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An Eigen value study on the variant of Murali-Lakshmanan-Chua circuit

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ABSTRACT

In this paper, the Eigen values of a simple second-order non-autonomous chaotic circuit namely the variant of the Murali-Lakshmanan-Chua's circuit are studied. The dynamical behavior of the circuit is obtained by means of a study on the Eigen values of the linearized Jacobian of the nonlinear differential equations. The trajectories of the Eigen values as functions of the dynamic parallel loss conductance explaining the supercritical hopf bifurcation exhibited by the autonomous system is presented.

KEY WORDS: Chaos, Eigen value, quasiperiodic

INTRODUCTION

The Eigen values of a system are important for analyzing the stability of the fixed points that characterize the system. Since the nature of the stability of the fixed points can be understood from the Eigen values, they form an important role in linear stability analysis. The Murali-Lakshmanan-Chua (MLC) circuit is the simplest second-order non-autonomous nonlinear electronic circuit consisting of a forced series RLC circuit with the Chua's diode connected in parallel to the capacitor. The dynamics exhibited by the system were studied experimentally, numerically (Murali *et al*., 1994; 1994b) and analytically by Murali *et al*. (Lakshmanan and Murali, 1995; Lakshmanan and Murali, 1996). An Eigen value study on the MLC circuit was done by Lindberg and Murali (1998) explaining the chaotic behavior of the circuit on the basis of the Eigen values of the linearized Jacobian of the nonlinear differential equations describing the circuit. The periodic and chaotic behavior of the MLC circuit was studied by varying the frequency and amplitude of the external voltage source. The frequency of the external voltage source thus varied is selected from the different Eigen values obtained by varying the dynamic nonlinear conductance g_{nl} of the nonlinear element G_p, namely the Chua's diode (Kennedy, 1992). The chaotic dynamics of some second-order non-autonomous and certain higher order nonlinear electronic circuits were studied numerically, experimentally and analytically by Lakshmanan and Murali (Lakshmanan and Murali, 1995; Lakshmanan and Murali, 1996). The variant of the Murali-Lakshmanan-Chua's (MLCV) circuit is another second-order non-autonomous chaotic circuit that exhibits quasiperiodic and chaotic motion in its dynamics. The bifurcations and chaotic phenomena exhibited by the MLCV circuit were studied by Thamilmaran *et al*. (2000), showing the quasiperiodicity, intermittency and period doubling routes to chaos. An investigation on the classification of bifurcations and routes to chaos in the circuit was made by Thamilaran and Lakshmanan (2002) experimentally, numerically. Furthermore, an explicit analytical solution to the normalized state equations of the circuit with a linear stability analysis of the fixed points along with the nature of the Eigen values was presented in the study.

In this paper, the types of Eigen values of the MLCV circuit are studied for different values of the nonlinear dynamical conductance "g_{nl}." The MLCV circuit consists of a nonlinear parallel loss conductor G_p (Chua's diode), connected in parallel to the LC network of a simple forced parallel RLC circuit as shown in Figure 1. The circuit equations are given as,

$$C\frac{dv}{dt}=\frac{1}{R}fsin(\omega t)\text{-}(\upsilon)\text{-}i_L\text{-}g(\upsilon) \tag{1a}$$

$$L\frac{di_L}{dt}=\upsilon \tag{1b}$$

where g (υ) is the mathematical form of the piecewise linear function given by

$$g(\upsilon)=\begin{cases} G_b\upsilon+(G_a-G_b) & if\ \upsilon>1 \\ G_a\upsilon & if\ |\upsilon|\leq 1 \\ G_b\upsilon-(G_a-G_b) & if\ \upsilon<-1 \end{cases} \tag{2}$$

where $G_a = -0.76$ mS, $G_b = -0.41$ mS and $B_p = \pm 1.0$ V are the respective values of the negative slopes of the inner and outer regions and the breakpoints in the (υ-i) characteristic curve of the nonlinear element (Kennedy, 1992) and f, ω are the amplitude and frequency of the external voltage source, respectively.

Being a simple forced parallel LCR circuit with the Chua's diode as the only nonlinear element, an eigen value study is carried on by varying the dynamic loss conductance gnl of the circuit. Because of the only nonlinear component G_p the trajectory of the Eigen values of the linearized Jacobian of the nonlinear differential equations may be found by means of simple linear frequency analysis varying the dynamic value g_{nl}, of the nonlinear parallel loss conductor G_p, in a certain choice of the circuit parameters L = 445 mH, R = 1475 Ω, and C = 10.15 nF.

Qualitative Analysis

When the dynamic parallel loss conductance g_{nl} is zero the circuit reduces to a simple parallel RLC oscillator with steady state oscillations of voltage and current, for an external applied voltage. In the presence of G_p, the admittance in the parallel LCG_p network in the circuit

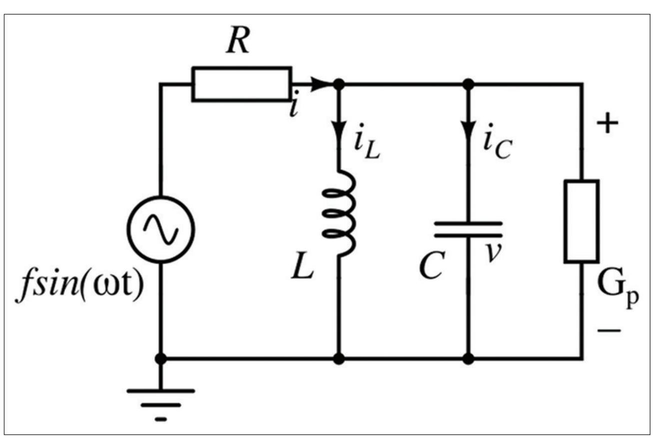

Figure 1: Schematic diagram of the variant of Murali-Lakshmanan-Chua circuit

is, $Y = G_p + j\ (\omega C - 1/\omega L)$, where ω is the frequency of the external applied voltage. Now let us calculate the Eigen values of the linearized Jacobian of the differential equations characterizing the circuit. The Jacobian of the circuit equations is,

$$J_0=\begin{pmatrix} -\left(\frac{Gp}{C}+\frac{1}{RC}\right) & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{pmatrix}$$

The Eigen values of the linearized Jacobian are either two real poles or a pair of complex poles in the complex frequency plane. The trajectory of the Eigen values for different values of the dynamic loss conductance, g_{nl} can be obtained in the complex frequency plane. The poles are the roots of the characteristic equation obtained using the Jacobian given above. The characteristic equation is,

$$\lambda^2 + 2\alpha\lambda + \omega^2_0 = 0 \tag{4}$$

where, $2\ \alpha = (G_p/C + 1/RC)$ and $\omega_0 = 1/\sqrt{LC}$ is the natural frequency of oscillation of the circuit. For some fixed values of the circuit parameters L, C, and R, the conditions imposed on G_p to get the different types of roots and their dynamics for Equation (4) is summarized in Table 1.

The roots of the characteristic Equation (4) are $\lambda_{1,2} = -\ \alpha \pm \sqrt{\alpha^2\ \omega^2_0}$. The roots may be either real or complex and the conditions for the corresponding values of G_p are explained as follows: For $\alpha^2 < \omega^2_0$, the roots are a pair of complex poles and the condition for G_p is such that $G_p < -\ 1/R \pm 2\ \sqrt{C/L}$. The value of the rela part is $\alpha = 1/2C\ (G_p + 1/R)$. When $|\ G_p| = 1/R$ the real part of the complex root becomes zero and the quilibrium point is an elliptic/center. If $|\ G_p| > 1/R$ then the equilibrium point is an unstable spiral and if $|\ G_p| < 1/R$ equilibrium point is a stablespiral within the limit of G_p.

Table 1: Conditions on G_p for different types of roots and their dynamics

Dynamics	Roots	Conditions on G_p
Stable spiral	$\lambda_{1,2}=-\alpha\pm\sqrt{\omega_0{}^2-\alpha^2}$ (complex conjugates)	$G_p<-1/R\pm2\sqrt{C/L}$ and $\lvert G_p\rvert<1/R$
Unstable spiral	$\lambda_{1,2}=-\alpha\pm\sqrt{\omega_0{}^2-\alpha^2}$ (complex conjugates)	$G_p<-1/R\pm2\sqrt{C/L}$ and $\lvert G_p\rvert>1/R$
Centre/elliptic	$\lambda_{1,2}=-\alpha\pm\sqrt{\omega_0{}^2-\alpha^2}$ (complex conjugates)	$G_p<-1/R\pm2\sqrt{C/L}$ and $\lvert G_p\rvert=1/R$
Stable/unstable node	$\lambda_{1,2}=-\alpha\pm\sqrt{\alpha^2-\omega_0{}^2}$ (real and unequal)	$G_p>-1/R\pm2\sqrt{C/L}$
Stable/unstable star	$\lambda_{1,2}=-\alpha$ (real and equal)	$G_p=-1/R\pm2\sqrt{C/L}$

For $\alpha^2 > \omega^2_0$, the roots are two unequal real poles and the condition on Gp is such that for $G_p > -1/R \pm 2\sqrt{C/L}$ and for $\alpha^2 = \omega^2_0$, the roots become a pair of real double poles and the corresponding values of G_p becomes, $G_p = -1/R \pm 2\sqrt{C/L}$. The above conditions for G_p give the range of conductance values of the nonlinear element that gives steady state oscillations, exponential growth and decay of signals in the circuit. The G_p values could be varied to obtain the exact numerical values of the roots corresponding to the different G_p values to predict the dynamics of the system.

Quantitative Analysis

A set of Eigen values obtained using Equation (4) for a specific set of parameters L = 445 mH, R = 1475 Ω and C = 10.15 nF is as given in Table 2. It is seen that for G_p going to "minus infinity" one of the real pole re1 becomes zero, and the other one re2 reaches a maximum value of +9.85221574e26 and the system becomes highly unstable. For G_p = 9.800190027837e-4 we get two real double poles in the right half plane of the complex frequency plane. Hence, the system becomes unstable for large negative values of G_p. Due to the maximum slope of −0.76 mS for G_p in origin, the poles are a pair of complex conjugates with a positive real part re = 4.04107873e3. Due to this large positive value of the real part, the autonomous system has an unstable limit cycle at the origin, and the signals are negatively damped. If the autonomous system is started up with an initial capacitor voltage of 1e-12 volt across the capacitor C, then the amplitude of oscillation of the signals must increase until the break point of the piecewise liner conductance G_p is reached, i.e., it is to be expected that the voltage of capacitor 'C' will rise to 1 volt when the complex pole pair for G_p = −0.41 mS, λ= −1.320030e-4 ± j6.8665998e-3, will take over and give rise to damped oscillations. Hence, the autonomous system is balanced between this growth and damped oscillations giving rise to a stable limit cycle in the origin Now, it could be well understood that as G_p is increased from G_p = −0.41 mS to G_p = −0.76 mS, the stable spiral becomes an unstable spiral enclosed by a limit cycle, undergoing a supercritical hopf bifurcation (Strogatz, 1994). For G_p = −0.677966104 mS, the system gives rise rise to a stable limit-cycle enclosing the origin.

Table 2: Eigen values as functions of G_p

G_p	re_1/re	re_2/im
−2.5058e	0.0	+2.468768401e12
−-2.5057e4	+2.44140625e-5	+2.46866988e12
−1.0	−2.24871561	+9.84553705e7
−9.8001900278374e-4	+1.48794532e4	+1.48794532e4
−9.800190027837e-4	+1.48794532e4	+j8.32408550e-3
−7.6e-4	+4.04107873e3	+j1.43201889e-4
−6.77966101e-4	−1.45519152e-11	+j1.48794532e4
−4.1e-4	−1.32003005e4	+j6.86659987e3
−3.759132006061e-4	−1.48794532e4	+j4.96752684e-3
−3.7591320060608e-4	−1.48794532e4	−1.48794532e4
0.0	−6.32969187e4	−3.49777103e3
9.80019002e-4	−1.61981474e5	−1.36681141e3
+1.0	−9.85889599e7	−2.24566857
+1.0e4	−9.85221741e11	−2.44140625e-4
+2.5125e4	−2.47536952e12	−2.44140625e-4
+2.5126e4	−2.47546804e12	0.0

Figure 2 shows the real Eigen values of the system as a function of the loss conductance g_p. The blue line shows the real Eigen values while the red line shows the real part of the complex conjugate Eigen values. The trajectories of the Eigen values are shown in Figure 3. In Figure 3a and b, the two real poles are pictured against each other for positive and negative values of g_{nl}, respectively. In Figure 3c it is seen how the complex pole pair leaves the real axis for g_{nl} = −0.980019 mS and returns back to the real axis for g_{nl} = −0.3759132 mS.

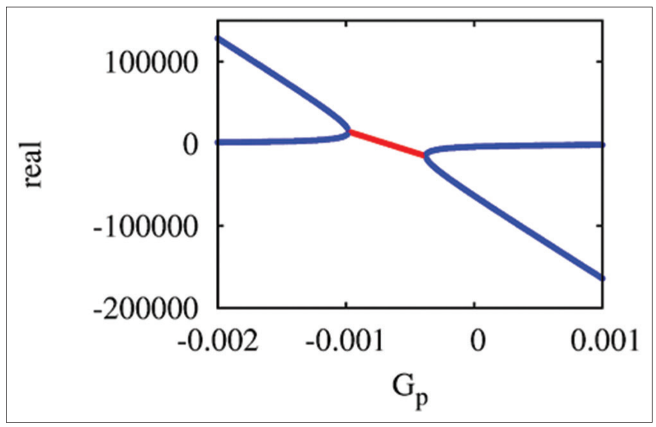

Figure 2: Real Eigen Values as Function of Gp

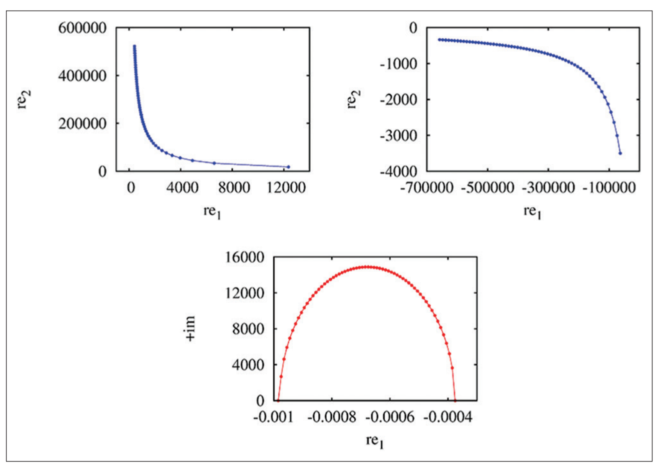

Figure 3: (a) Real Poles Trajectory for Positive Values of gnl. x-axis: re2, y-axis: re2, (b) Real Poles Trajectory for Negative Values of gnl. x-axis: re1,y-axis: re2, (c) Complex Pole Pair Trajectory. xaxis: Real Part, y-axis: Positive Imaginary Part.

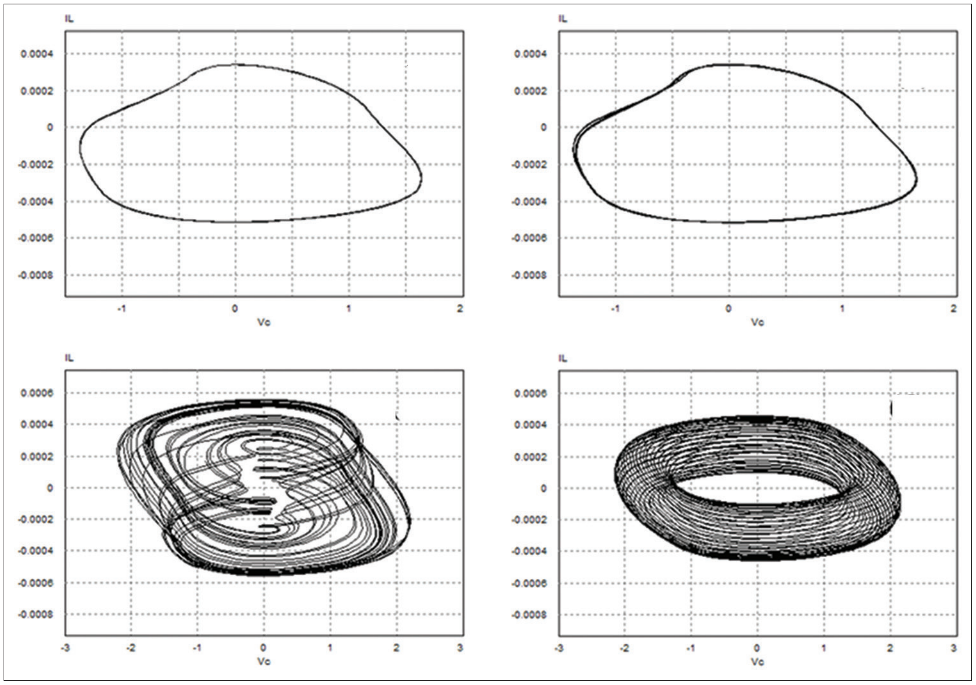

Figure 4: (a) Reverse Period-doubling and Quasiperiodic Routes to Chaos. (a) f = 0.457 V: Period-1 limit cycle; (b) f = 0.455 V: Period – 2 Limit Cycle: (c) f = 0.43V: Chaotic Attractor; (d) f = 0.25 V: Quasiperiodic Attractor

The trajectory of the complex pole pair moves from the right half plane to the left half plane as g_{nl} is increased from − 0.980019 mS and it crosses the imaginary axis for $g_{nl} = -0.6779661e{-}3$ where the real part fo the complex pole approaches zero and the complex pole becomes 14879.4532556 rps corresponding to the oscillatory frequency 2368.13853613 Hz. This is the value of G_p at which the system makes steady state oscillations and the reactance in the circuit becomes nearly zero.

Reverse Period-doubling and Quasiperiodic Routes to Chaos

With knowledge about the Eigen values of the system we may choose the frequency of the excitation deliberately to obtain the quasiperiodic and chaotic behavior by varying the amplitude of the independent voltage source. In the following PSIM, 9.0 is used for the simulations. With the discussions from previous sections, it is known that the circuit exhibits a stable limit cycle motion at the origin, under the autonomous case. However, it also gives rise to reverse period-doubling and quasiperiodic routes to chaos, in the non-autonomous case, as shown in the PSIM simulation of the circuit, by Figure 4.

The frequency of the external periodic force is fixed at $\omega = 7012$ rps and its amplitude is varied as the control parameter. A period-1 limit cycle obtained for f = 0.457 V undergoes hopf bifurcation and becomes a period-2 limit cycle for f = 0.455 V as shown in Figure 4a and b respectively. With a further decrease in amplitude, the chaotic attractor and torus are obtained for f=0.43 V and f = 0.25 V respectively, as shown in Figure 4c and d. Hence, the stable limit cycle obtained in the autonomous case gives rise to chaotic and quasiperiodc attractors in the non-autonomous case.

CONCLUSION

The behavior of the variant of the MLC circuit is investigated by means of a study on the Eigen values of the linearized Jacobian of the nonlinear differential equations. It is found that the autonomous system exhibiting supercritical hopf bifurcation in its dynamics gives rise to quasiperiodic and chaotic behavior in the non-autonomous circuit.

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