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Darcy – weish bach friction factor through porous rectanguler open – channels

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Abstract

Open channel flows widely occur in the design of irrigation canal net works.Most of the investigation on open channel flows are experimental studies only.This may be due to geometric complexity of channels and channel networks.The present paper deals with an analytical investigation on the flow through an open porous straight channel of rectangular cross-section under a constant pressure gradient.The momentum equation takes care of the convective acceleration and the viscous stresses generated in the flow region. The basic flow equation is solved (i) analytically employing Fourier series technique yielding exact solution and (ii) approximately by Galerkin technique that would facilitate fast numerical computation of the result. Expression for the Darcy-Weish Bach Friction factor has been computed for a wide spectrum of the values of the channel depth and porosity coefficient of the flow medium and its variation graphically illustrated. The results obtained by the two methods support each other.

Keywords: Open channel flows, Darcy-Weish Bach Friction factor, Fluid flow

INTRODUCTION

Fluid flows through pipes and channels have been a subject of intensive study by both experimental and analytical scientists-Hydraulic Engineers and Mathematicians for the last several centuries because of their wide applications. In a pipe flow, the pipe is completely filled with the fluids. As such it is subjected to hydraulic pressure head only and is not influenced by the atmospheric pressure.

In contrast to this, the channel flow, quite often referred as open channel flow has a free surface which is in contact with the atmosphere and so the atmosphere pressure influences the flow in the channel. Open channel flows are widely employed in the design of irrigation canal net works. Most of the investigations on open channel flows are experimental studies only. This may be due to geometric complexity of channels and channel networks. Many experimental studies on friction losses, wall shears, seepage and evaporation in flows through channels of different non-circular cross-sections are presented in the treatises of Bakhmeteff.B.A[1], Posey.C.S[2], Ven Te Chow[3] on the subject of Open Channels flows. The drop in the flow rate due to friction in channels is invariably expressed in terms of a friction factor which is referred in the literature as Darcy-Weish Bach friction factor (f).

An Emperical relation has been given to find the variation of the friction factor with equivalent hydraulic radius of the channels. The treatises cited above amply illustration such relation. Recently Prabhakar Reddy[4] studied analytically for the “Darcy’s Weish Bach’ friction factor for a wide spectrum of channel – cross sections Rectangular, Trapezoidal, Isosceles triangular, general triangle, semicircle, circular segments and parabola.

In this paper a problem is investigated analytically and approximately by Galerkin technique when fluid is viscous, incompressible homogenous and the channel is of rectangular cross section and flow medium is porous. The momentum equation adopted is the one given by Yamamoto.K and Yoshida.K[5] and Yamamoto.K and Iumamura [6] which takes into consideration of convective acceleration and the viscous stresses generated in the flow region. The channel walls are taken to be impermeable by allowing no cross flow into the channel a constant pressure gradient down the channel length. The basic flow equation is solved (i) analytically employing Fourier series technique yielding exact solution and (ii) approximately by Galerkin technique that would facilitate fast numerical computation of the result.

MATHEMATICAL FORMULATION

Consider a uniform free surface flow of a homogenous incompressible fluid of constant density ρ =□g and constant viscosity μ through a straight open channel of depth H. Consider a rectangular Cartesian frame O (XYZ) with the origin on the free surface, the axis of Z parallel to the flow direction, Y – axis vertically upwards (consequently the X and Y axis in the plane of the channel cross section – perpendicular to its length (i.e. the Z – direction).

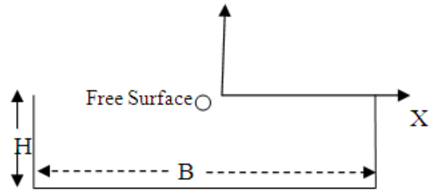

Fig 1. Channel cross section

With reference to this frame of reference the velocity is given by

$$\vec{V} = (o, o, W(X, Y))$$

which evidently satisfies the continuity equation

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div $\overline{V} = 0$ (1)

where $\overline{V}$ is the fluid velocity
The momentum equation in the Z – direction reduces to

$$\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} - \frac{W}{K} = \frac{-C}{\mu} \quad (2)$$

Here μ is the coefficient of viscosity of the fluid and K the porosity coefficient of the medium. Further C stands for the pressure gradient $C = -\frac{\partial P}{\partial Z}$, where P stands for the pizometric head in the channel.

Also the velocity W(x, y) satisfies the following boundary condition:

$w/_{Y=-H} = 0$ (3.1)
$w/_{Y=\pm B/2} = 0$ (3.2)
on the free surface which is stress free: $\frac{\partial W}{\partial Y}/_{Y=0} = 0$ (3.3)

Darcy – weishbach friction factor

The average velocity $\overline{W}$ in the channel defined by the equation

$$\overline{W} = \iint_A W dA \quad (4)$$

the integration being carried over the channel cross – section area A. The Darcy – Weish Bach friction factor in the channel(p – 8 of [3]) is given by the definition equation $f = \frac{8Rgs}{(\overline{W})^2} = \frac{K_1}{R^*}$ (5)
where R represents the Hydraulic radius

$$R = \frac{\text{Channel cross-section area}}{\text{Channel wetted perimeter}}, \quad s = \frac{\mu c}{\rho g} \text{ and } R^* = \frac{\gamma s^2}{Rg\mu^2} \quad (6)$$

Further K_1, is the parameter characteristic of the Darcy – Weish Bach Friction Factor associated with the channel cross – section.We introduce non-dimensional quantities as per the following scheme:

$$\left.\begin{array}{ll} X = Bx & Y = By,\ H = Bh \\ W = \frac{B^2 C}{\mu} w, & K = \frac{B^2}{\sigma} \end{array}\right\} \quad (7)$$

Flow rate Q = $\iint W dX dY$

Q = $\frac{B^4 C}{\mu} q$

Mean velocity

$$\overline{W} = \frac{Q}{BH} = \frac{B^2 C}{\mu h} . q$$

$$\overline{W} = \frac{B^2 C}{\mu} . \overline{w} \quad (8)$$

And q = $\iint w dx dy$ and

where $\overline{w} = \frac{q}{h}$

The basic equation and the boundary conditions in terms of the non – dimensional quantities introduced above is rewritten by

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} - \sigma^2 w = -1 \text{ within the rectangle} \quad (9)$$

on the bottom

$$w/_{y=-h} = 0 \quad (10)$$

on the sides

$$w/_{x=\pm\frac{1}{2}} = 0 \quad (11)$$

and on the free surface

$$\frac{\partial w}{\partial y}/_{y=0} = 0 \quad (12)$$

ANALYTICAL SOLUTION

Let the velocity field *w* (x, y) be split as

$w = w_1(y) + w_2(x, y)$ (13)

Substituting (13) in the (9) and separating the terms, we get the equation for $w_1(y)$

$$\frac{d^2 w_1}{dy^2} - \sigma^2 w_1(x, y) = -1 \quad (14)$$

with the conditions:

$$w_1(y)/_{y=-h} = 0 \quad (14.1)$$

$$\frac{dw_1}{dy}/_{y=-b} = 0 \quad (14.2)$$

and the equation for $w_2(x, y)$

$$\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2} - \sigma^2 w_2(x, y) = 0 \quad (15)$$

with the conditions

$$w_2(x, y)/_{y=-h} = 0 \quad (15.1)$$

$$\frac{\partial w_2}{\partial y}/_{y=0} = 0 \quad (15.2)$$

$$w_2/_{x=\pm\frac{1}{2}}(1/2, y) = -w_1(y) \quad (15.3)$$

The solution of the equation (14) satisfies the conditions (14.1) and (14.2) is

$$w_1 = \frac{1}{\sigma^2}\left[1 - \frac{Cosh(\sigma y)}{Cosh(\sigma h)}\right] \quad (16)$$

Let the solution of the equation (15) be assumed as

$$w_2 = \sum_{n=0}^{\infty} f_n\,(x).Cos\left(\frac{2n+1)}{2h}\right)\Pi y \quad (17)$$

where $f_n(x)$ satisfies the equation

$$f_n^{11}(x) - r_n^2 f_n(x) = 0 \quad (18)$$

with $r_n^2 = \left(\frac{(2n+1)\Pi}{2h}\right)^2 + \sigma^2$ (19)

The general solution of eq(18) is

$f_n(x)$ = A_n *Cosh* (r_n *x*) + B_n *Sinh* (r_n *x*) (20)

where A_n and B_n are constants to be determined
w is symmetric about the Y – axis, w_2 is also symmetric about Y – axis this leads to $B_n = 0$
Hence

$$w_2(x,y) = \sum_{n=0}^{\infty} A_n Cosh(r_n x) Cos\left(\frac{2n+1)}{2h}\right)\Pi y \quad (21)$$

The constants {A_n} can be determined by employing the boundary condition (11)

$$w_1(y) + w_2(\pm 1/2, y) = 0$$

$$w_2\left(\frac{1}{2}, y\right) = -w_1(y)$$

$$\sum_{n=0}^{\infty} A_n Cosh(r_n x) Cos\left(\frac{2n+1)}{2h}\right)\Pi y = \frac{1}{\sigma^2}\left[\frac{Cosh(\sigma y)}{Cosh(\sigma h)} - 1\right] \quad (22)$$

Expanding the R.H.S. as a Fourier series in the interval (- ½, ½) and comparing the like terms in the equation (22) we get

$$A_n = \frac{2(-1)^{n+1}}{h} \times \left[\frac{1}{\lambda_n (r_n)^2 Cosh(r_n/2)}\right] \quad (23)$$

The non dimensional flow rate

$$q = \iint w\,dxdy = \frac{1}{\sigma^3}\{\sigma h - \tanh(\sigma h)\} + \left\{\frac{4}{h}\frac{\sum(-1)^{2n+1}}{\lambda_n^2 \gamma_n^3}\tanh(\lambda_n/2)\right\} \quad (24)$$

and the average velocity

$$\bar{w} = \frac{1}{h}q = \frac{1}{h}\left\{\frac{1}{\sigma^3}\{\sigma h - \tanh(\sigma h)\} + \frac{4}{h}\sum_{n=0}^{\infty}(-1)^{2n+1}\frac{\tanh(\gamma_n/2)}{\lambda_n^2\gamma_n^3}\right\} \quad (25)$$

The average velocity in the dimensional form

$$\overline{W} = \frac{B^2 C}{\mu}\bar{w}$$

$$= \frac{B^2 C}{\mu h}\left\{\frac{1}{\sigma^3}\{\sigma h - \tanh(\sigma h)\} + \frac{4}{h}\sum_{n=0}^{\infty}(-1)^{2n+1}\frac{\tanh(\gamma_n/2)}{\lambda_n^2\gamma_n^3}\right\} \quad (26)$$

Using this, the Darcy-Weish Bach friction factor can be expressed as

$f = \frac{8Rgs}{\overline{W}^2}$, Here $s = \frac{\mu c}{\rho g}$, $R = \frac{Bh}{2h+1}$

$$f = \frac{8\mu c}{\rho} \times \frac{Bh}{(2h+1)} \times \frac{\mu^2 h^2}{B^4 C^2}\left\{\frac{1}{\sigma^3}\{\sigma h - \tanh(\sigma h)\} + \frac{4}{h}\sum_{n=0}^{\infty}(-1)^{2n+1}\frac{\tanh(\gamma_n/2)}{\lambda_n^2\gamma_n^3}\right\}^{-2} \quad (27)$$

Where, $\gamma_n^2 = \sigma^2 + \lambda_n^2 = \sigma^2 + \{(2n+1)\pi/2h\}^2$ (28)

For simplicity of the expression for the friction factor

$$F^* = \frac{f}{\frac{8\mu^3}{\rho C B^3}} = \frac{h^3}{(2h+1)}\left\{\frac{1}{\sigma^3}\{\sigma h - \tanh(\sigma h)\} + \frac{4}{h}\sum_{n=0}^{\infty}(-1)^{2n+1}\frac{\tanh(\gamma_n/2)}{\lambda_n^2\gamma_n^3}\right\}^{-2} \quad (29)$$

F* is computed for wide spectra of h and σ :
h = 0.1, 0.2, 0.3, 0.5, 1, 5, 10 ....
σ = 0.1, 0.2, 0.3, 0.5, 1, 5, 10 ....
and the results are illustrated in Figs 2 and 3.

APPROXIMATION SOLUTIONS EMPLOYING GALERKIN'S TECHNIQUE

Galerkin's one term approximation

Consider the function $\phi(x, y) = (x^2 - 1/4)\, y^2 (y + h)$ (30)

Let $w^{(1)}(x,y) = A_0^{(1)}(x^2 - 1/4)y^2(y+h)$ (31)

where $A_0^{(1)}$ is suitable constant
This evidently satisfy all the boundary conditions (10, 11, 12)
The error in the momentum equation by assuming (31) as solution is

$$\in^{(1)} = \frac{\partial^2 w^{(1)}}{\partial x^2} + \frac{\partial^2 w^{(1)}}{\partial y^2} - \sigma^2 w^{(1)} + 1$$

$$= A_0^{(1)}[2(y^3 + hy^2) + (x^2 - 1/4)(6y + 2h) - \sigma^2(y^3 + y^2 h)(x^2 - 1/4)] + 1 \quad (32)$$

The constant $A_0^{(1)}$ is so chosen that $<\in^{(1)}.\phi^{(1)}> = 0$

i.e. $$\int_{y=0}^{y=-h}\int_{x=-\frac{1}{2}}^{x=\frac{1}{2}} \in^{(1)}.\phi^{(1)}\,dxdy = 0 \quad (33)$$

This yields $A_0^{(1)} = \dfrac{-175}{4h\,(14 + 10\,h^2 + \sigma^2 h^2)}$

.

Using this, we get the flow rate $q^{(1)} = \iint w^{(1)}dxdy$ integrated over the channel Cross section

$$q^{(1)} = \left(\frac{h^4}{72}\right) A_0^{(1)}$$

Mean velocity $\overline{W} = \left(\frac{B^2 C}{\mu}\right)\frac{q}{h}$

$$= \left(\frac{B^2 C}{\mu}\right) \times \frac{1}{h}\left(\frac{h^4}{72}\right) A_0^{(1)}$$

Friction factor $f = \frac{8Rgs}{(\overline{W})^2}$ where $R = \frac{Bh}{2h+1}$ and $s = \frac{\mu c}{\rho g}$

$$= \left(\frac{8^5 \times 9^2 \times \mu^3}{\rho C B^3}\right) \times \frac{1}{25(2h+1)h^3\left[\frac{1}{5}+\frac{h^2}{7}+\frac{h^2\sigma^2}{35}\right]^2}$$

$$F^* = \frac{f}{\left(\frac{8\mu^3}{\rho C B^3}\right)} = \left(\frac{8^4 \times 9^2}{25}\right) \times \frac{1}{(2h+1)h^3}\left[\frac{1}{5}+\frac{h^2}{7}+\frac{h^2\sigma^2}{35}\right]^{-2}$$

F* is computed for wide spectra of values of h and σ ;
h = 0.1, 0.2, …..1, 5, …. and σ = 0.1, 0.2, …..1, 5, ….
and the results are illustrated in figs 4 & 6

Galerkin's two terms approximation

Let

$$w^{(2)}(x, y) = \left\{A_0^{(2)} + A_1^{(2)} x + B_1^{(2)} y\right\}(x^2 - 1/4) y^2 (y+h) \qquad (34)$$

where $A_0^{(2)}$, $A_1^{(2)}$, $B_1^{(2)}$ are constant to be decided.
This evidently satisfy all the boundary conditions (10, 11, 12)
The error in the momentum equation by assuming (34) to be solution is

$$\in^{(2)} = \frac{\partial^2 w^{(2)}}{\partial x^2} + \frac{\partial^2 w^{(2)}}{\partial y^2} - \sigma^2 w^{(2)} + 1$$

$$\begin{aligned}\in^{(2)} = {} & A_0^{(2)}[(2y^3+2y^2) + (6y+2h)(x^2-1/4) - \sigma^2(y^3+y^2h)(x^2-1/4) \\ & + A_1^{(2)} x[(6y^3+6y^2h) + (6y+2h)(x^2-1/4) - \sigma^2(y^3+y^2h)(x^2-1/4)] \\ & + B_1^{(2)} y[(2y^3+2y^2h) + (12y+6h)(x^2-1/4) - \sigma^2(x^2-1/4)(y^3+y^2h)] + 1\end{aligned} \qquad (35)$$

The constants $A_0^{(2)}, A_1^{(2)}, B_1^{(2)}$ are to be chosen so that

$< \in^{(2)} . \phi^{(2)} > = 0$ (36)

$< \in^{(2)} . x\phi^{(2)} > = 0$ (37)

and $< \in^{(2)} . y\phi^{(2)} > = 0$ (38)

From (36), (37), (38) we get

$$A_0^{(2)} = \frac{1}{40h}\left[\frac{5\left(\frac{1}{175}+\frac{h^2}{126\times 3}+\frac{h^2\sigma^2}{252\times 15}\right) - \frac{1}{2}\left(\frac{1}{25}+\frac{h^2}{42}+\frac{h^2\sigma^2}{420}\right)}{\frac{1}{5}\left(\frac{1}{5}+\frac{h^2}{7}+\frac{h^2\sigma^2}{70}\right)\left(\frac{1}{175}+\frac{h^2}{126\times 3}+\frac{h^2\sigma^2}{252\times 15}\right) - \frac{1}{8}\left(\frac{1}{25}+\frac{h^2}{42}+\frac{h^2\sigma^2}{420}\right)^2}\right]$$

$$A_1^{(2)} = 0$$

$$B_1^{(2)} = \frac{1}{120h}\left[\frac{\frac{5}{3}\left(\frac{1}{25}+\frac{h^2}{42}+\frac{h^2\sigma^2}{420}\right) - \frac{2}{5}\left(\frac{1}{5}+\frac{h^2}{7}+\frac{h^2\sigma^2}{70}\right)}{\frac{1}{5}\left(\frac{1}{5}+\frac{h^2}{7}+\frac{h^2\sigma^2}{70}\right)\left(\frac{1}{125}+\frac{h^2}{126\times 3}+\frac{h^2\sigma^2}{252\times 15}\right) - \frac{1}{8}\left(\frac{1}{25}+\frac{h^2}{42}+\frac{h^2\sigma^2}{420}\right)^2}\right]$$

Using these in [34], we get the flow rate

$$q^{(2)} = \iint w^{(2)} dxdy = \frac{h^4}{72} A_0^{(2)} - \frac{h^5}{120} B_1^{(2)}$$

$$\frac{h^4}{360}\left[5A_0^{(2)} - 3B_1^{(2)} h\right]$$

and this in the dimensional form as

$$Q^{(2)} = \frac{B^4 C}{\mu}\iint w dxdy = \frac{B^4 C}{\mu} q^{(2)}$$

and the mean velocity

$$\overline{W} = \frac{Q^{(2)}}{BH} = \frac{B^2 C}{\mu h} q^{(2)}$$

$$\frac{B^2 C h^3}{360\mu}\left[5A_0^{(2)} - 3B_1^{(2)} h\right]$$

Further $R = \frac{Bh}{2h+1}$, $s = \frac{\mu c}{\rho g}$

Hence the friction factor $f = \frac{8Rgs}{(\overline{W})^2}$

$$f = \left(\frac{8^5 \times 9^2 \times \mu^3}{\rho C B^3}\right)\frac{5^2 \times (75)^2}{(2h+1)h^3}\left[\frac{\frac{1}{5}\left(\frac{1}{5}+\frac{h^2}{7}+\frac{h^2\sigma^2}{70}\right)\left(\frac{1}{175}+\frac{h^2}{378}+\frac{h^2\sigma^2}{1260}\right) - \frac{1}{8}\left(\frac{1}{25}+\frac{h^2}{42}+\frac{h^2\sigma^2}{420}\right)^2}{-\frac{17}{14}+\frac{13h^2}{36}+\frac{47h^2\sigma^2}{315}}\right]$$

and

$$F^* = \left(\frac{f}{\frac{8\mu^3}{\rho C B^3}}\right)$$

$$= \left(8^4 \times 9^2 \times 5^2 \times (75)^2\right)\frac{1}{(2h+1)h^3}\left[\frac{\frac{1}{5}\left(\frac{1}{5}+\frac{h^2}{7}+\frac{h^2\sigma^2}{70}\right)\left(\frac{1}{175}+\frac{h^2}{378}+\frac{h^2\sigma^2}{1260}\right) - \frac{1}{8}\left(\frac{1}{25}+\frac{h^2}{42}+\frac{h^2\sigma^2}{420}\right)^2}{-\frac{17}{14}+\frac{13h^2}{36}+\frac{47h^2\sigma^2}{315}}\right]^2$$

F* is computed for wide spectra of h & σ ;
Numerical computation have been carried out for the variation of the friction factor F* vrs a wide spectrum of the depth h and porosity parameter σ
h = 0.1, 0.2, …..1, 5, ….
σ = 0.1, 0.2, …..1, 5, ….
The results are illustrated in fig 5 & 7.

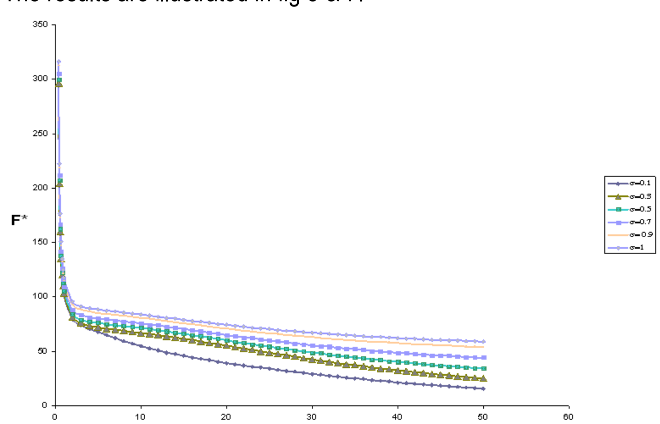

Fig 2. Variation of the Friction Factor F* with the channel depth h for different porosity coef. σ =0.1, 0.2 etc.

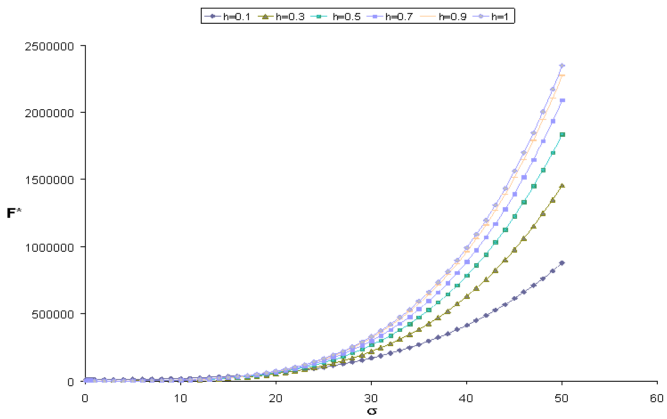

Fig 3. Variation of the Friction Factor F* Vs the porosity σ for different channel depth h.

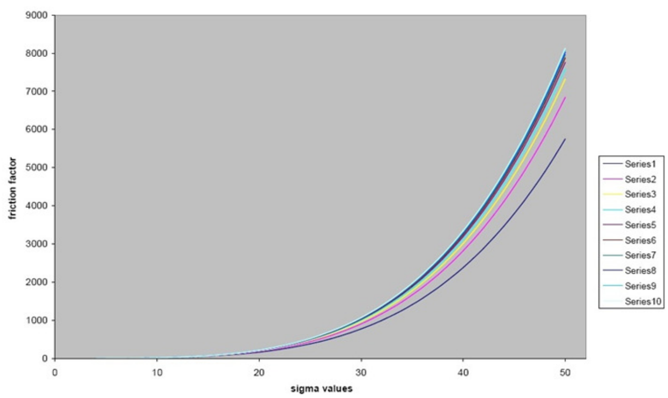

Fig 4. Friction factor Vs the porosity σ h = 1 to 10] Rectangular – I Approx.

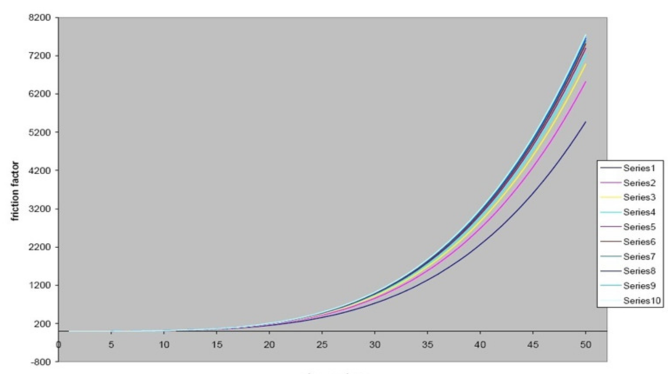

Fig 5. Friction factor Vs the porosity σ h = 1 to 10] Rectangular – II Approx.

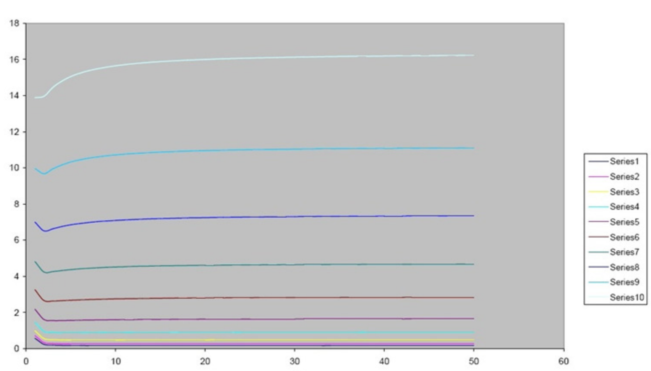

Fig 6. riction factor Vs height σ = 1 to 10] Rectangular – I Approx.

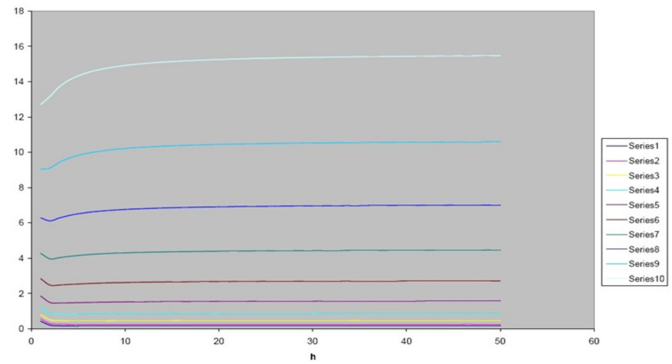

Fig 7. Friction factor Vs height σ= 1 to 10] Rectangular – II Approx

CONCLUSION

From the figs. 3, 4 and 5 it is observed that the friction factor increases with porosity coefficient σ increases. The figs.2, 6 and 7 shows that for h<1 not much variation of friction factor for different values of σ, the friction factor linearly falls. As channel depth h is increased. Beyond that it is asymptotic to the h axis rising as the porosity parameter increases. From figs.2, 6 and 7 the channel depth has no significant influence on friction factor upto the porosity coefficient σ = 20.5 approximately. Beyond that the friction factor increases as the porosity σ increases and increases with the channel depth.

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