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Mathematical model of gonorrhea in a hetero sexuals with time dependent population

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Abstract

This paper deals with a mathematical model of gonorrhea among hetero-sexual. This model composed of males and females is characterizes growth rates of promiscuous population (P), Infective male population (I_1), and Infective female population (I_2). In all three equilibrium points are identified for the system under investigation, the criterion for the asymptotic stability of all three possible equilibrium points is derived of those, purely healthy state is stable under the condition $F.M$ <1 and co-existence state is stable with the condition $F.M$ >1.

Keywords: Mathematical model, Gonorrhea, population

INTRODUCTION

Mathematical model of infectious diseases is a large sub field of mathematical biology. Mathematical models provide an explicit framework to understand biological systems that cannot be observed directly. Seminal mathematical work was done by Hethcote,York [5] on gonorrhea transmission of the disease in the form of a set of three simultaneous nonlinear first order differential equations. Cook and York [4] developed a model for gonorrhea involving susceptibles, infectives and removals. The model of gonorrhea for heterogeneous population was given by Braun [3].Stability of gonorrhea is given by Beretta and Capasso [2]. N.C Sreenivas and N.Ch. Pattabhi Ramacharyulu [8] investigated stability of time delay gonorrhea. R.Ramakishore, N.Ch.PattabhiRamacharyulu [6] derived the stability criteria for gonorrhea in heterogeneous population by considering time dependent population as variable.

As gonorrhea symptoms can be identified earlier in male then the females [5]. Male infective cure rate is greater that for females. The present investigation is an analytical study of gonorrhea among Hetero-sexual population. We have identified biologically feasible equilibria for the system namely (1) Trivial study state (2) Disease free steady state (3) Endemic equilibrium state.

Notation adopted

$P(t)\rightarrow$ Total number of promiscuous individuals in the population.
$P_1(t)\rightarrow$ Number of promiscuous males in the population.(αP)
$P_2(t)\rightarrow$ Number of promiscuous females in the population. ($(1-\alpha)P$)
$I_1(t)\rightarrow$ Number of infective males in population.
$I_2(t)\rightarrow$ Number of infective females in population.
$a_1\rightarrow$ Natural growth rate of total promiscous population.
$a_{11}\rightarrow$ Natural self inhibition coefficient of total promiscuous population.
$b_1\rightarrow$ Infective rate in susceptible male population.
$b_2\rightarrow$ Infective rate in susceptible female population.
$c_1\rightarrow$ Cure rate in infective male population.
$c_2\rightarrow$ Cure rate in infective female population.
$k\rightarrow$ Carrying Capacity (a_1/a_{11}) for the total population.
$M\rightarrow$ Maximal Male contact rate.
$F\rightarrow$ Maximal Female contact rate.

here a_1, a_{11}, c_1, c_2, b_1, b_2 are assumed to be non negative constants and $0<\alpha<1$.

Basic Equations

The model equations for Hetero-Sexuals are governed by the following system of nonlinear ordinary differential equations.

I. Equation for the logistic growth rate of promiscuous population (P)

$$\frac{dP}{dt}=(a_1-a_{11}P)P \tag{3.1}$$

II. Equation for growth rate of Infective male population (I^1)

$$\frac{dI_1}{dt}=b_1(\alpha P-I_2)I_1-c_1I_1 \tag{3.2}$$

III. Equation for growth rate of Infective female population (I_2)

$$\frac{dI_2}{dt}=b_2(\alpha P-I_1)I_2-c_2I_2 \tag{3.3}$$

Equilibrium Points

The system has three equilibrium points:

1. Trivial steady state

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$\overline{P}=0,\ \bar{I}_1=0,\ \bar{I}_2=0$

2. Disease free steady state

$\overline{P}=\frac{a_1}{a_{11}}(say\ k),\ \bar{I}_1=0,\ \bar{I}_2=0$

3. Endemic equilibrium state

$\overline{P}=k,\ \bar{I}_1=\frac{b_1b_2\alpha(1-\alpha)P^2-c_1c_2}{b_1b_2\alpha(1-\alpha)P+c_1b_2},\ \bar{I}_2=\frac{b_1b_2\alpha(1-\alpha)P^2-c_1c_2}{b_1b_2\alpha P+c_2b_1}$

Stability Criteria of Equilibrium States

let u , v and w are small perturbations from any of equilibrium levels say $(\overline{P},\bar{I}_1,\bar{I}_2)$ of P(t),I_1(t),I_2(t) respectively.

i.e. $P=\overline{P}+u,\ I_1=\overline{I_1}+v,\ I_2=\overline{I_2}+\mathrm{w}$

after neglecting higher order terms of u, v, w, we get the system of linearized perturbed equations are given by $\frac{dX}{dt}=AX$

where A = $\begin{bmatrix} a_1-2a_{11}\overline{P} & 0 & 0 \\ b_1\overline{I_2}\alpha & -(b_1\overline{I_2}+c_1) & b_1(\alpha\overline{P}+\overline{I_1}) \\ b_2\overline{I_1}(1-\alpha) & b_2((1-\alpha)\overline{P}+\overline{I_2}) & -(b_2\overline{I_1}+c_2) \end{bmatrix}$

and $X=[u,v,w]^T$

Theequilibrium state is stable if all eigen values of the characteristic matrix A are negative or have negative real parts according as the roots are real or complex.

Trivial Steady state

i.e. $\overline{P}=0,\ \bar{I}_1=0,\ \bar{I}_2=0$

Corresponding linearized perturbed equations are

$$\frac{du}{dt}=a_1u,\quad \frac{dv}{dt}=-c_1v,\quad \frac{dw}{dt}=-c_2w \qquad (5.1.1)$$

and A= $\begin{bmatrix} a_1 & 0 & 0 \\ 0 & -c_1 & 0 \\ 0 & 0 & -c_2 \end{bmatrix}$

The characteristic roots are a_1, $-c_1$, $-c_2$ which are all non negative. Hence this equilibrium point is repulsive in u-t plane and attracting in v-t and w-t plane and it is a saddle point. Hence it is unstable.

By solving the equations (5.1.1) we get

$$u=u_0e^{a_1t},\quad \mathrm{v}=v_0e^{-c_1t},\quad \mathrm{w}=w_0e^{-c_2t}$$

Where u_o, v_o, w_o are the initial values of u, v and w respectively.

Trajectories of the perturbations:

Trajectories of above equations in u-v and v-w planes are given by

$$\left[\frac{u}{u_0}\right]^{\frac{1}{a_1}}=\left[\frac{v}{v_0}\right]^{\frac{-1}{c_1}} \text{ and } \left[\frac{v}{v_0}\right]^{\frac{-1}{c_1}}=\left[\frac{w}{w_0}\right]^{\frac{-1}{c_2}}$$

$$\Rightarrow \left[\frac{u}{u_0}\right]^{c_1c_2}=\left[\frac{v}{v_0}\right]^{-a_1c_2}=\left[\frac{w}{w_0}\right]^{-a_1c_1}$$

Disease Free Steady State

i.e $\overline{P}=k,\ \bar{I}_1=0,\ \bar{I}_2=0$

Corresponding linearized perturbed equations are

$$\frac{du}{dt}=-a_1u,\quad \frac{dv}{dt}=-c_1v+b_1\alpha kw,\quad \frac{dw}{dt}=b_2(1-\alpha)kv-c_2w \qquad (5.2.1)$$

here A= $\begin{bmatrix} a_1 & 0 & 0 \\ 0 & -c_1 & b_1\alpha k \\ 0 & b_2(1-\alpha)k & -c_2 \end{bmatrix}$

Characteristic roots of which are

$-a_1,\ \frac{-1}{2}[R_1+(c_1+c_2)],\ \frac{1}{2}[R_1-(c_1+c_2)]$

Where $R_1=\sqrt{(c_1-c_2)^2+4b_1b_2\alpha(1-\alpha)k^2}$ 5.2.1

Case 1 :- If $R_1>c_1+c_2$

i.e. $\frac{b_1b_2\alpha(1-\alpha)k^2}{c_1c_2}>1$ This can be interpreted as *F.M* > 1

here F = $\frac{b_1\alpha k}{c_2}$ is maximal female contact rate

and M = $\frac{b_2(1-\alpha)k}{c_1}$ is maximal male contact rate

roots are $-a_1,\ \frac{-1}{2}[R_1+(c_1+c_2)],\ \frac{1}{2}[R_1-(c_1+c_2)]$

hence the state is unstable By solving the equations (5.2.1) we get

$$u=u_0e^{-a_1t},\quad \mathrm{v}=A_1e^{\lambda_1t}+A_2e^{\lambda_2t},\quad \mathrm{w}=A_3e^{\lambda_1t}+A_4e^{\lambda_2t} \qquad (5.2.1.1)$$

Where $A_1=\frac{v_0(c_2+\lambda_1)+b_1\alpha kw_0}{(\lambda_1-\lambda_2)}$, $A_2=\frac{v_0(c_2+\lambda_2)+b_1\alpha kw_0}{(\lambda_2-\lambda_1)}$,

$A_3=\frac{w_0(c_1+\lambda_1)+b_1(1-\alpha)kv_0}{(\lambda_1-\lambda_2)}$ $A_4=\frac{w_0(c_1+\lambda_2)+b_2(1-\alpha)kv_0}{(\lambda_2-\lambda_1)}$,

$\lambda_1=\frac{-1}{2}[R_1+(c_1+c_2)]$, $\lambda_2=\frac{1}{2}[R_1-(c_1+c_2)]$

Trajectories of perturbed equations

Trajectories of above equations are given by

$$\left[\frac{u}{u_0}\right]^{\frac{-1}{a_1}}=\left[\frac{v}{A_1A_2}\right]^{\frac{-1}{c_1+c_2}} \text{ and } \left[\frac{v}{A_1A_2}\right]^{\frac{-1}{c_1+c_2}}=\left[\frac{w}{A_3A_4}\right]^{\frac{-1}{c_1+c_2}}$$

$$\Rightarrow \left[\frac{u}{u_0}\right]^{-(c_1+c_2)^2}=\left[\frac{v}{A_1A_2}\right]^{-a_1(c_1+c_2)}=\left[\frac{w}{A_3A_4}\right]^{-a_1(c_1+c_2)}$$

Where A_1, A_2, A_3, A_4 are same as above mentioned.

Case 2 :- If $R_1 = c_1 + c_2$

i.e. $\frac{b_1 b_2 \alpha(1-\alpha)k^2}{c_1 c_2} = 1$ This can be interpreted as *F.M*= 1The characteristic roots are $-a_1$, $-(c_1+c_2)$, 0 hence the state is unstable.

By solving the equations (5.2.1) we get

$$u = u_0 e^{-a_1 t}, \quad v = A_1 e^{-(c_1+c_2)t} + A_2, \quad w = A_3 e^{-(c_1+c_2)t} + A_4 \qquad (5.2.2.1)$$

Trajectories of perturbed equations

Trajectories of above equations are given by

$$\left[\frac{u}{u_0}\right]^{-(c_1+c_2)^2} = \left[\frac{v}{A_1A_2}\right]^{-a_1(c_1+c_2)} = \left[\frac{w}{A_3A_4}\right]^{-a_1(c_1+c_2)}$$

Where A_1, A_2, A_3, A_4 are same as above mentioned.

Case 3 :- If $R_1 < c_1 + c_2$

i.e. $\frac{b_1 b_2 \alpha(1-\alpha)k^2}{c_1 c_2} < 1$ This can be interpreted as *F.M* < 1

The characteristic roots are

$-a_1$, $\frac{-1}{2}[R_1 + (c_1+c_2)]$, $\frac{-1}{2}[(c_1+c_2) - R_1]$ hence the state is stable.

Hence we can state following theorem

Theorem-1: *The system (3.1), (3.2), (3.3) is stable around the disease free study state (k, 0, 0) when F.M* < 1.

By solving the above equations we get

$$u = u_0 e^{-a_1 t}, \quad v = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}, \quad w = A_3 e^{\lambda_1 t} + A_4 e^{\lambda_2 t} \qquad (5.2.3.1)$$

where *A_1, A_2, A_3, A_4* are same as above.

Trajectories of perturbations:

Trajectories of above equations are given by

$$\left[\frac{u}{u_0}\right]^{\frac{-1}{a_1}} = \left[\frac{v}{A_1A_2}\right]^{\frac{-1}{c_1+c_2}} \quad \text{and} \quad \left[\frac{v}{A_1A_2}\right]^{\frac{-1}{c_1+c_2}} = \left[\frac{w}{A_3A_4}\right]^{\frac{-1}{c_1+c_2}}$$

$$\Rightarrow \left[\frac{u}{u_0}\right]^{-(c_1+c_2)^2} = \left[\frac{v}{A_1A_2}\right]^{-a_1(c_1+c_2)} = \left[\frac{w}{A_3A_4}\right]^{-a_1(c_1+c_2)}$$

Where A_1, A_2, A_3, A_4 are same as above mentioned.

Endemic equilibrium state

i.e. $\bar{P} = k$, $\bar{I}_1 = \frac{b_1 b_2 \alpha(1-\alpha)P^2 - c_1 c_2}{b_1 b_2 \alpha(1-\alpha)P + c_1 b_2}$, $\bar{I}_2 = \frac{b_1 b_2 \alpha(1-\alpha)P^2 - c_1 c_2}{b_1 b_2 \alpha P + c_2 b_1}$

This exist only when $b_1 b_2 \alpha(1-\alpha)k^2 - c_1 c_2 > 0$ i.e F.M > 1.

Corresponding linearized perturbed equations are

$$\frac{du}{dt} = -a_1 u, \quad \frac{dv}{dt} = (b_1 \alpha \overline{I_2})u - M_1 v + N_1 w,$$

$$\frac{dw}{dt} = (b_2(1-\alpha)\overline{I_1})u + N_2 v - M_2 w \qquad (5.3.1)$$

weith $A = \begin{bmatrix} a_1 & 0 & 0 \\ b_1 \alpha \overline{I_2} & -M_1 & N_1 \\ b_2(1-\alpha)\overline{I_1} & N_2 & -M_2 \end{bmatrix}$

here $M_1 = \frac{\alpha k b_2 [b_1(1-\alpha)k + c_1]}{b_2 \alpha k + c_2}$,

$$M_2 = \frac{(1-\alpha)k b_1 [b_2 \alpha k + c_2]}{b_1 \alpha k + c_1}$$

$$N_1 = \frac{c_1 b_1 [b_2 \alpha k + c_2]}{b_2[(1-\alpha)b_1 k + c_1]}, \quad N_2 = \frac{c_2 b_2 [b_1(1-\alpha)k + c_1]}{b_1[b_2 \alpha k + c_2]}$$

and roots are $-a_1$, $\frac{-1}{2}[R_2 + (M_1+M_2)]$, $\frac{1}{2}[R_2 - (M_1+M_2)]$

here $R_2 = \sqrt{(M_1 - M_2)^2 + 4c_1c_2}$

in this case $R_2 < M_1 + M_2$ since

$b_1 b_2 \alpha(1-\alpha)k^2 - c_1 c_2 > 0$

all roots are negative, hence the state is stable.

Hence we can state following theorem

Theorem-2: *The system (3.1), (3.2), (3.3) is stable around the Endemic equilibrium state when F.M* > 1.

By solving the equations (5.3.1) we get

$$u = u_0 e^{-a_1 t}, \quad v = A_5 e^{-a_1 t} + A_6 e^{\lambda_1 t} + A_7 e^{\lambda_2 t},$$

$$w = A_8 e^{-a_1 t} + A_9 e^{\lambda_1 t} + A_{10} e^{\lambda_2 t} \qquad (5.3.2)$$

Here $A_5 = \frac{a_1(a_1 v_0 - a) + b}{(a_1+\lambda_1)(a_1+\lambda_2)}$, $A_6 = \frac{\lambda_1(\lambda_1 v_0 + a) + b}{(a_1+\lambda_1)(\lambda_1-\lambda_2)}$,

$$A_7 = \frac{\lambda_2(\lambda_2 v_0 + a) + b}{(a_1+\lambda_2)(\lambda_2-\lambda_1)}, \quad A_8 = \frac{a_1(a_1 w_0 - p) + q}{(a_1+\lambda_1)(a_1+\lambda_2)},$$

$$A_9 = \frac{\lambda_1(\lambda_1 w_0 + p) + q}{(a_1+\lambda_1)(\lambda_1-\lambda_2)}, \quad A_{10} = \frac{\lambda_2(\lambda_2 w_0 + p) + q}{(a_1+\lambda_2)(\lambda_2-\lambda_1)}.$$

$$a = v_0(M_2 + a_1) + L_1 + N_1 w_0,$$

$$b = v_0 M_2 a_1 + M_2 L_1 + N_1 w_0 a_1 + N_1 L_2$$

$$p = w_0(M_1 + a_1) + L_2 + N_2 v_0,$$

$$q = w_0 M_1 a_1 + M_1 L_2 + N_2 v_0 a_1 + N_1 L_1$$

$$\lambda_1 = -\frac{1}{2}[R_2 + (M_1+M_2)],$$

$$\lambda_2 = -\frac{1}{2}[(M_1+M_2) - R_2]$$

Trajectories of the perturbed equations

Trajectories of above equations are given by

$$\left[\frac{u}{u_0}\right]^{\frac{-1}{a_1}} = \left[\frac{v}{A_5A_6A_7}\right]^{\frac{-1}{M_1+M_2+a_1}} \quad \text{and} \quad \left[\frac{v}{A_5A_6A_7}\right]^{\frac{-1}{M_1+M_2+a_1}} = \left[\frac{w}{A_8A_9A_{10}}\right]^{\frac{-1}{M_1+M_2+a_1}}$$

$$\left[\frac{u}{u_0}\right]^{\frac{-1}{a_1}} = \left[\frac{v}{A_5A_6A_7}\right]^{\frac{-1}{M_1+M_2+a_1}} = \left[\frac{w}{A_8A_9A_{10}}\right]^{\frac{-1}{M_1+M_2+a_1}}$$

Where A_5, A_6, A_7, A_8, A_9, A_{10} are same as above mentioned.

A Numerical approach

Solving equation (3.1) and substituted in (3.2) and (3.3) then we get

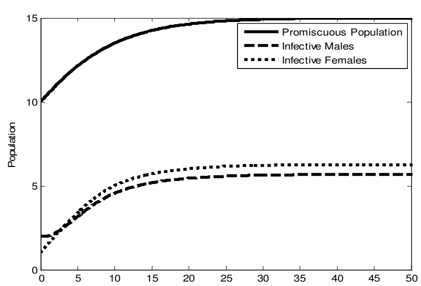
$$\frac{dI_1}{dt} = b_1\left[\frac{\alpha P_0 k}{P_0 + (k - P_0)e^{-a_{11}kt}} - I_1\right]I_2 - c_1 I_1 \qquad \text{(I)}$$

$$\frac{dI_2}{dt} = b_2\left[\frac{(1-\alpha) P_0 k}{P_0 + (k - P_0)e^{-a_{11}kt}} - I_2\right]I_1 - c_2 I_2 \qquad \text{(II)}$$

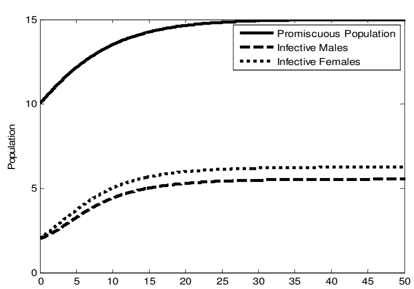
Numerical solutions of these equations is obtained by employing Runge-Kutta method of fourth order with initial conditions $I_1(t_0) = I_{10}$ and $I_2(t_0) = I_{20}$. The interval is to assume d to range over (0, 50) to investigate the behavior of males and females of this model.

Here we have considered values for all parameters of this model, among all the possible Cases nine interesting cases are illustrated below.

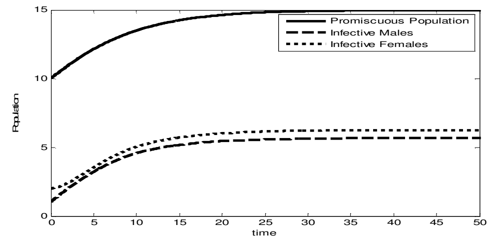
S.No	a_{11}	alp	P_0	k	I_{10}	I_{20}	c_1	c_2	b_1	b_2
1	0.01	0.5	10	15	2	1	0.2	0.1	0.1	0.09
2	0.01	0.5	10	15	2	2	0.2	0.1	0.09	0.09
3	0.01	0.5	10	15	1	2	0.2	0.1	0.1	0.09
4	0.01	0.5	10	10	2	1	0.2	0.1	0.1	0.09
5	0.01	0.5	10	10	2	2	0.2	0.1	0.09	0.09
6	0.01	0.5	10	10	1	2	0.2	0.1	0.15	0.17
7	0.01	0.5	10	2	2	1	0.2	0.1	0.1	0.09
8	0.01	0.5	10	2	2	2	0.2	0.1	0.1	0.09
9	0.01	0.5	10	2	1	2	0.2	0.1	0.1	0.09



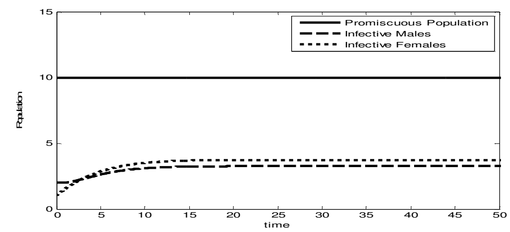
1.(a11=0.01,alp=0.5,p0=10,k=15,I10=2,I20=1,c1=0.2,c2=0.1,b1=0.1,b2=0.09)
(p0 < k, I10 > I20, c1 > c2, b1 > b2, c1 > b1, c2 > b2)



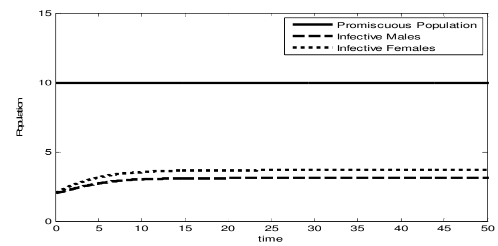
2.(a11=0.01,alp=0.5,p0=10,k=15,I10=2,I20=2,c1=0.2,c2=0.1,b1=0.09,b2=0.09)
(p0 < k, I10 = I20, c1 > c2, b1 = b2, c1 > b1, c2 > b2)



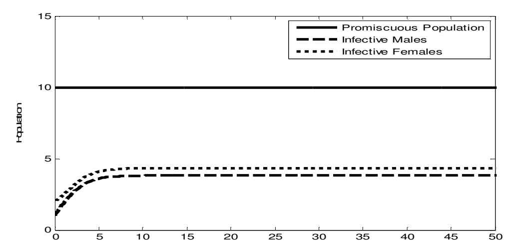
3.(a11=0.01,alp=0.5,p0=10,k=15,I10=1,I20=2,c1=0.2,c2=0.1,b1=0.1,b2=0.09)
(p0 < k, I10 < I20, c1 > c2, b1 > b2, c1 > b1, c2 > b2)



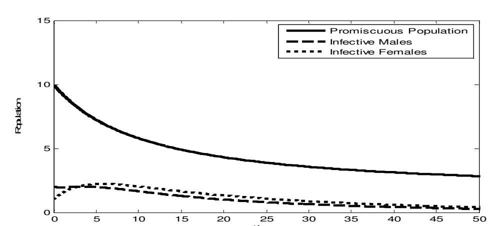
4.(a11=0.01,alp=0.5,p0=10,k=10,I10=2,I20=1,c1=0.2,c2=0.1,b1=0.1,b2=0.0)
(p0 = k, I10 > I20, c1 > c2, b1 > b2, c1 > b1, c2 > b2)



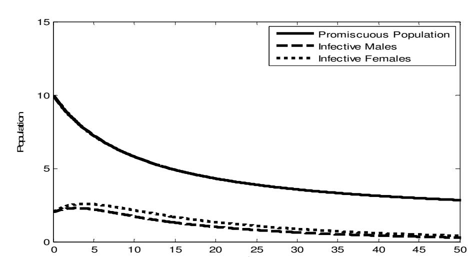
5.(a11=0.01,alp=0.5,p0=10,k=10,I10=2,I20=2,c1=0.2,c2=0.1,b1=0.09,b2=0.09)
(p0 = k, I10 = I20, c1 > c2, b1 = b2, c1 > b1, c2 > b2)



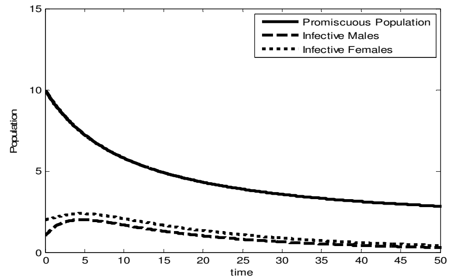
6.(a11=0.01,alp=0.5,p0=10,k=10,I10=1,I20=2,c1=0.2,c2=0.1,b1=0.15,b2=0.17)
(p0 = k, I10 < I20, c1 > c2, b1 < b2, c1 > b1, c2 > b2)



7.(a11=0.01,alp=0.5,p0=10,k=2,I10=2,I20=1,c1=0.2,c2=0.1,b1=0.1,b2=0.09)
(p0 > k, I10 > I20, c1 > c2, b1 > b2, c1 > b1, c2 >b2)



8.(a11=0.01,alp=0.5,p0=10,k=2,I10=2,I20=2,c1=0.2c2=0.1,b1=0.1,b2=0.09)
(p0 > k, I10 = I20, c1 > c2, b1 > b2, c1 > b1, c2 > b2)



9.(a11=0.01,alp=0.5,p0=10,k=2,I10=1,I20=2,c1=0.2,c2=0.1,b1=0.1,b2=0.09)
(p0 > k, I10 < I20, c1 > c2, b1 > b2, c1 > b1, c2 > b2)

Conclusions

1. Initial infective males are greater than initial infective female and cure rate and infective rate of males are greater than females: (p0 < k, I10 > I20, c1 > c2, b1 > b2, c1 > b1, c2 > b2) In this case number of infective males more than number of infective females up to sometime after that infective female exists more than infective males.

2. Initial infective males are equal to initial infective females and cure rate of males are greater than females and infective rate is equal in both: (p0 < k, I10 = I20, c1 > c2, b1 = b2, c1 > b1, c2 > b2) Here infective females are exists more than males throughout the time even they are equal in their number initially.

3. Initial infective males are less than initial infective females and cure rate and infective rate of males are greater than females: (p0 < k, I10 < I20, c1 > c2, b1 > b2, c1 > b1, c2 > b2) In this case infective females exist more than infective males throughout the time.

4. Initial infective males are greater than initial infective females and cure rate and infective rate of males are greater than females: (p0 = k, I10 > I20, c1 > c2, b1 > b2, c1 > b1, c2 > b2) In this case infective male dominates females up to some time, after that infective females dominates males throughout the time.

5. Initial infective males are equals to initial infective females and cure rate of males greater than females and infective rate of males and females are some: (p0 = k, I10 = I20, c1 > c2, b1 = b2, c1 > b1, c2 > b2) In this case infective female exist more than males constantly even they are equal initially.

6. Initial number of infective males less than initial number of infective females and cure rate of males greater than females and infective rate of males is less than females: (p0 = k, I10 < I20, c1 > c2, b1 < b2, c1 > b1, c2 > b2) Here infective male dominates female throughout the time.

7. Initial infective males are greater than initial infective females and cure rate and infective rate of males are greater than females: (p0 > k, I10 > I20, c1 > c2, b1 > b2, c1 > b1, c2 >b2) In this case infective male dominates females up to some time, after that infective females dominates males throughout the time.

8. Initial infective males are equal to initial infective females and cure rate and infective rate of males greater than females: (p0 > k, I10 = I20, c1 > c2, b1 > b2, c1 > b1, c2 > b2) Here infective females are exists more than males throughout the time even they are equal in their number initially.

9. Initial infective males are less than initial infective females and cure rate and infective rate of males greater than females: (p0 > k, I10 < I20, c1 > c2, b1 > b2, c1 > b1, c2 > b2) Here infective male dominates male and after some time both equal in their number and exist both together.

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