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# A case study in three species ecological ammensalism

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**Abstract**
In this paper, A case in three species ecosystem is investigated numerically.This system is formed by a set of three first order non-linear simultaneous equations in $N_1$, $N_2$ and $N_3$.The relations among the natural growth rate of enemy species and the dominance reversal time between predator and enemy are established. In addition to it, the interactions among the three species are also discussed .Some observations are presented by Numerical study.

**AMS Classification:** 92 D 25, 92 D 40
**Keywords:** Ecosystem, prey, predator, Ammensal, Enemy, Normal steady state, stability.

## INTRODUCTION

Computational ecology is the scientific discipline, which concentrates on the study of ecological systems by using computational techniqques/numerical methods in the area of mathematical models.Efficient models may improve the understanding capacity of the natural world models by revealing how the dynamics of species populations are often based on biological requirements.It includes the fundamentals of applied mathematics,Computer scinces,Biolody and Genetics etc.for enhancing the knowledge to investigate the real life situations. Mathematical modeling of ecosystems was initiated in 1925 by Lotka [13] and in 1931 by Volterra [16]. The general concepts of modeling have been presented in the treatises of Meyer [14], Paul Colinvaux [15] Kapur [9,10] and several authors.. The ecological symbiosis can be broadly classified as Prey-predation, competition, mutualism, commensalism, Ammensalism. N.C. Srinivas [17] studied the competitive ecosystems of two and three species with limited and unlimited resources. Lakshminarayan and Pattabhiramacharyulu [11, 12] investigated Prey-predator Ecological models with a partial cover for the prey and alternate food for the predator. Recently Acharyulu and Pattabhi Ramacharyulu [1-8] investigated some remarkable results "on the stability of an enemy-Ammensal species pair with manifold conditions.

The present paper deals with the numerical study of three species system: Ammensal–prey, predator and enemy. Eight equilibrium points are obtained which are based on the model equations and these are spread over three distinct classes: (i) Fully washed out (ii) Semi/partially washed out and (iii) Co-existent states. The relations among the natural growth rate of enemy and the dominance reversal time($t_{23}^*= t^*$) are found by utilizing runge-kutta method of fourth order. More over the interactions among the three species are identified and some conclusions are obtained with the coherence of dominance reversal time.

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### Notation Adopted

| | |
|---|---|
| $N_1$ (t) | The population size of the Prey-Ammansal Species |
| $N_2$ (t) | The population size of the predator striving of the prey $N_1$ |
| $N_3$ (t) | The Population size of the enemy to the prey $N_1$ |
| $a_i$ | The natural growth rates of $N_i$, i = 1,2,3 |
| $a_{ii}$ | The rate of decrease of $N_i$ due to insufficient resources of, $N_i$ i = 1,2,3 |
| $a_{12}$ | The rate of decrease of the prey ($N_1$) due to inhibition by the predator ($N_2$) |
| $a_{13}$ | The rate of increase of the Ammansal ($N_1$) due to its successful promotion by enemy ($N_3$) |
| $a_{21}$ | The rate of increase of the predator ($N_2$) due to its successful attacks on the prey ($N_1$) |
| $K_i = a_i / a_{ii}$ | Carrying capacities of $N_i$ ,i = 1, 2, 3. |
| $\alpha = a_{13}/a_{11}$ | Co-efficient of Ammensalism. |
| $P = a_{12}/a_{11}$ | Co-efficient of prey inhibition (suffering) |
| $Q = a_{21}/a_{22}$ | Co-efficient of predator consumption of the prey. |

### Basic Equations

The model equations for a three species ecosystem are given by the following system of non-linear ordinary differential equations.

**i.** Equation for the growth rate of Prey-Ammensal species ($N_1$):

$$\frac{dN_1}{dt} = a_{11}N_1 ( K_1 - N_1 - PN_2 - \alpha N_3 \qquad (1)$$

ii. Equation for the growth rate of predator species ($N_2$):

$$\frac{dN_2}{dt} = a_{22} N_2 ( K_2 - N_2 + Q N_1 ) \qquad (2)$$

iii. Equation for the growth rate of enemy species ($N_3$):

$$\frac{dN_3}{dt} = a_{33} N_3 ( K_3 - N_3 ) \quad (3)$$

Further the variables $N_1$, $N_2$ and $N_3$ are non-negative and the model parameters $a_1$, $a_2$, $a_3$ $a_{11}$, $a_{22}$, $a_{33}$, $a_{12}$, $a_{13}$, $a_{21}$, $K_1$, $K_2$, $K_3$,$\alpha$, P, and Q are all assumed to be non-negative constants.

**Equilibrium States**

The system under investigation has eight equilibrium states given by $\frac{dNi}{dt} = 0$ ;i=1,2,3.

A. Fully washed out state:

$$\overline{N_1} = 0, \overline{N_2} = 0; \overline{N_3} = 0 \quad (4)$$

B. States in which two of the three species are washed out and the third is not.

$$\overline{N_1} = 0;\ \overline{N_2} = 0;\ \overline{N_3} = K_3 \quad (5)$$

$$\overline{N_1} = 0;\ \overline{N_2} = K_2;\ \overline{N_3} = 0 \quad (6)$$

$$\overline{N_1} = K_1;\ \overline{N_2} = 0;\ \overline{N_3} = 0 \quad (7)$$

C. Only one of the three species is washed out and the other two are not

$$\overline{N_1} = 0;\ \overline{N_2} = K_2;\ \overline{N_3} = K_3 \quad (8)$$

$$\overline{N_1} = K_1 - \alpha K_3;\ \overline{N_2} = 0;\ \overline{N_3} = K_3 \quad (9)$$

This state would exist when $K_1 > \alpha K_3$

$$\overline{N_1} = \frac{K_1 - PK_2}{1+PQ};\ \overline{N_2} = \frac{QK_1 + K_2}{1+PQ}; \overline{N_3} = 0 \quad (10)$$

This state would exist only when $K_1 > PK_2$

D. The co-existent state or normal steady state

$$\overline{N_1} = \frac{K_1 - PK_2 - \alpha K_3}{1+PQ};\quad \overline{N_2} = \frac{QK_1 + K_2 - Q\alpha K_3}{1+PQ};\quad \overline{N_3} = K_3 \quad (11)$$

This would exist only when $K_1 > PK_2 + \alpha K_3$ and $K_3 > K_1/\alpha Q$

**Change in the growth rate of enemy species($a_3$) with out influencing the dominance reversal time($t_{23}^* = t^*$) in a finite specific interval**

The values of the parameters are conceived as below:
Fixed parameters: $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5
Varying parameter: $a_3$ =0.0919, 0.1919, 0.2919, 0.3919, 0.4919, 0.5919, 0.6919, 0.7919, 0.8919, and 0.9919
The values are tabled as in Table-1.

Table 1.

| Case | $a_1$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $a_2$ | $a_{22}$ | $a_{21}$ | $a_3$ | $a_{33}$ | $N_{10}$= $N_{20}$= $N_{30}$ | t* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 0.0919 | 3.0521 | 0.5 | * |
| 2 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 0.1919 | 3.0521 | 0.5 | * |
| 3 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 0.2919 | 3.0521 | 0.5 | * |
| 4 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 0.3919 | 3.0521 | 0.5 | * |
| 5 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 0.4919 | 3.0521 | 0.5 | * |
| 6 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 0.5919 | 3.0521 | 0.5 | * |
| 7 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 0.6919 | 3.0521 | 0.5 | * |
| 8 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 0.7919 | 3.0521 | 0.5 | * |
| 9 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 0.8919 | 3.0521 | 0.5 | * |
| 10 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 0.9919 | 3.0521 | 0.5 | * |

The corresponding graphs are illustrated from Figure((1) to Figure(10) as shown below.

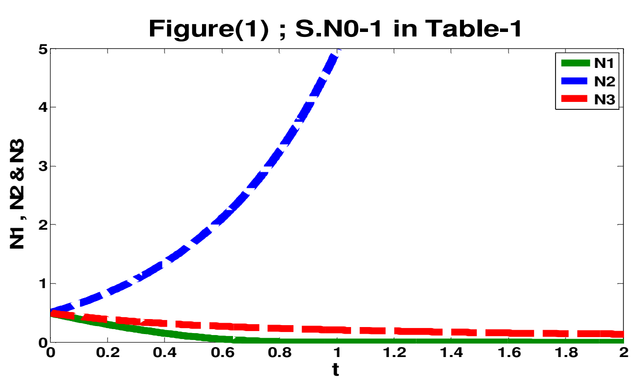


Fig 1. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =0.0919, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

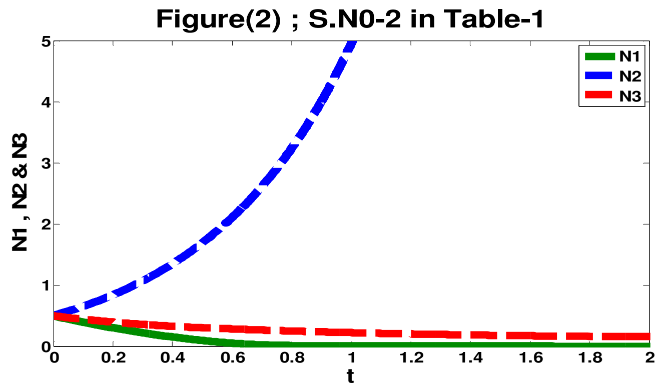


Fig 2. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =0.1919, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

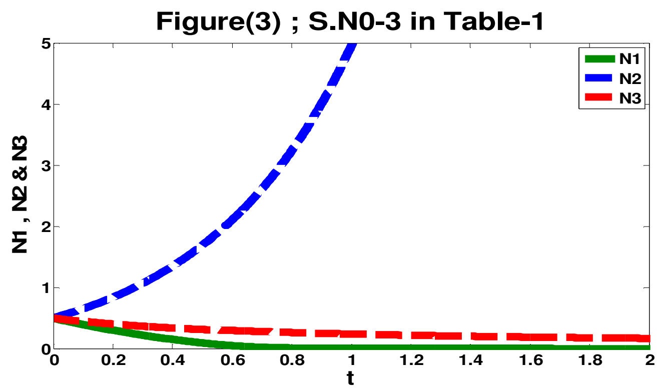


Fig 3. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =0.2919, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

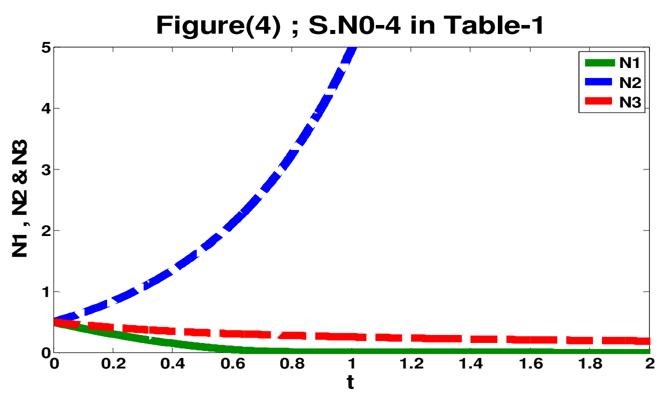


Fig 4. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =0.3919, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

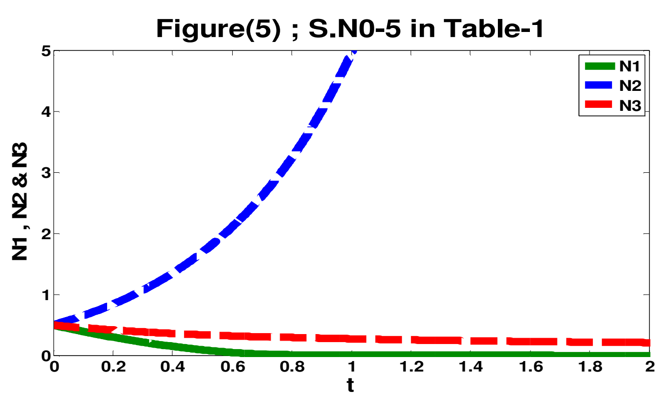


Fig 5. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =0.4919, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

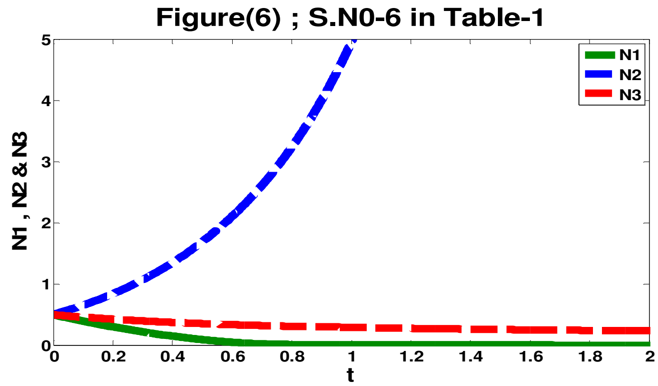


Fig 6. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =0.5919, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

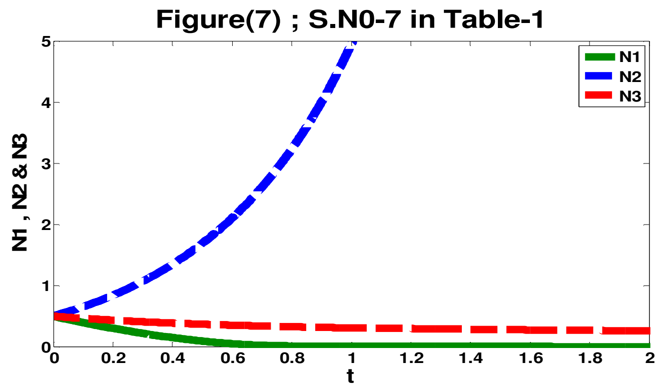


Fig 7. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =0.6919, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

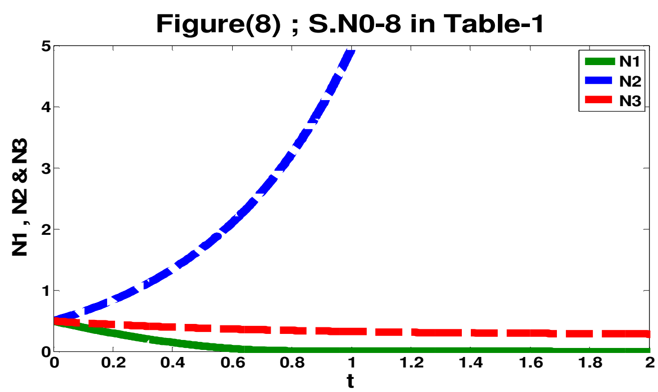


Fig 8. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =0.7919, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

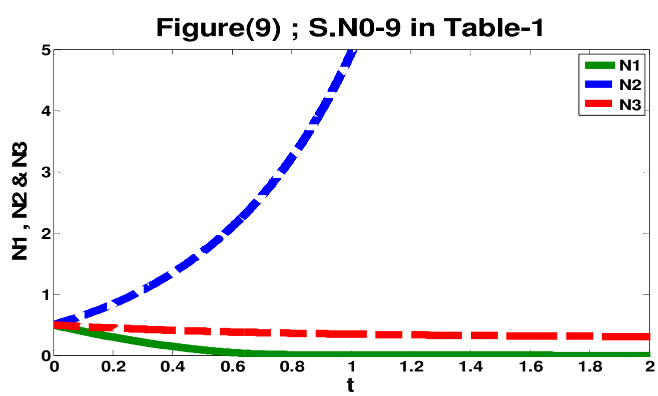


Fig 9. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =0.8919, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

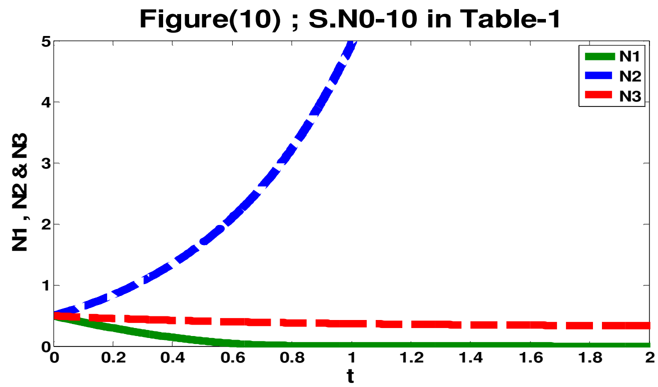


Fig 10. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =0.9919, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

## CONCLUSIONS

**The following observationa are derived with the help of above numerical study.**

I. The predator flourishes with an exponential growth rate and will not be influenced by Prey-Ammensal and enemy.

II. The enemy has a little bit growth rate than Prey-Ammensal,but finally it converges to wards the equilibrium point.

III. The prey-Ammensal has no growth rate through out the interval.In course of time,it is observed that,it declines to become extinct.

IV. No dominance reversal time is identified among three species in the considered interval.In this case the three species have no influence on one and other.They flourish or decline only with their natural growth rates but not on their biological conditions.

**The relation between the growth rate of enemy species ($a_3$) and the dominance reversal time($t_{23}^*$)**

The values of the parameters are considered as below: Fixed parameters: $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735,$a_2$=2.1263,$a_{22}$= 3.8963, $a_{21}$=5.0198, $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5.
Varying parameter: $a_3$ =1.1719, 2.1719, 3.1719, 4.1719, 5.1719, 6.1719, 7.1719, 8.1719, 9.1719, 10.1719, 11.1719, 12.1719, 13.1719, 14.1719, 15.1719, 16.1719, 17.1719, 18.1719, 19.1719, and 20.1719
The values are specified as in Table-2.

Table 2.

| Case | $a_1$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $a_2$ | $a_{22}$ | $a_{21}$ | $a_3$ | $a_{33}$ | $N_{10}$= $N_{20}$= $N_{30}$ | $t_{23}$* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 1.1719 | 3.0521 | 0.5 | * |
| 2 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 2.1719 | 3.0521 | 0.5 | 0.045 |
| 3 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 3.1719 | 3.0521 | 0.5 | 0.11 |
| 4 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 4.1719 | 3.0521 | 0.5 | 0.478 |
| 5 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 5.1719 | 3.0521 | 0.5 | 0.515 |
| 6 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 6.1719 | 3.0521 | 0.5 | 0.611 |
| 7 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 7.1719 | 3.0521 | 0.5 | 0.754 |
| 8 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 8.1719 | 3.0521 | 0.5 | 0.813 |
| 9 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 9.1719 | 3.0521 | 0.5 | 0.827 |
| 10 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 10.1719 | 3.0521 | 0.5 | 0.868 |
| 11 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 11.1719 | 3.0521 | 0.5 | 0.908 |
| 12 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 12.1719 | 3.0521 | 0.5 | 0.956 |
| 13 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 13.1719 | 3.0521 | 0.5 | 0.991 |
| 14 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 14.1719 | 3.0521 | 0.5 | 1.027 |
| 15 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 15.1719 | 3.0521 | 0.5 | 1.072 |
| 16 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 16.1719 | 3.0521 | 0.5 | 1.09 |
| 17 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 17.1719 | 3.0521 | 0.5 | 1.13 |
| 18 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 18.1719 | 3.0521 | 0.5 | 1.151 |
| 19 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 19.1719 | 3.0521 | 0.5 | 1.169 |
| 20 | 1.8495 | 2.4889 | 3.735 | 2.0619 | 2.1263 | 3.8963 | 5.0198 | 20.1719 | 3.0521 | 0.5 | 1.195 |

The corresponding graphs are obtained from Figure(11) to Figure(30) with the help of MATLAB.

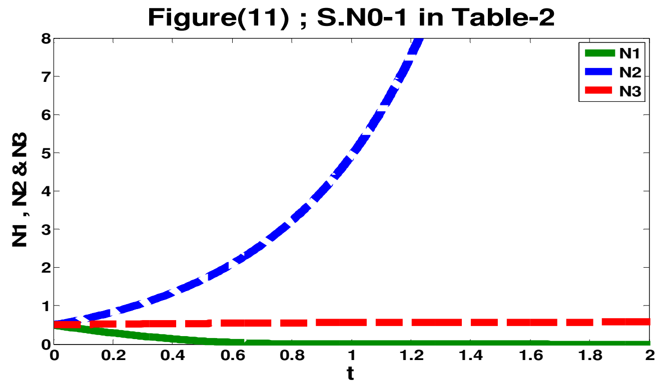


Fig 11. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =1.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

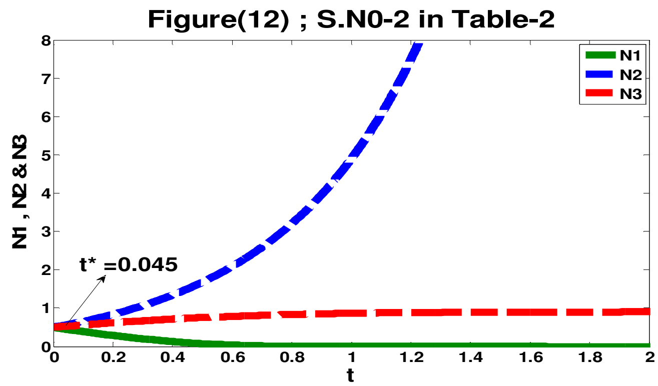


Fig 12. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =2.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

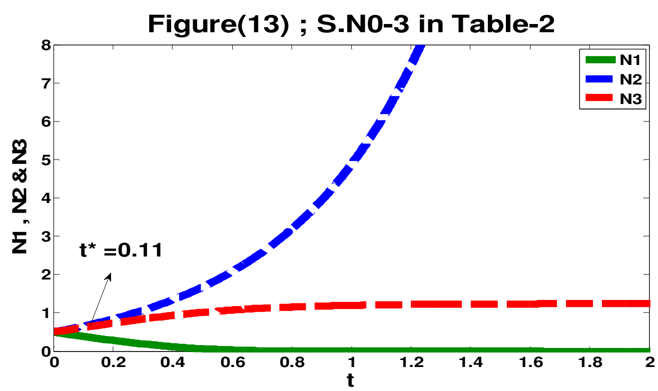


Fig 13. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =3.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

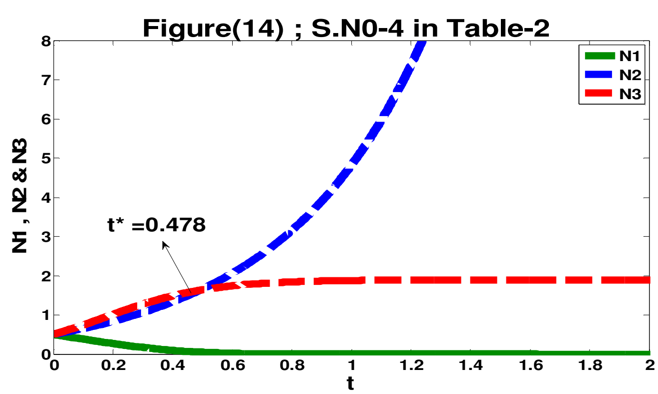


Fig 14. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =4.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

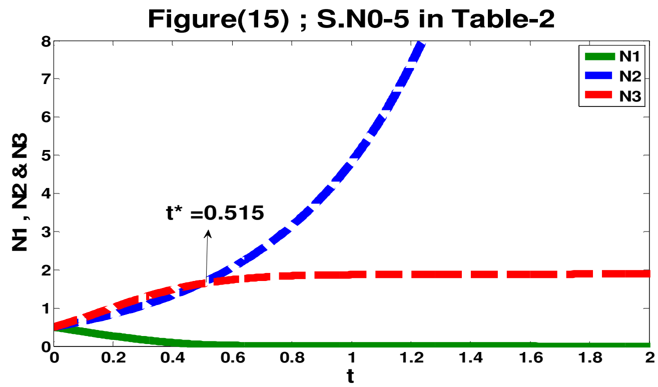


Fig 15. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =5.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

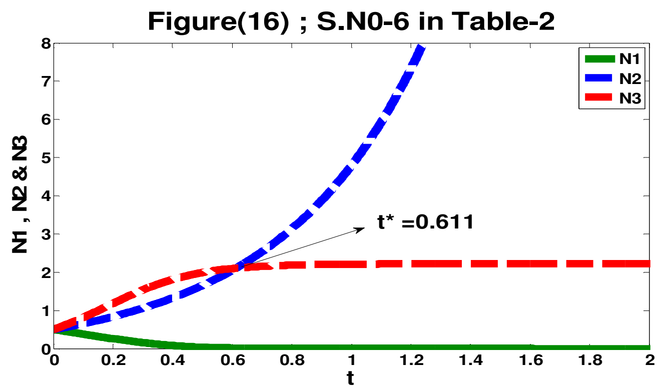


Fig 16. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =6.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

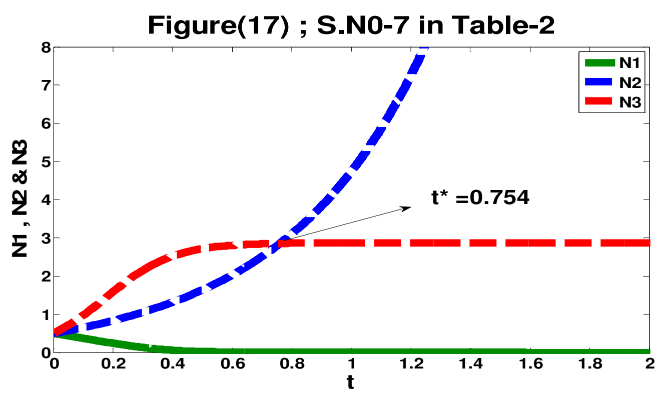


Fig 17. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =7.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

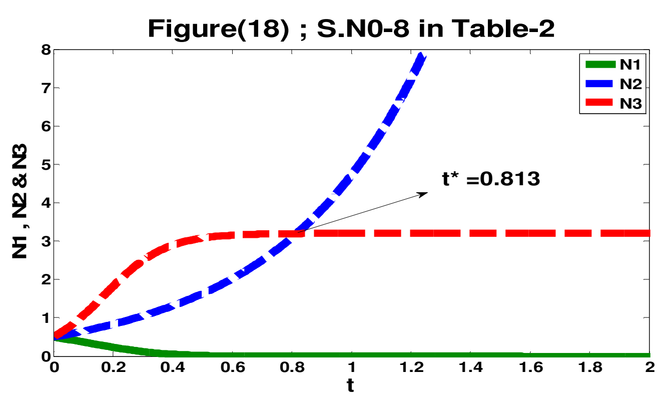


Fig 18.Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =8.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

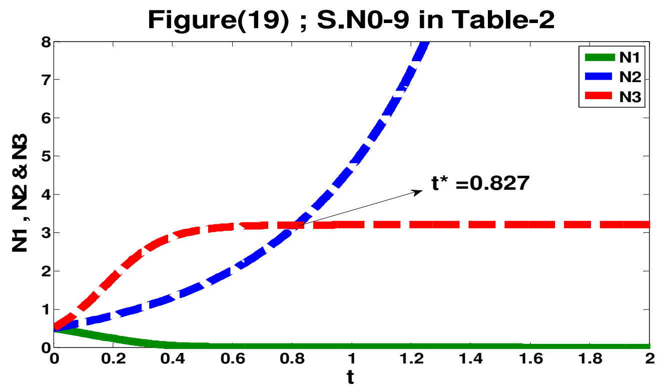


Fig 19. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =9.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

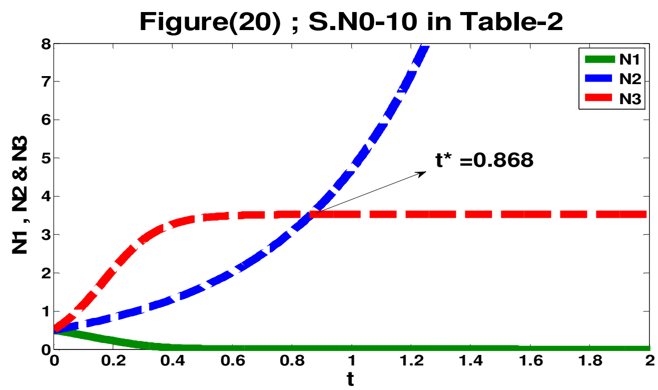


Fig 20. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =10.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

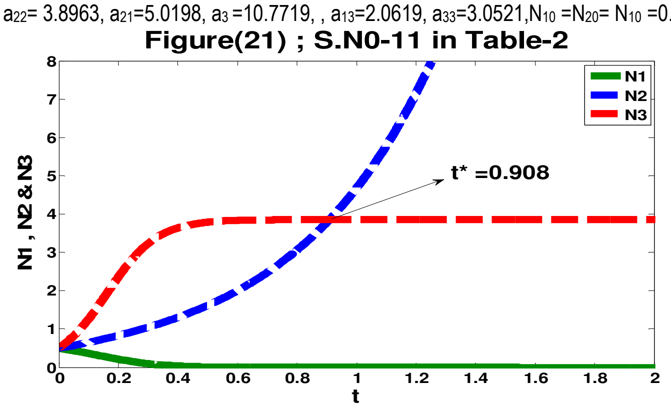


Fig 21. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =11.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

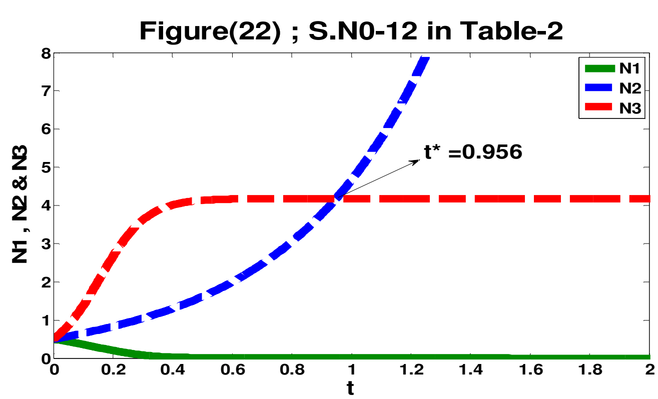


Fig 22. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =12.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

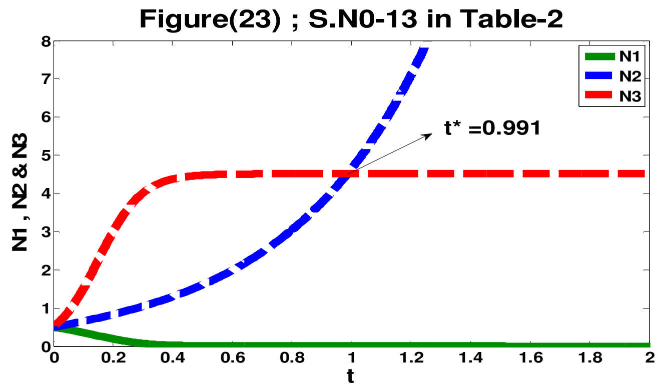


Fig 23. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =13.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

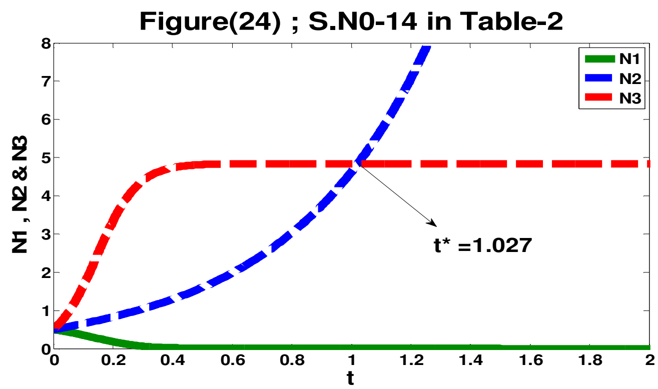


Fig 24. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =14.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

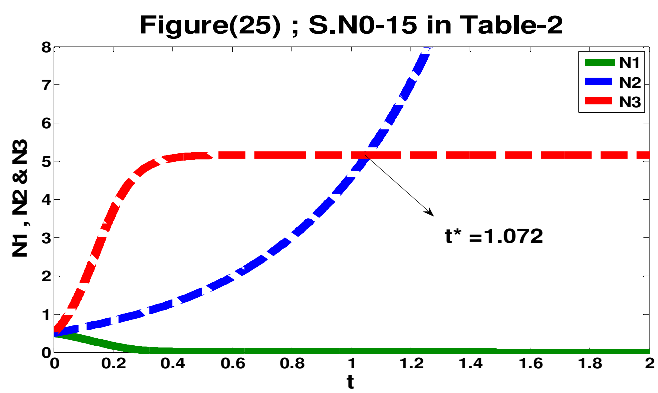


Fig 25. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =15.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

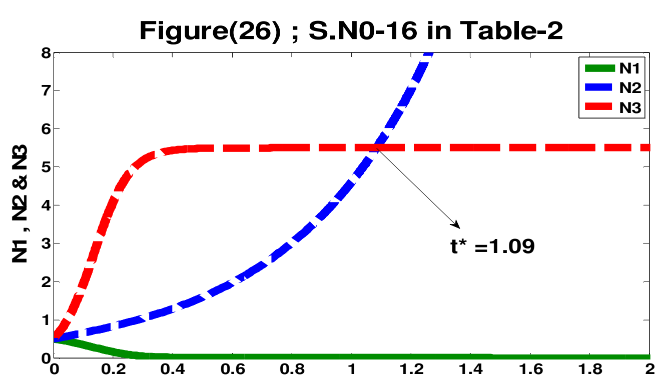


Fig 26. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =16.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

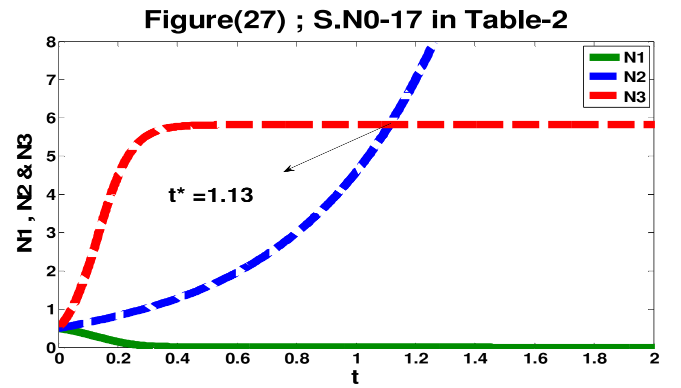


Fig 27. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =17.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

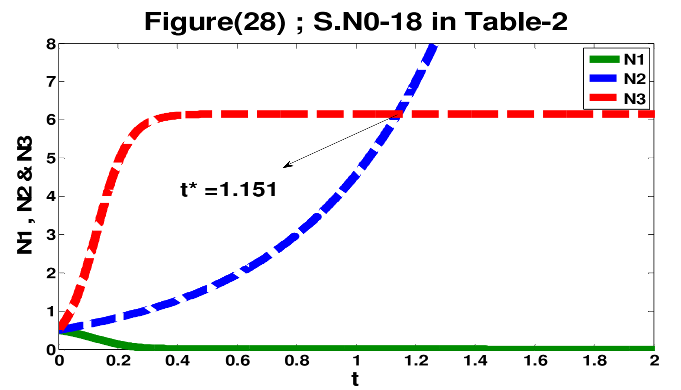


Fig 28. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =18.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

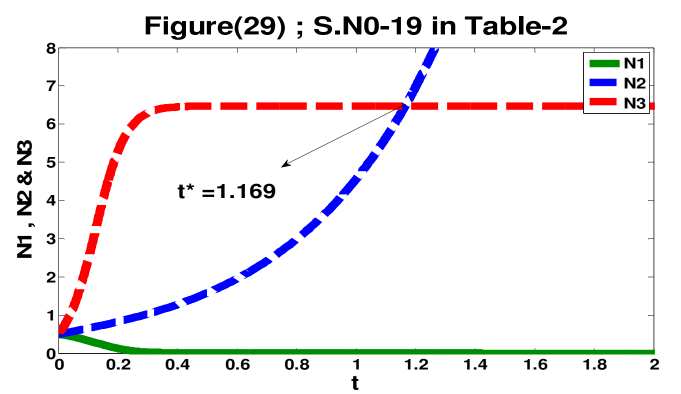


Fig 29. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =19.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

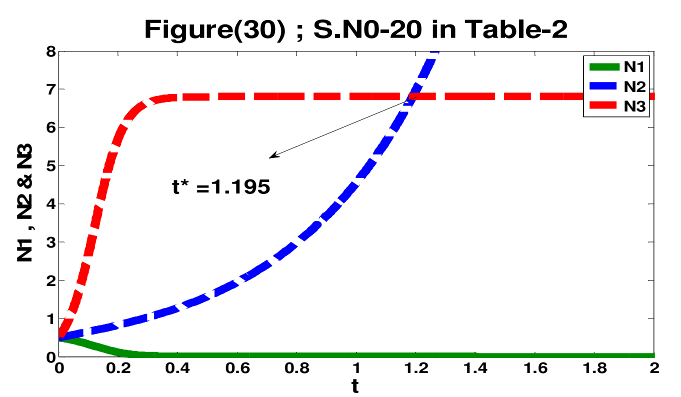


Fig 30. Variation of $a_3$ vs t*,when $a_1$=1.8495, $a_{11}$=2.4889, $a_{12}$=3.735, ,$a_2$=2.1263, $a_{22}$= 3.8963, $a_{21}$=5.0198, $a_3$ =20.7719, , $a_{13}$=2.0619, $a_{33}$=3.0521,$N_{10}$ =$N_{20}$= $N_{10}$ =0.5

The carrying capacity of enemy is defined by the ratio of the natural growth rate of enemy species and the decreased rate of enemy species (due to its own insufficient resources). The values of Carrying capacity of Ammensal-prey species in respect with the derived numerical solutions are tabulated in Table-3 along with the corresponding values of dominance reversal time($t^* = t_{23}^*$).

Table 3.

| S.NO | Carrying Capacity of enemy species($K_3$) | Dominance reversal time($t_{23}^*$) (between predator & enemy) |
|---|---|---|
| 1 | 0.5805 | * |
| 2 | 0.9081 | 0.045 |
| 3 | 1.2358 | 0.11 |
| 4 | 1.5634 | 0.478 |
| 5 | 1.8911 | 0.515 |
| 6 | 2.2187 | 0.611 |
| 7 | 2.5464 | 0.754 |
| 8 | 2.8740 | 0.813 |
| 9 | 3.2016 | 0.827 |
| 10 | 3.5293 | 0.868 |
| 11 | 3.8569 | 0.908 |
| 12 | 4.1846 | 0.956 |
| 13 | 4.5122 | 0.991 |
| 14 | 4.8399 | 1.027 |
| 15 | 5.1675 | 1.072 |
| 16 | 5.4952 | 1.09 |
| 17 | 5.8228 | 1.13 |
| 18 | 6.1504 | 1.151 |
| 19 | 6.4781 | 1.169 |
| 20 | 6.8057 | 1.195 |

## CONCLUSIONS

**I.** The Prey-Ammensal declines towards the interval and there is no appreciable growth rate in Prey-Ammensal.

**II.** Prey-Ammensal Species increases with sufficient growth rate and not effected by the remaining two species in any manner.

**III.** Enemy fights against and tries to influence on Prey - Ammensal species in the course of time.But prey - Ammensal will not be effected by enemy species and becomes a cause to be neutrally stable.It is also observed that enemy will be at a constant distance from equilibrium point.

## OVER ALL CONCLUSIONS

| Criterion | Conclusion |
|---|---|
| The natural growth of enemy species increases in a three species ecosystem | The enemy gradually increases up to some level and then it becomes neutrally stable with a constant distance. |
| | The carrying capacity of enemy increases |
| | The dominance reversal time $t^*_{23}$ between predator and enemy increases step by step |
| | The predator flourishes with exponential growth rate |

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