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PHYSICS

EXPERIMENTAL AND NUMERICAL REALIZATION OF CHAOTIC INTERMITTENCY PHENOMENA IN A FOURTH-ORDER AUTONOMOUS ELECTRIC CIRCUIT

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Abstract

In this work a fourth-order autonomous electric circuit. This circuit, which is capable of realizing the behavior of every member of the Chua's family, consists of just five linear elements (resistor, inductors, and capacitors) and a smooth cubic nonlinear resistor. The route followed is a transition from regular behavior to chaos and then to intermittency phenomena through boundary condition, as the system parameter is varied. The chaotic intermittency phenomena, characterized by two positive Lyapunov exponents are described by set of four coupled first-order ordinary differential equations. This has been investigated extensively using laboratory experiments and numerical analysis.

Keywords: Fourth-order autonomous electric circuit; Smooth cubic nonlinearity; Chaotic intermittency; Lyapunov exponents

Introduction

In the present report the behavior of a fourth-order autonomous nonlinear electric circuit has been studied. The electrical circuit consists of just five linear elements (resistor, inductors and capacitors) and a cubic nonlinear resistor exhibiting symmetrical piecewise-linear *v-i* characteristics. The route is followed is a transition from regular behavior to chaos and then to intermittency route to chaos through boundary condition, as the system parameter resistor (*R*) is varied [1]. Most chaotic and bifurcation effect cited in the literature have been observed in electrical circuits. They include the period-doubling route to chaos, the intermittency route to chaos, and the quasi-periodicity route to chaos and of course the crisis [2]. This popularity is attributed to the advantages which electric circuits offer to experimental chaos studies, synchronization and chaos communication system with robustness against various interferences including multi-user access.

Experimental Realization

Fourth-order autonomous electric circuit

The fourth-order autonomous nonlinear electric circuit we have studied is presented in Fig. 1. It consists of one active element such as cubic nonlinear resistor with an odd symmetric piecewise-linear *v-i* characteristic [3]. This fourth-order circuit is based on a third-order autonomous piecewise-linear circuit introduced by Chua and Lin, capable to realize every member of Chua's circuit family. The cubic nonlinear resistor (N_R) was realized as shown in Fig. 2. The system of differential equations governing the dynamical behavior of the circuit of Fig. 1 is the following one:

$$\begin{aligned} C_1 \frac{dV_1}{dt} &= i_{L_1} \\ C_2 \frac{dV_2}{dt} &= -i_{L_2} - i_{L_1} - i_N \\ L_1 \frac{di_{L_1}}{dt} &= V_2 - V_1 - i_{L_1} R - i_N R \\ L_2 \frac{di_{L_2}}{dt} &= V_2 \end{aligned} \quad (1)$$

With the current i_N flowing through the cubic nonlinear resistor (N_R) given by the following expression:

$$i_N = f(V_1) = aV_1 + bV_1^3 \quad (2)$$

Fig. 1: Fourth-order autonomous electric circuit

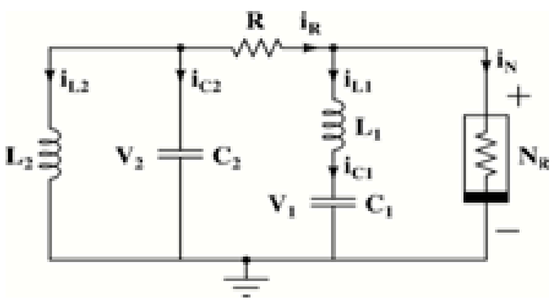

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Fig. 2: Cubic nonlinear resistor (NR)

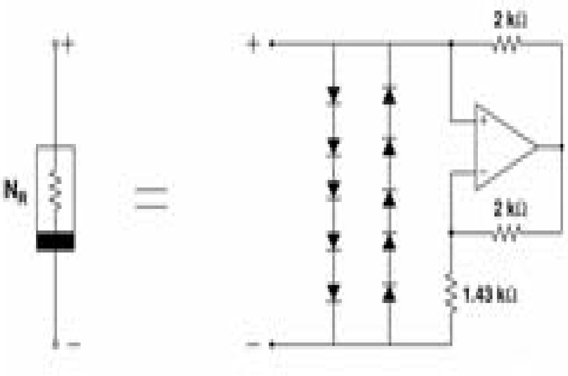
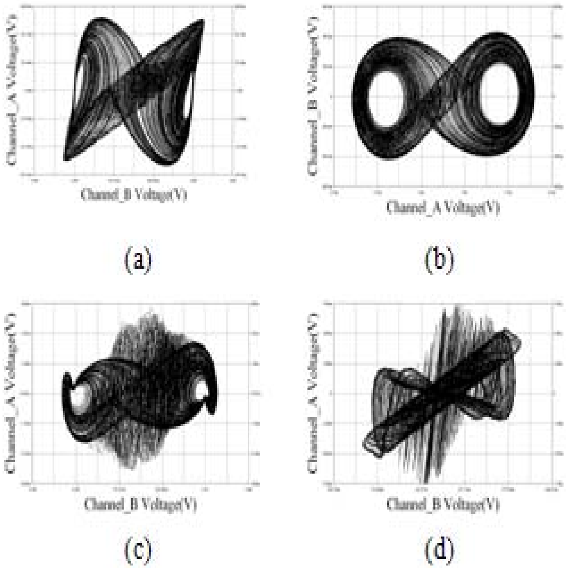


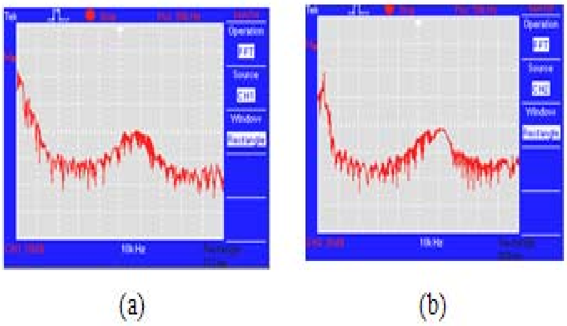
Fig. 3: Experimental realization of the intermittency chaotic attractor



For our present experimental study we have chosen the following typical values of the circuit in Fig. 1: C_1 = 10*nF*, C_2 = 100*nF*, L_1 = 31*mH* and L_2 = 20*mH*. The cubic nonlinear resistance characteristics are chosen the following typical value of the circuit as shown in Fig. 2: $a < 0$, $b > 0$ and $Bp = 1.0V$. Hence the variable resistor R is assumed to be the control parameter [4-6]. By decreasing the value of R from 3500Ω to 2000Ω, the circuit behavior of Fig. 1 is found to transit from a periodic limit cycle to chaos and then to intermittency route to chaos through boundary crisis etc.,. When the value of R is decreased be low 3500Ω, particularly in the range R = 3100Ω the system displays a intermittency route to chaos [7-9]. The projection on to different planes formed by the V_1, V_2, i_{L1} and i_{L2} axis plane of cathode ray oscilloscope are shown Fig. 3.

The distribution of power in a signal *x(t)* is the most commonly quantified by means of the power density spectrum or simply power spectrum. It is the magnitude-square of the Fourier transform of the signal *x(t)*. It can be detected the presence of intermittency route to chaos when the spectrum is broad banded, the power spectrum corresponding to the voltage $V_1(t)$ and $V_2(t)$ waveforms across the capacitors C_1 and C_2 for the intermittency regimes is shown in Fig. 4 which resembles broad-band spectrum noise.

Fig. 4: Experimental power spectrum of the signal $V_1(t)$ and $V_2(t)$



Numerical Realization

Fourth-order autonomous electric circuit

For convenient numerical analysis of the experimental system given by Eq.(1)

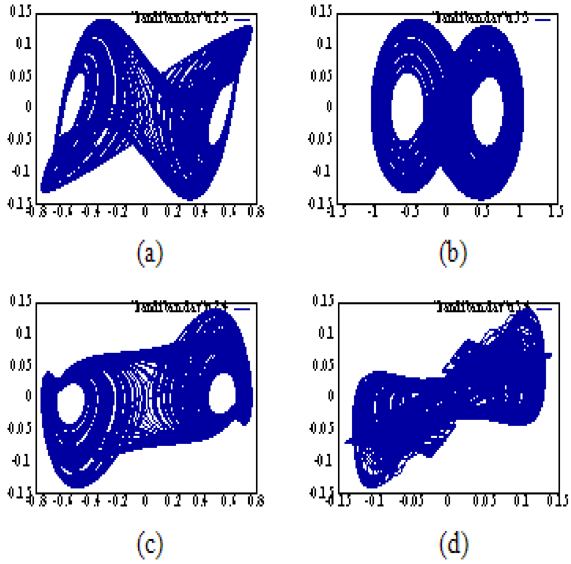
we rescale the parameter $x_1 = \sqrt{bR}\,V_1$, $x_2 = \sqrt{bR}\,V_2$, $x_3 = \sqrt{bR^3}\,i_{L_1}$ $x_4 = \sqrt{bR^3}\,i_{L_2}$, $t = \tau R C_2$, $\alpha = 1 + aR$, $\upsilon_1 = \dfrac{C_2}{C_1}$, $\upsilon_2 = \dfrac{C_2}{C_2}$, $\beta_1 = \dfrac{C_2 R^2}{L_1}$ and $\beta_2 = \dfrac{C_2 R^2}{L_2}$ and then redefine τ as T. Then the normalized equation of the fourth-order autonomous nonlinear electric circuit (Fig.1) is:

$$\begin{aligned}
\dot{x}_1 &= \upsilon_1 (x_3) \\
\dot{x}_2 &= -\upsilon_2 (x_4 + x_3 + (\alpha - 1)x_1 + x_1^{\,3}) \\
\dot{x}_3 &= \beta_1 (x_2 - x_3 - \alpha\, x_1 - x_1^{\,3}) \\
\dot{x}_4 &= \beta_2 (x_2)
\end{aligned} \tag{3}$$

Where $g(x_1) = (\alpha - 1)x_1 + x_1^{\,3}$ (4)

As seen in Eq.(3) and (4) can exhibits chaos and intermittency route to chaos due to the existence of the nonlinear term $g(x_1)$ which is a piecewise-linear function with three segments. The dynamics of Eq.(3) and (4) now depends upon the parameters υ_1, υ_2, β_1, β_2, α, $a < 0$, and $b > 0$. The experimental results have been verified by numerical simulation of the normalized Eq.(3) and (4) using standard Runge-Kutta integration routine for a specific choice of system parameters employed in the laboratory experiments. When the value of R is decreased be low, particularly in the range R = 3100Ω the system displays a intermittency route to chaotic motion. The projection of intermittency chaotic attractor on to different planes is shown in Fig.5.

Fig. 5: Numerical realization of the intermittency chaotic attractor



Conclusions

In this study, we have designed and investigated fourth-order autonomous electric circuits which have symmetrical piecewise-linear elements. We can confirm intermittency chaotic attractor on computer simulation or circuit experiments. We consider that such complex chaotic waveforms with chaotic intermittency are expected to be utilized for realization of chaos communication systems with robustness against various interferences including multi-user access because of its quick decay of correlation function.

References

1. Matsumoto T., L.O. Chua and K. Kobaiashi, *IEEE Trans. Circuits syst*., 33, 1143-1147, (1986).
2. Tamasevicius A., A. Namajunas and A. Cenys, *Electron. Lett.,* 32, 957-958, (1996).
3. Thamilmaran K., M. Lakshmanan, and A. Venkatesan, *Int. J. Bifurcation and Chaos*, 14, 221-243, (2004).
4. Balachandran V. and G. Kandiban, *Indian J. Pure and Appl. Phys*., 47, 823-827, (2009).
5. King G.P. and S.T. Gaito, *Phys. Rev. A*, 46, 3093-3099, (1992).
6. Liu X., J.Wang and L. Huang, *Int J Bifurcation & Chaos*, 17, 2705-2722, (2007).
7. Kilic R. and F.Yildirim, *Int J Bifurcation & Chaos*, 17, 607-616, (2007).
8. Barboza R., *Int J Bifurcation & Chaos,* 18, 1151-1159, (2008).
9. Barboza R. and L.O. Chua, *Int J Bifurcation & Chaos,*18,943-955,(2008).