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MATHEMATICS

A STOCHASTIC MODEL FOR THE EXPECTED TIME TO RECRUITMENT IN A SINGLE GRADED MANPOWER SYSTEM WITH TWO THRESHOLDS USING BIVARIATE POLICY

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Abstract
In this paper, an organization subjected to random exit of personnel due to policy decisions taken by the organization is considered. There is an associated loss of manpower if a person quits. As the exit of personnel is unpredictable, a new recruitment policy involving two thresholds - one is optional and the other one mandatory is suggested to enable the organization to plan its decision on recruitment. Based on shock model approach, a mathematical model is constructed using an appropriate bivariate policy of recruitment. The analytical expression for the mean and variance of the time to recruitment are obtained when i) the loss of manhours process forms a sequence of independent and identically distributed continuous random variables ii) the inter-decision times are independent and identically distributed continuous random variables and iii) the optional threshold level as well as the mandatory threshold level are also continuous random variables. The results are numerically illustrated and analyzed by assuming specific distributions.

Keywords: Manpower planning, Bivariate recruitment policy, Shock models, Mean and variance of the time to recruitment

Introduction

Exits of personnel which is in other words known as wastage is an important aspect in the manpower planning. Many models have been discussed using different types of distributions. Such models could be seen in Grinold and Marshall(1977) and Barthlomew and Forbes (1979). In Elangovan and Sathiyamoorthi(1998) the expected time to recruitment is obtained when the inter-decision times are independent and identically distributed random variables. In Saavithri and Srinivasan(2001) the mean time to recruitment is obtained on some univairiate polices. In Venkatesh and Srinivasan(2007a,2007b) the mean time to recruitment is obtained using univariate max policy of recruitment under different conditions on the threshold. In all the above cited works, the problem of time to recruitment in a single graded marketing organization involves only one threshold value. Since the number of exits in a policy decision making epoch is unpredictable and the time at which the cumulative loss of man hours crossing a single threshold is probabilistic, the organization has left with no choice except making recruitment immediately upon the threshold crossing or the total number of decisions crossing its corresponding threshold whichever is earlier. In this paper, this limitation is removed by considering the following new bivariate recruitment policy involving two thresholds in which one is optional and the other is mandatory for cumulative loss of manhours and a threshold for total number of decisions. If the cumulative loss of manpower crosses the optional threshold, the organization may or may not go for recruitment. However, recruitment is necessary whenever the cumulative loss of manpower crosses the mandatory threshold or the total number of decisions crossing its corresponding threshold whichever is earlier. In view of this policy, the organization can plan its decision upon the time for recruitment. Recently in Esther Clara and Srinivasan (2008), using new univariate policy of recruitment for a single graded system involving optional and mandatory exponential thresholds, the mean and variance of the time to recruitment are obtained when the inter-decision times are independent and identically distributed random variables. In Esther Clara and Srinivasan(2009a) the corresponding results are obtained when the threshold distributions have SCBZ property. In Esther Clara and Srinivasan (2009b) the mean time to recruitment is obtained when i) the inter decision times are exchangeable constantly correlated exponential random variables and ii) one threshold has SCBZ property and the other follows exponential distribution and vice versa. The objective of the present paper is to obtain the expected time to recruitment using new bivariate policy of recruitment. This paper is organized as follows. In section 2, model description is given and an analytical expression for the mean time to recruitment is derived. In section 3, the main results

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are numerically illustrated by assuming specific distributions and relevant conclusions are made.

Model description

Consider an organization taking decisions at random epochs in (0, ∞) and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. Let X_i be the loss of manpower due to the i[th] decision epoch, i=1,2,3,… forming a sequence of independent and identically distributed random variables. It is assumed that the inter-decision times are independent and identically distributed exponential random variables with parameter λ having probability density function (distribution function) $f(.)(F(.))$. Let $f_k(.)(F_k(.))$ be k fold convolution of $f(.)(F(.))$. Let $f^*(.)$ be the Laplace transform of $f(.)$. The loss of manpower process and the process of inter-decision times are assumed to be statistically independent. Let $Y(Z)$ be a positive continuous random variable denoting the optional (mandatory) threshold following exponential distribution with parameter $\mu_1(\mu_2)$ such that $Z > Y$. It is assumed that Y, Z and X_i , i=1,2,3,… are independent. Let A be the positive constant threshold for the total number of decisions. The bivariate recruitment policy employed in this paper is, *Recruitment is necessary whenever the total number of decisions exceeds the corresponding threshold or the total loss of manhours exceeds the mandatory threshold level whichever is earlier. The organization may or maynot go for recruitment whenever the optional threshold is crossed.* Let p be the probability that the organization is not going for recruitment whenever the total loss of manpower crosses the optional threshold level Y . Let W be a continuous random variable denoting the time for recruitment in the organization with probability density function $l(.)$ and cumulative distribution function $L(.)$. From Renewal theory, Karlin and Taylor (1981), we note that $V_k(t) = F_k(t) - F_{k+1}(t)$ where $F_0(t) = 1$. Let $E(W)$ be the expected time for recruitment and $V(W)$ be the variance of the time for recruitment.

Main Results

The survivor function of W is given by

$P[W > t] = \sum_{k=0}^{A} [$ Probability that exactly k decisions are taken in, k=0,1,2,…×(Probability that the total number of exits in these k-decisions does not cross the optional level Y or the total number of exits in these k-decisions crosses the optional level Y but lies below the mandatory level Z and the organization is not making recruitment)$]$

$$P[W > t] = \sum_{k=0}^{A} V_k(t) P\left[\sum_{i=1}^{k} X_i < Y\right] + \sum_{k=0}^{A} V_k(t) P\left[\sum_{i=1}^{k} X_i \geq Y\right] \times P\left[\sum_{i=1}^{k} X_i < Z\right] \times p \tag{1}$$

Conditioning upon X_i and using the law of total probability, it can be shown that

$P\left[\sum_{i=0}^{k} X_i < Y\right]$ = Probability that the system does not fail, after k epochs of exits

$$= \int_0^\infty e^{-\mu_1 x} g_k(x) dx = \left[g^*(\mu_1)\right]^k \tag{2}$$

$$\text{Similarly, } P\left[\sum_{i=0}^{k} X_i < Z\right] = \left[g^*(\mu_2)\right]^k \tag{3}$$

Using (2) and (3) in (1) and on simplification

$$P[W > t] = 1 + \left[g^*(\mu_1) - 1\right] \sum_{k=1}^{A-1} F_k(t) \left[g^*(\mu_1)\right]^{k-1} + p \times \left[g^*(\mu_2) - 1\right] \sum_{k=1}^{A-1} F_k(t) \left[g^*(\mu_2)\right]^{k-1}$$

$$- p \times \left[g^*(\mu_1) g^*(\mu_2) - 1\right] \sum_{k=1}^{A-1} F_k(t) \left[g^*(\mu_1) g^*(\mu_2)\right]^{k-1} \tag{4}$$

Since $L(t) = 1 - P[W > t]$, from (4)

$$L(t) = \left[1 - g^*(\mu_1)\right] \sum_{k=1}^{A-1} F_k(t) \left[g^*(\mu_1)\right]^{k-1} + p \times \left[1 - g^*(\mu_2)\right] \sum_{k=1}^{A-1} F_k(t) \left[g^*(\mu_2)\right]^{k-1}$$

$$- p \times \left[1 - g^*(\mu_1) g^*(\mu_2)\right] \sum_{k=1}^{A-1} F_k(t) \left[g^*(\mu_1) g^*(\mu_2)\right]^{k-1}$$

$$\therefore l(t) =$$

$$\left[1 - g^*(\mu_1)\right] \sum_{k=1}^{A-1} f_k(t) \left[g^*(\mu_1)\right]^{k-1} + p \times \left[1 - g^*(\mu_2)\right] \sum_{k=1}^{A-1} f_k(t) \left[g^*(\mu_2)\right]^{k-1}$$

$$- p \times \left[1 - g^*(\mu_1) g^*(\mu_2)\right] \sum_{k=1}^{A-1} f_k(t) \left[g^*(\mu_1) g^*(\mu_2)\right]^{k-1}$$

and

$$l^*(s) = \left[1 - g^*(\mu_1)\right] f^*(s) \left\{\frac{1 - \left[f^*(s) g^*(\mu_1)\right]^{A-1}}{1 - f^*(s) g^*(\mu_1)}\right\} + p\left[1 - g^*(\mu_2)\right] f^*(s) \left\{\frac{1 - \left[f^*(s) g^*(\mu_2)\right]^{A-1}}{1 - f^*(s) g^*(\mu_2)}\right\}$$

$$-p\left[1-g^*(\mu_1)g^*(\mu_2)\right]f^*(s)\left\{\frac{1-\left[f^*(s)g^*(\mu_1)g^*(\mu_2)\right]^{A-1}}{1-f^*(s)g^*(\mu_1)g^*(\mu_2)}\right\} \quad (5)$$

Since $f^*(s)=\lambda/\lambda+s$ (6)

It is known that

$$E(W)=\left[\frac{-d}{ds}l^*(s)\right]_{s=0} \quad (7)$$

$$E(W^2)=\left[\frac{d^2}{ds^2}l^*(s)\right]_{s=0} \quad (8)$$

$$V(W)=E(W^2)-[E(W)]^2 \quad (9)$$

Using (5) and (6) in (7) and (8), we get

$$E(W)=\left\{\frac{1-A\left[g^*(\mu_1)\right]^{A-1}+(A-1)\left[g^*(\mu_1)\right]^A}{\lambda\left[1-g^*(\mu_1)\right]}\right\}+p\left\{\frac{1-A\left[g^*(\mu_2)\right]^{A-1}+(A-1)\left[g^*(\mu_2)\right]^A}{\lambda\left[1-g^*(\mu_2)\right]}\right\}$$

$$-p\left\{\frac{1-A\left[g^*(\mu_1)g^*(\mu_2)\right]^{A-1}+(A-1)\left[g^*(\mu_1)g^*(\mu_2)\right]^A}{\lambda\left[1-g^*(\mu_1)g^*(\mu_2)\right]}\right\} \quad (10)$$

$$E(W^2)=$$

$$\frac{2\left(1-\left[g^*(\mu_1)\right]^{A-1}\right)}{\lambda^2\left[g^*(\mu_1)\right]^2}-\frac{(A-1)\left[g^*(\mu_1)\right]^{A-1}\left\{1+\lambda^A+A\left[1-g^*(\mu_1)\right]\right\}}{\lambda^2\left[g^*(\mu_1)\right]}$$

$$+p\left[\frac{2\left(1-\left[g^*(\mu_2)\right]^{A-1}\right)}{\lambda^2\left[g^*(\mu_2)\right]^2}-\frac{(A-1)\left[g^*(\mu_2)\right]^{A-1}\left\{1+\lambda^A+A\left[1-g^*(\mu_2)\right]\right\}}{\lambda^2\left[g^*(\mu_2)\right]}\right]$$

$$-p\left[\begin{array}{l}\dfrac{2\left(1-\left[g^*(\mu_1)g^*(\mu_2)\right]^{A-1}\right)}{\lambda^2\left[g^*(\mu_1)g^*(\mu_2)\right]^2}\\ -\dfrac{(A-1)\left[g^*(\mu_1)g^*(\mu_2)\right]^{A-1}\left\{1+\lambda^A+A\left[1-g^*(\mu_1)g^*(\mu_2)\right]\right\}}{\lambda^2\left[g^*(\mu_1)g^*(\mu_2)\right]}\end{array}\right]$$

(11)

Assume that X_i follow exponential distribution with parameter β then, $g^*(\mu)=\beta/\beta+\mu$ (12)

Using (12) in (10) and (11) we get the simplified mean and variance of the time to recruitment.

When p decreases to zero, our results agree with the results for a single graded manpower system by Bhuvaneshwari and Srinivasan (2007) with mandatory threshold only.

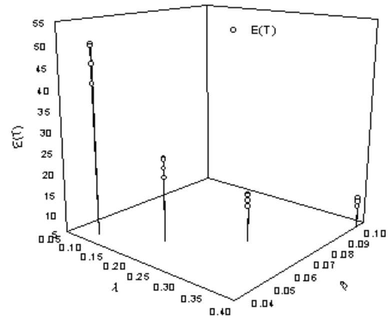
Numerical Illustrations and Conclusions

The following table gives the mean and variance of the time to recruitment μ_1=0.0133, μ_2=0.02, p=0.5 and A=15 are fixed varying λ and β simultaneously.

Table 1

λ/β		0.0400	0.0500	0.0667	0.1
0.1	E(T)	41.5459	45.9555	50.0136	50.4968
	V(T)	3.2120×10^{17}	4.5304×10^{17}	7.2301×10^{17}	1.4514×10^{18}
0.2	E(T)	20.7730	22.9777	25.0068	25.2584
	V(T)	4.9012×10^{12}	6.9128×10^{12}	1.1032×10^{13}	2.2147×10^{13}
0.3	E(T)	13.8486	15.3185	16.6712	16.8323
	V(T)	7.4618×10^{9}	1.0524×10^{10}	1.6796×10^{10}	3.3717×10^{10}
0.4	E(T)	10.3865	11.4889	12.5034	12.6242
	V(T)	7.4786×10^{7}	1.0548×10^{8}	1.6834×10^{8}	3.3793×10^{8}

Figure 1: Mean time to recruitment graph for table 1



From table 1, we observe the following:

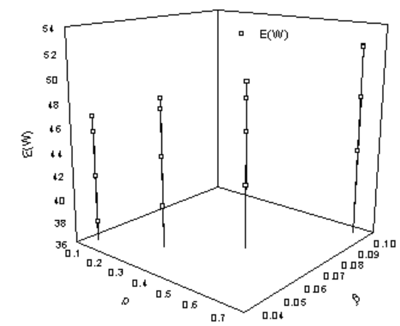
1. When the average inter-decision times (λ) decreases as well as the average loss of manhours (β) decreases simultaneously the mean and variance of the time to recruitment decrease.
2. As λ alone increases, mean and variance of the time to recruitment decreases and when β alone increases the mean and variance of the time to recruitment increases. The same conclusions can be made from the figure 1.

The following table gives the mean and variance of the time to recruitment μ_1=0.0133, μ_2=0.02, λ=0.5 and A=15 are fixed varying p and β simultaneously.

Table 2

p/β		0.0400	0.0500	0.0667	0.1
0.1	E(W)	37.7799	41.7425	45.5604	46.8868
	V(W)	3.2120×10^{17}	4.5304×10^{17}	7.2301×10^{17}	1.4514×10^{18}
0.3	E(W)	39.6629	43.8490	47.7870	48.6918
	V(W)	3.2120×10^{17}	4.5304×10^{17}	7.2301×10^{17}	1.4514×10^{18}
0.5	E(W)	41.5459	45.9555	50.0136	48.6918
	V(W)	3.2120×10^{17}	4.5304×10^{17}	7.2301×10^{17}	1.4514×10^{18}
0.7	E(W)	43.4289	48.0620	52.2403	52.3018
	V(W)	3.2120×10^{17}	4.5304×10^{17}	7.2301×10^{17}	1.4514×10^{18}

Figure 2: Mean time to recruitment graph for table 2



From table 2, we observe the following:

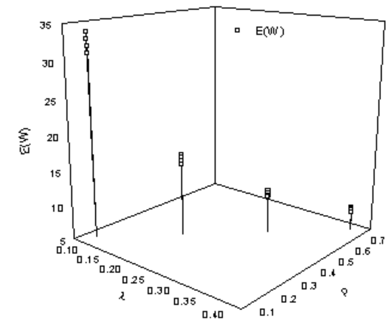
1. When the probability (p) for not going for recruitment when the optional threshold level is crossed increases as well as the average loss of manhour increases simultaneously the mean and variance of the time to recruitment increase.
2. As p alone increases, mean and variance of the time to recruitment increases and when β alone increases the mean and variance of the time to recruitment increases. The same conclusions can be made from the figure 2.

The following table gives the mean and variance of the time to recruitment μ_1=0.0133, μ_2=0.02, β=0.25 and A=15 are fixed varying λ and p simultaneously.

Table 3

λ/p		0.1	0.3	0.5	0.7
0.1	E(W)	33.9243	32.9900	32.0557	31.1214
	V(W)	7.8384×10^{18}	7.8384×10^{18}	7.8384×10^{18}	7.8384×10^{18}
0.2	E(W)	16.9621	16.4950	16.0278	15.5607
	V(W)	1.1960×10^{14}	1.1960×10^{14}	1.1960×10^{14}	1.1960×10^{14}
0.3	E(W)	11.3081	10.9967	10.6852	10.3738
	V(W)	1.8209×10^{11}	1.8209×10^{11}	1.8209×10^{11}	1.8209×10^{11}
0.4	E(W)	8.4811	8.2475	8.0139	7.7803
	V(W)	1.8250×10^{7}	1.8250×10^{7}	1.8250×10^{7}	1.8250×10^{7}

Figure 3: Mean time to recruitment graph for table 3



From table 2, we observe the following:

1. When the probability (p) for not going for recruitment when the optional threshold level is crossed increases as well as the average inter-decision time (λ) decreases simultaneously the mean and variance of the time to recruitment decrease.
2. As p alone increases, mean and variance of the time to recruitment decreases and when λ alone increases the mean and variance of the time to recruitment decreases. The same conclusions can be made from the figure 3.

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