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DETERMINATION OF EXPECTED TIME TO RECRUITMENT IN MANPOWER PLANNING

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Abstract

Manpower planning in a sense, according to Barthlomew [1], is concerned with matching the supply of people with the jobs available. Recruitment is required to compensate the wastage of manpower; wastage refers to the loss of manpower in the organization. Recruitment cannot be made frequently since it involves cost. The threshold level is the maximum amount of wastage that can be permitted in the organization beyond which the organization reaches a point of breakdown. It is similar to the concept of shock model and cumulative damage discussed by Esary et al. [2]. In this paper we made an attempt to determine the expected time to recruitment, assuming that the threshold as exponentiated exponential distribution which introduced by Gupta and Kundu [3].
Key Words: Expected time; Manpower planning; Recruitment; Threshold; Wastage.

Introduction

The purpose of manpower planning is to best match future manpower needs (demand) and resources (supply). The total flow out of the system is termed wastage. An individual propensity to leave depends on many factors including current length of service, age, salary, place of residence in relation to workplace, sex, marital status and the general employment situation. Such models were discussed by Grinold and Marshall [4] and Bartholomew and Forbes [5].

Esary et al. [2] discussed that any component or device when exposed to shocks which cause damage to the device or system is likely to fail when the total accumulated damage exceeds a level called the threshold. The rate of accumulation of damage determines the life time of the component or device.

Cumulative damage process (CDP) is related to the shock models in reliability theory. The basic idea is that accumulating random amount of damages due to shocks in successive epoch leads to the breakdown of the system when the total damage crosses a random threshold level. Assuming (in terms of man hours) an organizational system the expected time to the breakdown of organization due to the depletion of manpower is studied, taking a threshold level which in other words can be called as the breakdown point of the system. The breakdown point or the level can also be interpreted as that point at which the immediate recruitment is necessitated to makeup the manpower loss suffered cumulatively on successive occasions. The threshold having SCBZ (Setting the clock back to zero) property in which the expected time to the breakdown/recruitment was discussed by Sathiyamoorthi and Parthasarathy [6].

The three parameter Generalized Exponential distribution (location, scale, shape) discussed by Gupta and Kundu [3] which has an increasing or decreasing failure rate depending on the shape parameter. Gupta and Kundu [7] also discussed about Exponentiated Exponential distribution with two parameters namely scale and shape parameter.

The distribution function, $F_E(x,\alpha,\lambda)$,

$$F_E(x,\alpha,\lambda) = \left(1 - e^{-\lambda x}\right)^{\alpha} \qquad \alpha,\lambda,x > 0$$

Therefore it has the density function

$$f_E(x,\alpha,\lambda) = \alpha\lambda\left(1 - e^{-\lambda x}\right)^{\alpha-1} e^{-\lambda x}$$

The corresponding survival function is

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$$S_E(x,\alpha,\lambda) = 1 - \left(1 - e^{-\lambda x}\right)^{\alpha}$$

When the shape parameter $(\alpha = 1)$, it represents the exponential distribution. Damodaran and Gopal [8] stated that, for the simplicity and for the single parameter distribution, the Generalised Exponential Distribution with shape parameter $(\alpha = 2)$ was considered. Then a random variable which has the density function defined as

$$f_E(x,\alpha,\lambda) = 2\lambda\left(1 - e^{-\lambda x}\right)e^{-\lambda x}$$

In this paper having the threshold which follows Exponentiated Exponential distribution is discussed with the shape parameter $(\alpha = 2)$ is been considered. The expected time and variance are obtained with numerical illustration.

Assumption of the Model

1. Exit of person from an organization takes place whenever the policy decisions regarding targets, incentives and promotions are made.
2. The exit of every person from the organization results in a random amount of depletion of manpower (in man hours).
3. The process of depletion is linear and cumulative.
4. The inter arrival times between successive occasions of wastage are i.i.d. random variables.
5. If the total depletion exceeds a threshold level Y which is itself a random variable, the breakdown of the organization occurs. In other words recruitment becomes inevitable.
6. The process, which generates the exits, the sequence of depletions and the threshold are mutually independent.

NOTATIONS

X_i : a continuous random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the i^{th} occasion of policy announcement, $i = 1,2,\ldots.k$ and X_i's are i.i.d and $X_i = X$ for all i.

Y : a continuous random variable denoting the threshold level having Exponentiated Exponential distribution.

$g(.)$: The probability density functions of X.

$g_k(.)$ The k- fold convolution of $g(.)$ i.e., p.d.f. of $\sum_{i=1}^{k} X_i$

T : a continuous r.v denoting time to breakdown of the system.

$g^*(.)$: Laplace transform of $g(.)$.

$g_k^*(.)$: Laplace transform of $g_k(.)$.

$h(.)$: The p.d.f. of random threshold level which has Exponentiated Exponential distribution and $H(.)$ is the corresponding c.d.f.

U : a continuous random variable denoting the inter-arrival times between decision epochs.

$f(.)$: p.d.f. of random variable U with corresponding c.d.f. $F(.)$.

$F_k(\mathrm{t})$: The k-fold convolution functions of $F(.)$.

$S(.)$: The survivor function i.e. $P\ [T > t]$.

$L(t) : 1 - S(t)$

$F_k(t)$: Probability that there are exactly 'k' policies decisions in $(0,t]$.

Results

Let Y be the random variable which has the cdf defined as

$$H_E(x,\alpha,\lambda) = \left(1 - e^{-\lambda x}\right)^2 \qquad \alpha,\lambda,x > 0$$

Therefore it has the density function

$$h_E(x,\alpha,\lambda) = 2\lambda\left(1 - e^{-\lambda x}\right)$$

The corresponding survival function is

$$S_E(x,\alpha,\lambda) = 1 - \left(1 - e^{-\lambda x}\right)^2$$

Now,

$P(X_1 + X_2 + \cdots + X_k < Y) =$ P [the system does not fail, after k epochs of exits].

$$P\left(\sum X_i < Y\right) = \int_0^\infty g_k(x)\bar{H}(x)dx$$

$$= \int_0^\infty g_k(x)\left[2e^{-\lambda x} - e^{-2\lambda x}\right]dx$$

$$= 2g_k^*(\lambda) - g_k^*(2\lambda)$$

$$= 2[g^*(\lambda)]^k - [g^*(2\lambda)]^k \qquad (1)$$

The survival function $S(t)$ which is the probability that an individual survives for a time t.

$S(t) = P(T > t) =$ Probability that the system survives beyond t.

$$= \sum_{k=0}^{\infty} P\ \{\text{there are exactly k instants of exists in } (0,t]$$

$$* \ P\ (\text{the system does not fail in } (0,t]\}$$

It is also known from renewal theory that

$$P(\text{exactly k policy decesions in } (0,t]) = F_k(t) - F_{K+1}(t) \qquad \text{with} \quad F_0(t) = 1$$

$$= \sum_{k=0}^{\infty} F_k(t)P(X_i < Y)$$

$$= 2\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\, g_k^*(\lambda) - \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\,[g_k^*(2\lambda)] \qquad (2)$$

Now

$P(T < t) = L(T)$ = The distribution function of T

$$= 1 - \left\{2\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)][g^*(\lambda)]^k - \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\,[g^*(2\lambda)]^k\right\}$$

$$= 1 - 2 + 2[1 - g^*(\lambda)]\sum_{k=1}^{\infty}F_k(t)[g^*(\lambda)]^{k-1} + 1 - [1 - g^*(2\lambda)]\sum_{k=1}^{\infty}F_k(t)[g^*(2\lambda)]^{k-1}$$

$$= 2[1 - g^*(\lambda)]\sum_{k=1}^{\infty}F_k(t)[g^*(\lambda)]^{k-1} - [1 - g^*(2\lambda)]\sum_{k=1}^{\infty}F_k(t)[g^*(2\lambda)]^{k-1} \qquad (3)$$

Taking laplace transform of $L(t)$, we get

$$L^*(S) = 2[1 - g^*(\lambda)]\sum_{k=1}^{\infty}[g^*(\lambda)]^{k-1}[f^*(s)]^k - [1 - g^*(2\lambda)]\sum_{k=1}^{\infty}[g^*(2\lambda)]^{k-1}\,[f^*(s)]^k \qquad (4)$$

Where $[f^*(s)]^k$ is laplace transform of $F_k(t)$ since the inter arrival times are i.i.d. The above equation can be rewritten as,

$$L^*(S) = 2[1 - g^*(\lambda)]f^*(s)\sum_{k=1}^{\infty}[g^*(\lambda)f^*(s)]^{k-1} - [1 - g^*(2\lambda)]f^*(s)\sum_{k=1}^{\infty}[g^*(2\lambda)f^*(s)]^{k-1} \quad (5)$$

$$E(T) = -\frac{d}{ds}L^*(S) \quad \text{given } s = 0$$

$$E(T^2) = \frac{d^2L^*(S)}{ds^2}$$

From which $V(T)$ can be obtained.

Let the random variable U denoting inter arrival time which follows exponential with parameter c. Now $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the above equation (5) we get,

Numerical Illustration

From table 1 and the corresponding figure 1.1 and 1.2 we could observe the difference in the values of $E\ (T)$ and $V\ (T)$ when the threshold distribution has Exponentiated exponential distribution with $\mu = 0.3 \text{ and } \lambda = 0.1$ as C increases $E\ (T)$ as well as $V\ (T)$ decreases.

From table 2 and the corresponding figure 2.1 and 2.2 we could observe the difference in the values of $E\ (T)$ and $V\ (T)$ when the threshold distribution has Exponentiated exponential distribution. If parameter value is increased by $\mu = 0.6 \text{ and } \lambda = 0.1$ as C increases $E\ (T)$ and $V\ (T)$ decreases. In both the cases the behavior is found to be same.

Table 1 $\mu = 0.3, \lambda = 0.1, c = 1,2 \ldots 10$

c	1	2	3	4	5	6	7	8	9	10
$E(T)$	0.055	0.0275	0.0183	0.0137	0.011	0.0091	0.0078	0.0068	0.0061	0.0055
$V(T)$	22.75	5.6875	2.5277	1.4218	0.91	0.6319	0.4642	0.3554	0.2808	0.2275

Table 2 $\mu = 0.6, \lambda = 0.1, c = 1,2 \ldots 10$

c	1	2	3	4	5	6	7	8	9	10
$E(T)$	0.1	0.05	0.0333	0.025	0.02	0.0166	0.0142	0.0125	0.0111	0.01
$V(T)$	65.5	16.375	7.277	4.093	2.62	1.819	1.336	1.023	0.808	0.655

$$L^*(S) = \frac{2[1 - g^*(\lambda)]\left(\frac{c}{c+s}\right)}{[1 - g^*(\lambda)]\left(\frac{c}{c+s}\right)} - \frac{[1 - g^*(2\lambda)]\left(\frac{c}{c+s}\right)}{[1 - g^*(2\lambda)]\left(\frac{c}{c+s}\right)}$$

$$= \frac{2c[1 - g^*(\lambda)]}{[c + s - g^*(\lambda)\, c]} - \frac{[1 - g^*(2\lambda)]c}{[c + s - g^*(2\lambda)\, c]}$$

$$E(T) = \frac{2}{c[1 - g^*(\lambda)]} - \frac{1}{c[1 - g^*(2\lambda)]} \quad \text{on simplification} \qquad (6)$$

$$E(T^2) = \frac{d}{ds}\left[-\frac{2[1 - g^*(\lambda)]c}{[c + s - g^*(\lambda)c]^2}\right] + \left[\frac{[1 - g^*(2\lambda)]c}{[c + s - g^*(2\lambda)c]^2}\right]$$

$$= \frac{4}{c^2[1 - g^*(\lambda)]^2} - \frac{2}{c^2[1 - g^*(2\lambda)]^2} \quad \text{on simplification}$$

$$V(T) = \frac{4}{c^2[1 - g^*(\lambda)][1 - g^*(\lambda)]} - \frac{3}{c^2[1 - g^*(2\lambda)]^2} \qquad (7)$$

$$g^*(.) \sim exp(\mu), \quad g^*(\lambda) = \frac{\mu}{\mu + \lambda}, \quad g^*(2\lambda) = \frac{\mu}{\mu + 2\lambda}$$

$$E(T) = \frac{2}{c\left[1 - \frac{\mu}{\mu + \lambda}\right]} - \frac{1}{c\left[1 - \frac{\mu}{\mu + 2\lambda}\right]}$$

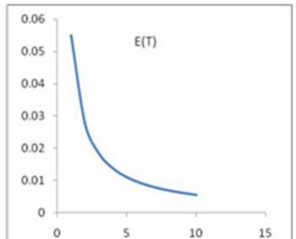

Fig 1.1

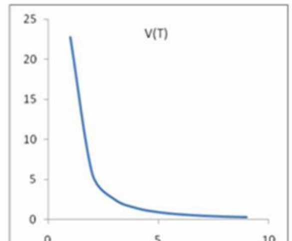

Fig 1.2

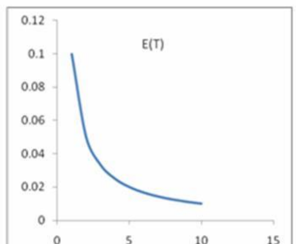

Fig 2.1

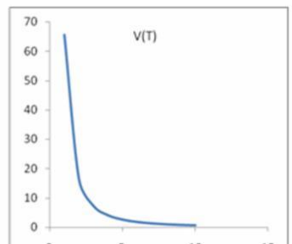

Fig 2.2

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