issues in any SBB implementation.

Tight convex under estimators are already available for many types of non convex term, including bilinear and trilinear products, linear fractional terms, and concave and convex univariate functions. However, terms which are piecewise concave and convex are not explicitly catered for a frequently occurring example of such a term is x^{2k+1}, where $(k \in N)$ and the range of x includes zero. A detailed analysis of the conditions required for concavity and convexity of polynomial functions has been given in [1]; however, the results obtained therein only apply to the convex underestimation of multivariate polynomials with positive variable values. For monomials of odd degree, where the variable ranges over both negative and positive values, no special convex envelopes have been proposed in the literature, and one therefore has to rely either on generic convex relaxations such as those given by Floudas and co-workers {Sec [2,3]}. on reformulation in terms of other types of terms for which convex relaxations are available.

Convex Relaxations

A relaxation cannot be used to solve a difficult problem directly because the solution of the original problem cannot, in general, be directly inferred from the solution of the solution.

Relaxations are however, very important in the field of deterministic global optimization. One of the most important tools in this field is the Branch and Bound algorithm, which uses a convex (or linear) relaxation at each step to calculate the Lower bound in a region. Convex relaxations for non convex problems are obtained by substituting the (non convex) objective function f(x) with a convex relaxation $\underline{f}(x)$ and the (non-convex) feasible region Ω with a convex set $\overline{\Omega}$ such that $\Omega \leq \overline{\Omega}$

Statement of the problem

In [1], the generation of convex envelopes for general univariate functions was discussed. Here we consider the monomial x^{2k+1} in the range $x \in [a,b]$ where $a < 0 < b$. Let c, d be the x-coordinates of the points C, D where the tangents from points A and B respectively meet the curve (see Figure 1). The shape of the convex under estimator of x^{2k+1} depends on the relative magnitude of b

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and c. In particular, if $c<b$ (as is the case in Figure 1), a convex under estimator can be formed from the tangent from $x=a$ to $x=c$ followed by the curve x^{2k+1} from $x=c$ to $x=b$. On the other hand, if $c>b$ (see Figure 2), a convex under estimator is simply the straight line passing through A and B.

The situation is similar for the concave over estimator of x^{2k+1} in the range $x\in[a,b]$. If $d>a$, the over estimator is given by the upper tangent from B to D followed by the curve x^{2k+1} from D to A, as shown in Figure 1. On the other hand, if $d<a$, the over estimator is just the straight line from A and B. It should be noted that the conditions $c>b$ and $d<a$ cannot both hold simultaneously.

Figure 1. Tightest (nonlinear) convex envelope of x^{2k+1}

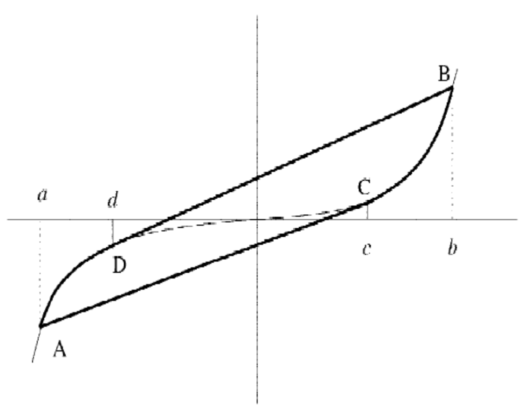
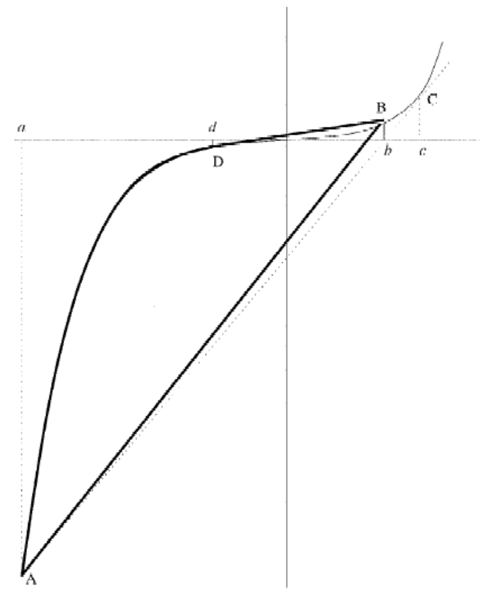


Figure 2. The case.When $c>b$



The tangent equations

The discussion in Section 1.1 indicates that forming the convex envelope of x^{2k+1} requires the determination of the tangents that pass through points A,C and B,D.

Considering the first of these two tangents and equating the slope of the line AC to the gradient of x^{2k+1} at $x=c$, we derive the tangency condition:

$$\frac{c^{2k+1}-a^{2k+1}}{c-a}=(2k+1)c^{2k}$$

(1.1)

Hence c is a root of the polynomial:

$$p^{k}(x,a)=(2k)x^{2k+1}-a(2k+1)x^{2k}+a^{2k+1}$$

(1.2)

It can be shown by induction on k that:

$$p^{k}(x,a)=a^{2k-1}(x-a)^{2}Q^{k}(\frac{x}{a})$$

(1.3)

where the polynomial $Q^k(x)$ is defined as:

$$Q^k(x) = 1 + \sum_{i=1}^{2k} ix^{i-1}$$

(1.4)

Thus, the roots of $p^k(x,a)$ can be obtained from the roots of $Q^k(x)$).

Unfortunately, polynomials of degree greater than 4 cannot generally be solved by radicals (what is usually called an "analytic solution"). This is the case for $Q^k(x)$ for $k > 2$.

For example, the Galois group of

$$Q^3(x) = 6x^5 + 5x^4 + 4x^3 + 3x^2 + 2x + 1$$

is isomorphic to s_5 (i.e., the symmetric group of order 5) which is not solvable since its biggest proper normal subgroup is A5, the smallest non-solvable group. For details on Galois theory and the solvability of polynomials [see 4].

Nonlinear convex envelope

If r^k is the real roots of $Q^k(x)$ for any $k \geq 1$ and this lies $[-1+\frac{1}{2k}, -0.5]$, then the tangent points c and d in Figure 1.1 are simply $c = r_k a$ and $d = r_k b$. The lower and upper tangent lines are given respectively by:

$$a^{2k+1} + \frac{c^{2K+1} - a^{2K+1}}{c-a}(x-a)$$

(1.5)

$$b^{2k+1} + \frac{d^{2K+1} - b^{2K+1}}{d-b}(x-b)$$

(1.6)

Hence, the convex/concave envelope for $z = x^{2k+1}$ when $x \in [a,b]$ and $a < 0 < b$:

$$l_k(x) \leq Z \leq U_k(x)$$

(1.7) is as follows:

If $c < b$, then:

$$l_k(x) = \begin{cases} a^{2k+1\{1+R_k(\frac{x}{a}-1)\}} if x < c \\ x^{2k+1} if x \geq c \end{cases}$$

(1.8)

Otherwise

$$l_k(x) = a^{2k+1} + \frac{b^{2k+1} - a^{2k+1}}{b-a}(x-a)$$

(1.9)

If d > a, then:

$$U_k(x) = \begin{cases} x^{2k+1} if x \leq d \\ b^{2k+1\{1+R_k(\frac{x}{b}-1)\}} if x > d \end{cases}$$

(1.10)

otherwise:

$$U_k(x) = a^{2k+1} + \frac{b^{2k+1} - a^{2k+1}}{b-a}(x-a)$$

(1.11)

where we have used the constant

$$R_k = \frac{r_k^{2k+1} - 1}{r_k - 1}$$

If the range of x is unbounded either below or above we take the limits of $l_k(x)$) and $U_k(x)$ as $a \to \infty$ or $b \to \infty$.

By construction, the above convex under estimators and over estimators of x^{2k+1} are continuous and differentiable everywhere.

Conclusion

We have proposed a convex nonlinear envelope for Logic Based Optimization. it is clear from the review present in this paper that convex envelope for monomials terms of odd degree when the range of the defining variable includes 0. i.e. when they are piecewise convex and concave thus convex under estimators and over estimators of x^{2k+1} are continuous and differentiable everywhere.Thus many engineering optimization problem can be formulated as non convex non linear programming problems.

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