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# A STOCHASTIC MODEL FOR MEAN TIME TO SEROCONVERSION OF HIV TRANSMISSION WITH CHANGE OF THRESHOLD UNDER CORRELATED INTERCONTACT TIMES

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## Abstract

This paper focuses on the study of a stochastic model for predicting seroconvesion time of HIV transmission with change of threshold under correlated intercontact times. The antigenic diversity threshold is an important aspect of consideration in the studies relating to HIV infection. Successive sexual contacts are the mode of transmission of HIV would result in acquiring more of HIV which contribute to the antigenic diversity of the antigen. As and when the cumulative antigenic diversity contributed due to successive contacts crosses the antigenic diversity threshold, seroconversion takes place. In developing this model the result of Gurland (1955) has been used. The mean time to seroconversion and its variance are derived and the numerical illustrations are provided.

**Key Words**: Human Immuno Deficiency Virus; Antigenic Diversity Threshold; Seroconversion; Acquired Immuno Deficiency Syndrome.

## Introduction

The concept of shock model and the cumulative damage process has been used to determine the expected time to seroconversion under different assumptions, especially regarding the threshold distribution and the distribution of inter arrival times between successive contacts. In the study of the HIV infection and its consequences the seroconversion of the infected is a vital event. If more and more of HIV are getting transmitted from the infected person to the uninfected, the antigenic variation would be on the increase. The time to seroconversion from the point of infection depends upon what is known as antigenic diversity, which acts against the immune ability of an individual. Every individual has a threshold level of antigenic diversity, beyond which the human immune system cannot withstand. If the antigenic diversity due to acquiring more and more of HIV due to homo or hetro sexual contacts exceeds the threshold level, the immune system of human body is completely suppressed which in turn leads to seroconversion. For a detailed study of antigenic diversity threshold and its estimation one can refer to Nowak and May (1991) and Stilianakis et al. (1994).

The antigenic diversity threshold is taken to be a random variable which has a change of distribution after a change point in the sense that antigenic diversity threshold will have a change in its behavior with the passage of time. This assumption is justified in the sense that he antigenic diversity threshold of an individual may undergo changes due to the ageing of a person, remedial intervention etc. So the model is developed taking these aspects into consideration. Also it has been assumed that the contributions to the antigenic diversity due to successive contacts are i.i.d random variables and the interarrival times between successive contacts are also identically independently distributed random variables. It may be observed that the interarrival times between successive contacts may not always the independently distributed by virtue of the fact that a person who has contacts with an infected partner may have psychological

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depressions and hence the time interval between the successive contacts, may undergo changes.

In this paper a stochastic model is developed under the assumption that the intercontact times between successive contacts are correlated random variables. Shock model with correlated intercontact times has been studied by Sathiyamoorthi (1979). In developing this model, the result of Gurland (1955) has been used. Using the same concept, time to seroconversion and its variance are obtained in this paper.

## Assumption of the model

1. Sexual contact is the only source of transmission.
2. When an uninfected individual has sexual contacts with a HIV infected partner, a random number of HIV gets transmitted.
3. An individual is exposed to a damage process acting on the immune system and damage is assumed to be linear and cumulative.
4. The intercontact times between successive contacts are not independent but are correlated.
5. The threshold is a random variable whose distribution undergoes a change after a change point.
6. The process which generates the contacts, the sequence of damages and threshold are mutually independent.

**Notations:**

$X_i$ = Increase in the antigenic diversity arising due to the HIV transmission during the $i^{th}$ contact. $X_1 X_2 \ldots X_k$ are continuous i.i.d. random variables, with p.d.f g (.) and c.d.f. G (.).

$Y$ = A random variable denoting the threshold with p.d.f. h (y) and c.d.f. H (y).

$U_i$ = is random variable denoting the intercontact times between contacts, with p.d.f. f(.) and c.d.f. f (.)

$g_k(.)$ = denotes the p.d.f of the random variable. $\sum_{i=1}^{k} X_i$

$\rho$ = is correlation coefficient between $X_i$ and $X_j$, $i \neq j$

$W_k(u)$ = the c.d.f. or $Z = \sum_{i=1}^{k} U_i$

$b$ = $C(1-\rho)$

$\eta(n,u) = \int_0^u e^{-\lambda} \lambda^{n-1} d\lambda$

$W_k(t)$ = probability of exactly k contacts in (0,t]

$$S(t) = P(T>t)$$

$$= \text{pr [that the seroconversion does not take place before t ]}$$

$$= \sum_{k=1}^{\infty} \text{Pr [seroconversion does not take place before t. ]} \times P\{\text{ exactly k contacts in (0, t]}\}$$

$$= \sum_{i=0}^{k} [W_k(t) - W_{k+1}(t)] \Pr\left[\sum_{i=0}^{k} X_i < Y\right] \quad \ldots(1)$$

Where $W_k(t) - W_{k+1}(t)$ is the probability that there are exactly k contacts in (0, t], by renewal theory.

It may be noted that

$$P\left[\sum_{i=1}^{k} X_i < Y\right] = \text{Prob \{the cumulative contribution to antigenic diversity on k contacts in (0,t] <Y\}}$$

$$= \int_0^{\infty} g_k(x) \overline{H(x)} \, dx$$

Where

$$\overline{H(x)} = 1 - H(x)$$

It is assumed that the random variable Y denoting the threshold is one which has a change of distribution namely from exponential with parameter $\theta_1$, to Erlang 2 with parameter $\theta_2$, at a change point denoted as $\tau$.

The p.d.f of Y is given as h(y) and

$$h(y) = \theta_1 e^{-\theta_1 Y} \text{ if } 0 < Y \leq \tau$$

$$= \theta_2^2 (y-\tau) e^{-\theta_2 (y-\tau)} e^{-\theta_1 \tau} \text{ if } y > \tau$$

Now the change point $\tau$ itself is taken to be random variable which follows exp($\lambda$).

Sureshkumar (2006) has derived p.d.f of Y as follows.

$$h(y) = \theta_1 e^{-\theta_1 y} e^{-\lambda y} + \int_0^y \theta_2^2 (y-\tau)(y-\tau) e^{-\theta_2 (y-\tau)} e^{\theta_1 \tau} \lambda e^{-\lambda \tau} d\tau$$

$$= \theta_1 e^{-(\theta_1+\lambda)\lambda} + \frac{\lambda \theta_2^2 y . e^{-\theta_2 y}}{\theta_1 + \theta_2 + \lambda} - \frac{\lambda \theta_2^2 y e^{-\theta_2 y}}{(\theta_1+\theta_2+\lambda)^2} + \frac{\lambda \theta_2^2}{(\theta_1-\theta_2+\lambda)^2} e^{-(\theta_1+\lambda)y}$$

on simplification ...(2)

The c.d.f of Y is given as

$$H(y) = 1 - \left\{\frac{\theta_1(\theta_1-\theta_2+\lambda)^2 + \lambda\theta^2}{(\lambda+\theta_1)(\theta_1-\theta_2+\lambda)^2}\right\} e^{-(\theta_1+\lambda)y} + \left\{\frac{\lambda\theta_2 - \lambda(\lambda+\theta_1-\theta_2)}{(\theta_1-\theta_2+\lambda)^2}\right\} e^{-\theta_2 y} + \frac{\lambda\theta_2}{(\theta_1-\theta_2+\lambda)} y e^{-\theta_2 y}$$

on simplification ...(3)

Hence using (3) we have

The survivor function $\overline{H(x)}$ = 1 – H(x) which is given by

$$\overline{H(x)} = m_1 e^{-(\theta_1+\lambda)x} - m_2 e^{-\theta_2 x} - m_3 y e^{-\theta_2 x}$$

where

$$m_1 = \frac{\theta_1(\theta_1-\theta_2+\lambda)^2 + \lambda\theta_2^2}{(\lambda+\theta_1)(\theta_1-\theta_2+\lambda)^2}$$

$$m_2 = \frac{\lambda\theta_2 - \lambda(\lambda+\theta_1-\theta_2)}{(\theta_1-\theta_2+\lambda)^2}$$

$$m_3 = \frac{\lambda\theta_2}{(\theta_1-\theta_2+\lambda)}$$

Now

$$\int_0^{\infty} g_k(x)\overline{H(x)}\, dx = \int_0^{\infty} g_k(x)\left[m_1 e^{-(\theta_1+\lambda)x} - m_2 e^{-\theta_2 x} - m_3 e^{-\theta_2 x}\right] dx$$

$$= m_1\, g_k^*(\theta_1+\lambda) - m_2 g_k^*(\theta_2) + m_3 \frac{d}{d\theta_2} g_k^*(\theta_2)$$

$$= m_1 \left[g^*(\theta_1+\lambda\right]^k - m_2 \left[g^*(\theta_2)\right]^k + m_3 \frac{d}{d\theta_2}\left[g^*(\theta_2)\right]^k \quad \ldots(4)$$

There fore

$$S(t) = m_1 \sum_{k=0}^{\infty} \left[W_k(t) - W_{k+1}(t)\right]\left[g^*(\lambda+\theta_1)\right]^k$$

$$- m_2 \sum_{k=0}^{\infty} [W_k(t) - W_{k+1}(t)] [g^*(\theta_2)]^k$$

$$+ m_3 \sum_{k=0}^{\infty} [W_k(t) - W_{k+1}(t)] \frac{d}{d\theta_2} [g^*(\theta_2)]^k$$

$= 1 - S(t) = P[T \le t]$

$$= 1 - m_1 + m_1 [1 - g^*(\theta_1 + \lambda)] \sum_{k=1}^{\infty} W_K(t) [g^*(\lambda + \theta_1)]^{k-1}$$

$$+ m_2 - m_2 [1 - g^*(\theta_2)] \sum_{k=1}^{\infty} W_K(t) [g^*(\theta_2)]^{k-1}$$

$$- m_3 \sum_{k=0}^{\infty} k W_K(t) [g^*(\theta_2)]^{k-1} \frac{d}{d\theta_2} [g^*(\theta_2)]$$

$$+ m_3 \sum_{k=0}^{\infty} k W_{k+1}(t) [g^*(\theta_2)]^{k-1} \frac{d}{d\theta_2} [g^*(\theta_2)] \qquad ...(5)$$

Let $U_1, U_2, \ldots, U_k$ represent the interarrival times between successive contacts which are correlated. Gurland (1955) has derived the cumulative distribution function of the sum, say $Z_k = \sum_{i=1}^{k} U_i$, when $U_i$' s from a sequence of exchangeable constantly correlated random variables each having exponential distribution with pdf

$$f(u) = \mu e^{-u\mu}, \ \mu > 0, 0 < u < \infty.$$

such that the correlation co-efficient between $U_i$ and $U_j$ ($i \ne j$) is $\rho$.

This cdf is given by

$W_k(u) = P[Z_k \le u]$

$$= (1-\rho) \sum_{i=0}^{\infty} \frac{(\rho k)^i \eta[k+i, u/b]}{[1-\rho+k\rho]^{i+1} (k+i-1)!} \qquad ...(6)$$

where $b = (1-\rho)/\mu$ and

$$\eta(k,u) = \int_0^u e^{-\mu} \mu^{k-1} d\mu$$

The Laplace transform of the density function of $Z_k$ is given by

$$W^*_k(s) = \frac{1}{(1+bs)^k \left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]} \qquad ... (7)$$

Hence taking Laplace transform of L (t) and substituting for $W^*_k$ (s) in (5) we get,

$$\ell^*(s) = 1 - m_1 + m_1 [1 - g^*(\theta_1 + \lambda)] \sum_{k=1}^{\infty} \frac{g^*(\theta_1+\lambda)^{k-1}}{(1+bs)^k} \left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]$$

$$+ m_2 - m_2 [1 - g^*(\theta_2)] \sum_{k=1}^{\infty} \frac{g^*(\theta_2)^{k-1}}{(1+bs)^k \left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]}$$

$$- m_3 \frac{d}{d\theta_2} g^*(\theta_2) \sum_{k=1}^{\infty} k g^*(\theta_2)^{K-1} \frac{1}{(1+bs)^k \left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]}$$

$$+ m_3 \frac{d}{d\theta_2} g^*(\theta_2) \sum_{k=1}^{\infty} k \frac{g^*(\theta_2)^{K-1}}{(1+bs)^{K+1} \left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]}$$

$$\ell^*(s) = 1 - m_1 + m_1 [1 - g^*(\theta_1 + \lambda)] \sum_{k=1}^{\infty} \frac{g^*(\theta_1+\lambda)^{K-1}}{(1+bs)^k \left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]}$$

$$+ m_2 - m_2 [1 - g^*(\theta_2)] \sum_{k=1}^{\infty} \frac{g^*(\theta_2)^{K-1}}{(1+bs)^{K+1} \left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]}$$

$$- m_2 g^I(\theta_2) \sum_{k=1}^{\infty} \frac{k g^*(\theta_2)^{K-1}}{(1+bs)^{K+1} \left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]}$$

$$\frac{d\ell^*(s)}{ds} = m_1 [1 - g^*(\theta_1 + \lambda)] \sum_{K=1}^{\infty} g^*(\theta_1 + \lambda)^{K-1} \left[ \frac{(-kb)}{(1+bs)^{K+1}} \frac{1}{\left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]} \right.$$

$$\left. - \frac{1}{(1+bs)^K} \frac{\frac{k\rho b}{(1-\rho)} \frac{1}{(1+bs)^2}}{\left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]} \right]$$

$$- m_2 [1 - g^*(\theta_2)] \sum_{K=1}^{\infty} g^*(\theta_2)^{K-1} \left[ \frac{-kb}{(1+bs)^{K+1}} \frac{1}{\left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]} \right.$$

$$\left. - \frac{1}{(1+bs)^K} \frac{\frac{k\rho b}{(1-\rho)} \frac{1}{(1+bs)^2}}{\left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]^2} \right]$$

$$+ m_3 g^{*'}(\theta_2)] \sum_{K=1}^{\infty} k g^*(\theta_2)^{K-1} \left[ \frac{-kb}{(1+bs)^{K+1}} \frac{1}{\left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]} \right.$$

$$\left. - \frac{1}{(1+bs)^{K+2}} \frac{k\rho b}{(1-\rho) \left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]^2} \right]$$

$$- m_3 g^{*'}(\theta_2)] \sum_{K=1}^{\infty} k g^*(\theta_2)^{K-1} \left[ \frac{-(k+1)b}{(1+bs)^{K+2}} \frac{1}{\left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]} \right.$$

$$\left. - \frac{1}{(1+bs)^{K+3}} \frac{\frac{k\rho b}{1-\rho}}{\left[1 + \frac{k\rho bs}{(1-\rho)(1+bs)}\right]^2} \right]$$

$$\left. \frac{d\ell^*(s)}{ds} \right|_{s=0} = m_1 [1 - g^*(\theta_1 + \lambda)] \sum_{K=1}^{\infty} g^*(\theta_1 + \lambda)^{K-1} \left( -kb - \frac{k\rho b}{1-\rho} \right)$$

$$- m_2 [1 - g^*(\theta_2)] \sum_{K=1}^{\infty} g^*(\theta_2)^{K-1} \left( -kb - \frac{k\rho b}{1-\rho} \right)$$

$$- m_3 \, g^{*'}(\theta_2)] \sum_{K=1}^{\infty} k g^*(\theta_2)^{K-1} \left( -kb - \frac{k\rho b}{1-\rho} \right)$$

$$+ m_3 g^{*'}(\theta_2)] \sum_{K=1}^{\infty} k g^*(\theta_2)^{K-1} \left( -(k+1)b - \frac{k\rho b}{1-\rho} \right)$$

Similarly, it can be shown that

$$E(T) = \left. \frac{-dl^*(S)}{ds} \right|_{s=0} = m_1 \left( \frac{\alpha + \lambda + \theta_1}{\lambda + \theta_1} \right) \frac{b}{1-\rho} - m_2 \left( \frac{\alpha + \theta_2}{\theta_2} \right) \frac{b}{1-\rho} - m_3 b \frac{\alpha}{\theta_2^2} \qquad ...(8)$$

on simplification

Similarly to find E ($T^2$)

$$\left.\frac{d^2 l^*(S)}{ds^2}\right|_{s=0} = m_1\left[1-g^*(\theta_1+\lambda)\right]\sum_{k=1}^{\infty} g^*(\theta_1+\lambda)^{k-1}\left[k(k+1)b^2+\frac{k^2\rho b^2}{1-\rho}+\frac{k(k+2)\rho b^2}{1-\rho}+\frac{2k^2\rho^2 b^2}{(1-\rho)^2}\right]$$

$$-m_1\left[1-g^*(\theta_2)\right]\sum_{k=1}^{\infty} g^*(\theta_2)^{k-1}\left[k(k+1)b^2+\frac{k^2\rho b^2}{1-\rho}+\frac{k(k+2)\rho b^2}{1-\rho}+\frac{2k^2\rho^2 b^2}{(1-\rho)^2}\right]$$

$$-m_3 g^{*'}(\theta_2)\sum_{k=1}^{\infty} g^*(\theta_2)^{k-1}\left[k(k+1)b^2+\frac{k^2\rho b^2}{1-\rho}+\frac{k(k+2)\rho b^2}{1-\rho}+\frac{2k^2\rho^2 b^2}{(1-\rho)^2}\right]$$

$$+m_3 g^{*'}(\theta_2)\sum_{k=1}^{\infty} g^*(\theta_2)^{k-1}\left[(k+1)(k+2)b^2+\frac{k(k+1)\rho b^2}{1-\rho}+\frac{k(k+3)\rho b^2}{1-\rho}+\frac{2k^2\rho^2 b^2}{(1-\rho)^2}\right]$$

Let g(.)~ exp ($\alpha$)

Then $g^*(\theta_1)=\dfrac{\alpha}{\alpha+\theta_1}$

Hence

$$[E(T)]^2 = m_1^2\left(\frac{\alpha+\lambda+\theta_1}{\lambda+\theta_1}\right)^2\frac{b^2}{(1-\rho)^2}+m_2^2\left(\frac{\alpha+\theta_2}{\theta_2}\right)^2\frac{b^2}{(1-\rho)^2}+m_3^2 b^2\frac{\alpha^2}{\theta_2^4}$$

$$-2m_1m_2\left(\frac{\alpha+\lambda+\theta_1}{\lambda+\theta_1}\right)\left(\frac{\alpha+\theta_2}{\theta_2}\right)\frac{b^2}{(1-\rho)^2}-2m_1m_3\left(\frac{\alpha+\lambda+\theta_1}{\lambda+\theta_1}\right)\frac{\alpha}{\theta_2^2}\frac{b^2}{(1-\rho)^2}$$

$$+2m_2m_3\frac{b^2}{(1-\rho)}\frac{\alpha}{\theta_2^2}\left(\frac{\alpha+\theta_2}{\theta_2}\right) \quad \text{on simplification} \qquad \ldots (9)$$

Hence

$$V(T) = m_1\left(\frac{\alpha+\lambda+\theta_1}{\alpha+\theta_1}\right)\left[2\left(\frac{\alpha+\lambda+\theta_1}{\lambda+\theta_1}\right)\frac{b^2(1+\rho^2)}{(1-\rho)^2}-\frac{2\rho^2 b^2}{(1-\rho)^2}\right]$$

$$-\; m_2\left(\frac{\alpha+\theta_2}{\theta_2}\right)\left[2\left(\frac{\alpha+\theta_2}{\theta_2}\right)\frac{b^2(1+\rho^2)}{(1-\rho)^2}-\frac{2\rho^2 b^2}{(1-\rho)^2}\right]$$

$$-\; m_3\frac{\alpha}{\theta_2^2}\left[4\left(\frac{\alpha+\theta_2}{\theta_2}\right)b^2-\frac{\rho b^2(3\rho-1)}{(1-\rho)^2}\right]$$

$$-\; m_1^2\left(\frac{\alpha+\lambda+\theta_1}{\lambda+\theta_1}\right)^2\frac{b^2}{(1-\rho)^2}-m_2^2\left(\frac{\alpha+\theta_2}{\theta_2}\right)^2\frac{b^2}{(1-\rho)^2}-m_3^2 b^2\frac{\alpha^2}{\theta_2^4}$$

$$+2m_1m_2\left(\frac{\alpha+\lambda+\theta_1}{\lambda+\theta_1}\right)\left(\frac{\alpha+\theta_2}{\theta_2}\right)\frac{b^2}{(1-\rho)^2}+2m_1m_3\left(\frac{\alpha+\lambda+\theta_1}{\lambda+\theta_1}\right)\frac{\alpha}{\theta_2^2}\frac{b^2}{(1-\rho)}$$

$$-2m_2m_3\frac{b^2}{(1-\rho)}\frac{\alpha}{\theta_2^2}\left(\frac{\alpha+\theta_2}{\theta_2}\right) \qquad \text{on simplification} \qquad \ldots (10)$$

Table-1

| $\alpha$= 0.1, $\theta_1$=1, $\theta_2$=1, $\rho$=0.7 | | |
|---|---|---|
| $\lambda$ | E(T) | V(T) |
| 1 | 1.0200 | 0.5655 |
| 2 | 2.0733 | 2.3255 |
| 3 | 3.1350 | 5.2729 |
| 4 | 4.2000 | 9.4063 |
| 5 | 5.2667 | 14.7253 |
| 6 | 6.3343 | 21.2295 |
| 7 | 7.4025 | 28.9189 |
| 8 | 8.4711 | 37.7934 |
| 9 | 9.5400 | 47.8530 |
| 10 | 10.6091 | 59.0976 |

Figure-1

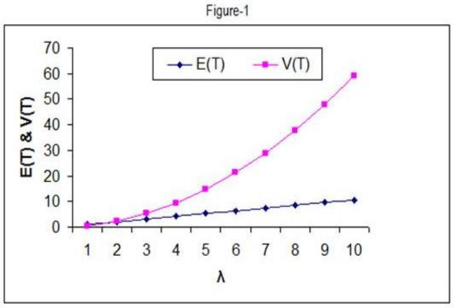



Table-2

| $\lambda$= 1, $\theta_1$=1, $\theta_2$=1, $\rho$=0.7 | | |
|---|---|---|
| $\alpha$ | E(T) | V(T) |
| 0.1 | 1.0200 | 0.5655 |
| 0.2 | 1.0400 | 0.5991 |
| 0.3 | 1.0600 | 0.6299 |
| 0.4 | 1.0800 | 0.6576 |
| 0.5 | 1.1000 | 0.6821 |
| 0.6 | 1.1200 | 0.7032 |
| 0.7 | 1.1400 | 0.7209 |
| 0.8 | 1.1600 | 0.7351 |
| 0.9 | 1.1800 | 0.7457 |
| 1 | 1.2000 | 0.7526 |

Figure-2

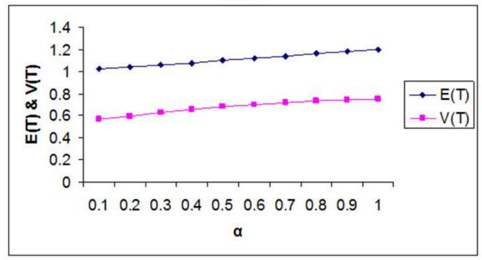



Table -3

| $\alpha$= 0.1, $\lambda$=1, $\theta_1$=1, $\theta_2$=1 | | |
|---|---|---|
| $\rho$ | E(T) | V(T) |
| 0.1 | 0.96 | 2.1985 |
| 0.2 | 0.97 | 1.8175 |
| 0.3 | 0.98 | 1.4992 |
| 0.4 | 0.99 | 1.2262 |
| 0.5 | 1.00 | 0.9856 |
| 0.6 | 1.01 | 0.7678 |
| 0.7 | 1.02 | 0.5655 |
| 0.8 | 1.03 | 0.3730 |
| 0.9 | 1.04 | 0.1857 |
| 1 | 1.05 | 0.1038 |

Figure- 3

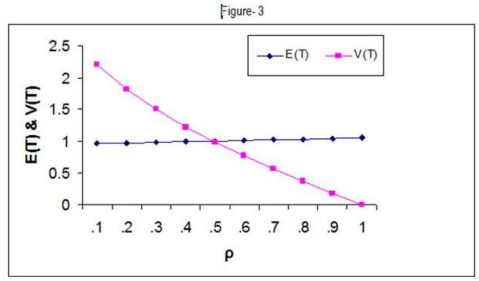



Table-4

| $\rho$ = 0.7, $\alpha$= 0.1, $\lambda$=1, $\theta_2$=1 | | |
|---|---|---|
| $\theta_1$ | E(T) | V(T) |
| 1 | 1.0200 | 0.5655 |
| 2 | 1.0350 | 0.3454 |
| 3 | 1.0317 | 0.2963 |
| 4 | 1.0275 | 0.2709 |
| 5 | 1.0240 | 0.2547 |
| 6 | 1.0212 | 0.2433 |
| 7 | 1.0189 | 0.2348 |
| 8 | 1.0171 | 0.2283 |
| 9 | 1.0156 | 0.2230 |
| 10 | 1.0143 | 0.2188 |

Figure-4

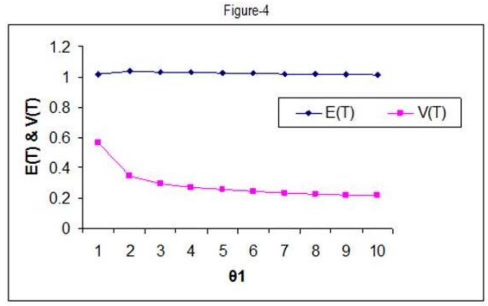
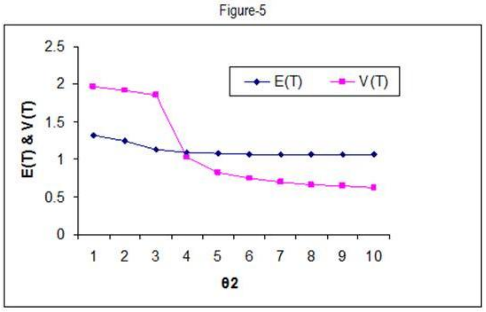


Table-5

| $\rho = 0.7, \alpha = 0.1, \lambda=1, \theta_1=1$ | | |
|---|---|---|
| $\theta_2$ | E(T) | V(T) |
| 1 | 1.3210 | 1.9655 |
| 2 | 1.2433 | 1.9123 |
| 3 | 1.1267 | 1.8541 |
| 4 | 1.0913 | 1.0332 |
| 5 | 1.0787 | 0.8307 |
| 6 | 1.0721 | 0.7426 |
| 7 | 1.0680 | 0.6939 |
| 8 | 1.0652 | 0.6631 |
| 9 | 1.0632 | 0.6420 |
| 10 | 1.0616 | 0.6266 |

Figure-5




### 10.2. Conclusions

If the value of λ which is the parameter of the exponential distribution of the random variable τ denoting the truncation point increase the expected time to seronversion increases as indication table (1) and figure (1), So, also in the variance of T.

ii) If α which is the parameter of the random variable X denoting the magnitude of increases in antigenic diversity increases then E (x) =1/α decreases. Hence there is an increase in E (T) and also its variance V (T). This is given table (2) and figure (2).

When 'ρ' which is the constant correlation between the interarrival times between successive contacts increases, there is a marginal increase in the value of E (T). But the variance V (T) is on the decrease as indicated in table (3) and figure (3).

iv) It can be seem that when $\theta_1$ which is the parameter of the exponential distribution denoting the threshold prior to the truncation point τ, and $\theta_2$ which is the parameter of the threshold distribution usually Erlang 2 after τ produce insignificant changes in E(T) and V (T) as indicated in table (4) and table (5) respectively.

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