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A multilateral model of ecological ammensalism - numerical approach

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Abstract

The paper concerns with the numerical study of a mathematical model of "A Multilateral Model of Ecological Ammensalism - Numerical Approach". The mathematical model comprises of Ammensal-enemy species pair with (i) a constant number of Ammensal is provided with cover to protect it from enemy(ii)the enemy is provided with alternate resources in addition to the Ammensal species (iii) both the species are immigrated and migrated. The model is organized by a couple of first order non linear ordinary differential equations. The numerical illustrations of the growth equations are computed by implementing the classical Runge-Kutta method of fourth order. The relations among the cover protected constant and the dominance reversal time is investigated. Some conclusions are obtained by the results. AMS Classification: 92 D 25, 92 D 40

Keywords: Non-linear system, Ammensal species, Enemy species, Carrying capacity, Dominance reversal time.

INTRODUCTION

Ecology covers all possible interactions among living organisms and their environmental influences. In particular, Physiological ecology concerns with the interactions between individual living beings and outside forces of the surroundings. In general, Community ecology involves mainly the biological interactions between multiple species. But ecosystem ecology deals with the movement of matter and energy between communities and the physical environment. Mathematical modeling mainly throws light on the effort of widening the areas through the techniques of mathematics for obtaining a better insight. It also helps to strengthen our capacity of understanding about the various phenomena which take place in nature. With this insight only we can formulate a new mathematical model. Applying the suitable mathematical techniques we can reach to a conclusion by reasoning regarding that new model. Mathematical modeling of ecosystems was originated in 1925 by Lotka [11]. The ecumenical concepts of modeling have been introduced in the treatises of Meyer [12], Paul Colinvaux [13], Kapur [7,8]. The ecological symbiosis can be broadly classed as Prey-Predation, Competition, Mutualism, Commensalism, Ammensalism, and so on. N.C. Srinivas [15] studied the competitive ecosystems of two species and three species with limited and unlimited resources. Lakshminarayan and Pattabhirama charyulu [9, 10] Investigated Prey-predator Ecological models with a partial cover for the prey and alternate food for the predator. Recently, stability analysis of competitive species was carried out by Archana Reddy, Pattabhi Ramachryulu and Gandhi [5], Bhaskara Rama Sarma and Pattabhiramacharyulu [6]. Mutualism between to species was examined by Ravindra Reddy [14]. Following this Phanikumar, Seshagirirao and Pattabhiramacharyulu [13] studied the commensalism of two species with limited resources.

Acharyulu K.V.L.N and Pattabhi Ramacharyulu.N.Ch. [1,2,3,4] found some productive results "on the stability of an enemy-Ammensal species pair with various resources".The present investigation is related to Numerical study of "A Multilateral Model of Ecological Ammensalism - Numerical Approach"*.* All possible solutions of this model are derived in a finite interval for tracing the dominating species over the other. The relation between cover protected constant (m) for Ammensal species and dominance reversal time (t*) is discovered while fixing all the other parameters. The figures are illustrated with the aid of Mat lab wherever needed. The obtained results are explicated along with the conclusions.

Notations Adopted:

N_1 (t) :The population rate of the species S_1 at time t
N_2 (t) :The population rate of the species S_2 at time t
a_i : The natural growth rates of S_i, i = 1, 2.
a_{ii} :The rate of decrease of S_i; due to its own insufficient resources ,i=1,2.
a_{12} : The inhibition coefficient of S_1 due to S_2 i.e The Ammensal coefficient.
a_{21} :The inhibition coefficient of S_2 due to S_1
$H_1(t)$: The replenishment or renewal of S_1 per unit time
$H_2(t)$: The replenishment or renewal of S_2 per unit time
K_i : The carrying capacity of N_i,
i = 1, 2 :The coefficient of Ammensalism
h_1 : a_{11} H_1 is rate of harvest of the Ammensal
h_2 : a_{22} H_2 is rate of harvest of the enemy.
m_1 :Rate of decrease of the Ammensal due to harvesting.
m_2 :Rate of decrease of the enemy due to harvesting.
M :a constant characterized the cover provided for the Ammensal species (or) Cover protected constant for Ammensal Species

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{22}, K_1, K_2, h_1, h_2 are assumed to be non-negative

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constants.

The Basic Model Equations

The model equations for a two species ecological Ammensalism is constucted by the following system of non-linear ordinary different equations.

$$\frac{dN_1(t)}{dt}=a_1(1-m_1)N_1(t)-a_{11}N_1^2(t)-a_{12}(1-m)N_1(t)N_2(t)+h_1(t)$$

$$\frac{dN_2(t)}{dt}=a_2(1-m_2)N_2(t)-a_{22}N_2^2(t)+h_2(t) \ \ and \ \ N_i(0)=N_{i0}\geq 0, i=1,2$$

Numerical Solutions of the Growth Rate Equations

The obtained numerical solutions of the mathematical model by using the fourth order Runge-Kutta method are tabulated in Table-1

Table 1.

S.No	a_1	a_{11}	a_{12}	a_2	a_{22}	N_{10}	N_{20}	t^*
1	1.969206	2.416238	2.580276	4.427895	3.967456	1.024753	0.938015	0.012
2	3.607222	3.898314	4.642054	0.966343	4.38967	1.894774	1.611143	0.014
3	1.853738	2.643878	2.510159	4.77379	3.568079	1.508225	1.068504	0.037
4	4.254771	1.635093	2.911426	3.714524	3.557857	2.316287	1.492998	0.052
5	3.261062	0.022569	3.643864	1.348226	1.444063	2.107758	1.675339	0.058
6	2.213688	0.45081	2.450809	2.769649	3.025465	1.424785	1.074088	0.070
7	0.387843	0.409059	2.510159	1.120492	3.568079	1.424785	1.068504	0.103
8	2.014261	2.34779	0.589065	4.436913	2.761909	1.938932	0.864447	0.105
9	1.593187	3.283874	4.228554	1.867148	4.419193	1.638284	0.768296	0.106
10	1.862352	3.30158	1.773829	2.933943	1.959451	1.387418	0.429724	0.203
11	1.592739	3.30158	1.773829	3.110636	1.959451	3.762642	0.429724	0.251
12	1.592739	3.312169	2.72137	1.150753	1.959451	1.048781	0.429724	0.275
13	1.592739	3.482609	1.773829	1.150753	1.952291	1.520614	0.449626	0.315
14	1.460899	3.574731	2.227686	1.73952	4.878643	3.652344	0.392454	0.339
15	1.592739	3.30158	1.762407	1.100115	1.959451	1.387418	0.429724	0.344
16	1.450382	3.644115	2.272611	1.73952	4.872066	1.937778	0.326983	0.360
17	1.458055	3.716833	2.227686	1.73952	1.751107	3.652344	0.326983	0.387
18	1.460899	3.644115	2.227686	1.73952	4.872066	3.652344	0.326983	0.391
19	1.669844	2.578302	0.652708	0.521215	0.24536	4.795719	0.651831	0.48
20	1.242465	2.687266	3.017212	2.081598	1.821775	0.509887	0.159696	0.504
21	1.242465	0.687956	0.314514	3.085399	1.679375	0.509887	0.159696	0.507
22	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	1.560

The solution curves are illustrated as

FIGURE (1) ; S.N0-1 in TABLE-1

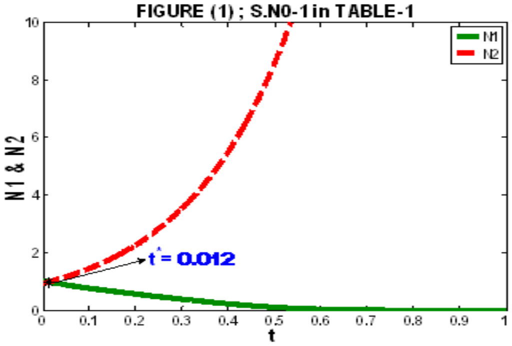


FIGURE (2) ; S.N0-2 in TABLE-1

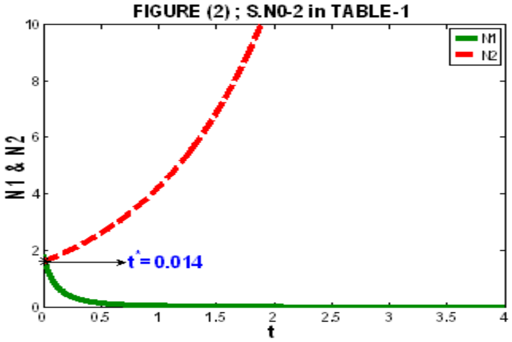


FIGURE (3) ; S.N0-3 in TABLE-1

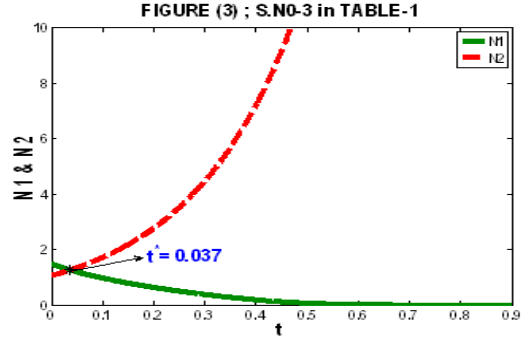


FIGURE (4) ; S.N0-4 in TABLE-1

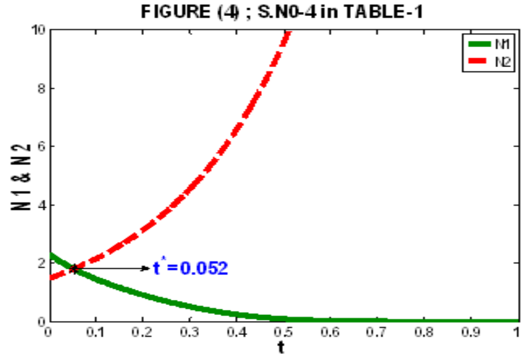


FIGURE (5) ; S.N0-5 in TABLE-1

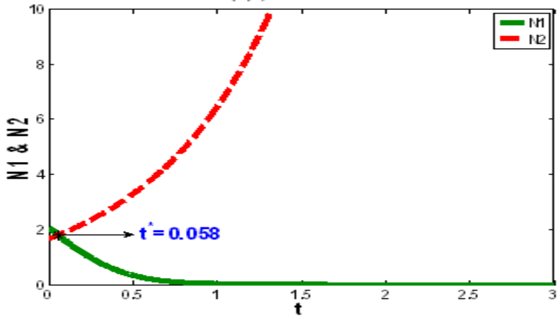


FIGURE (6) ; S.N0-6in TABLE-1

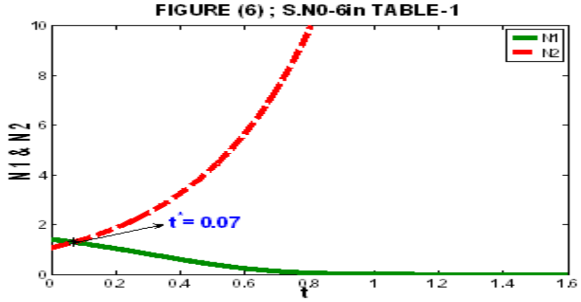


FIGURE (7) ; S.N0-7 in TABLE-1

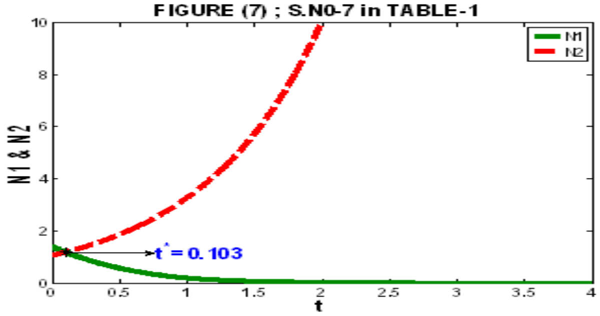


FIGURE (8) ; S.N0-8 in TABLE-1

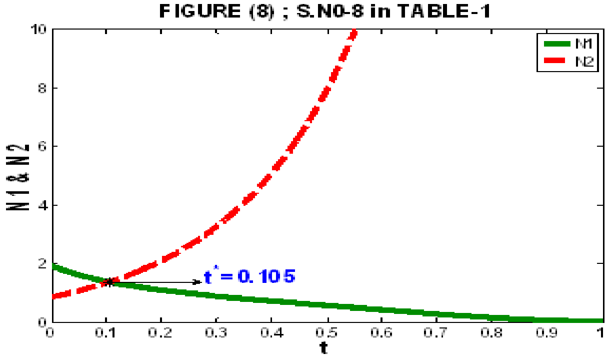


FIGURE (9) ; S.N0-9 in TABLE-1

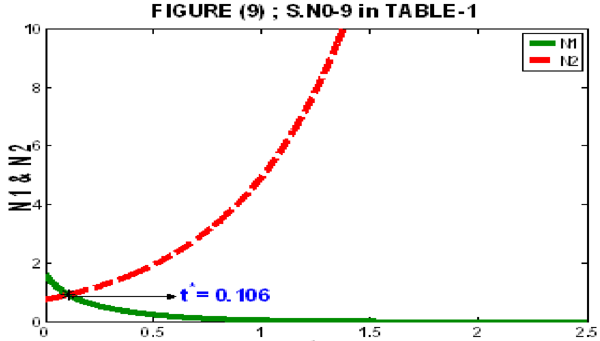


FIGURE (10) ; S.N0-10 in TABLE-1

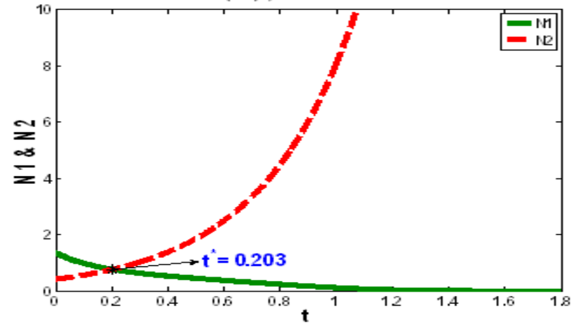


FIGURE (11) ; S.N0-11 in TABLE-1

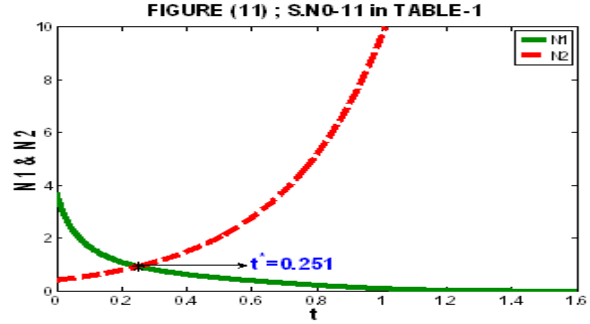


FIGURE (12) ; S.N0-12 in TABLE-1

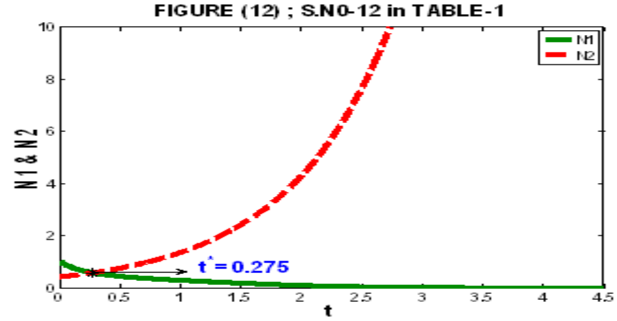


FIGURE (13) ; S.N0-13 in TABLE-1

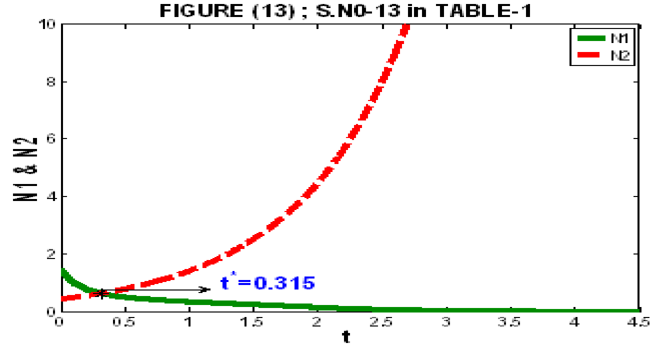


FIGURE (14) ; S.N0-14 in TABLE-1

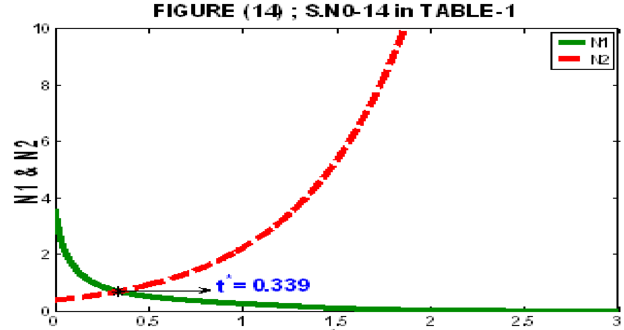


FIGURE (15) ; S.N0-15 in TABLE-1

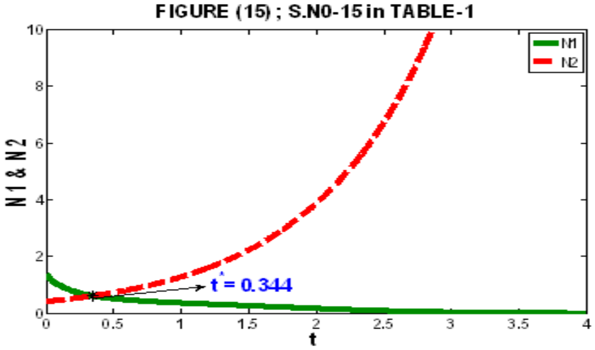


FIGURE (16) ; S.N0-16 in TABLE-1

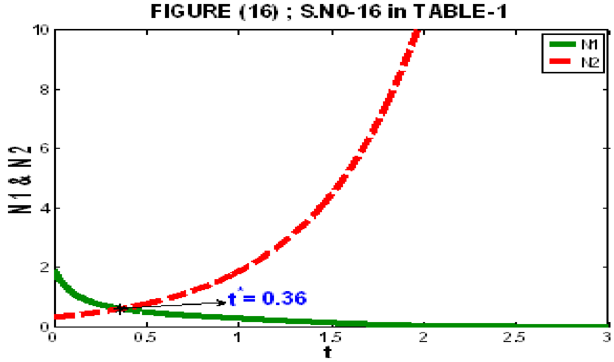


FIGURE (17) ; S.N0-17 in TABLE-1

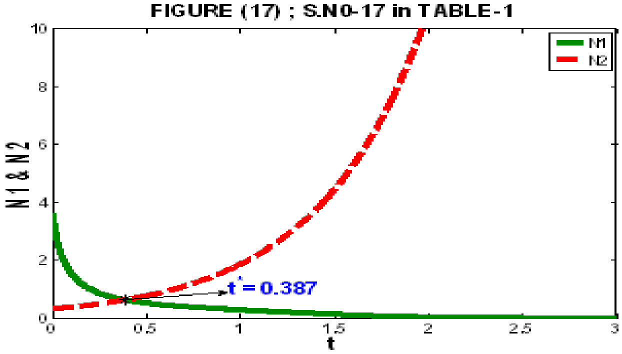


FIGURE (18) ; S.N0-18 in TABLE-1

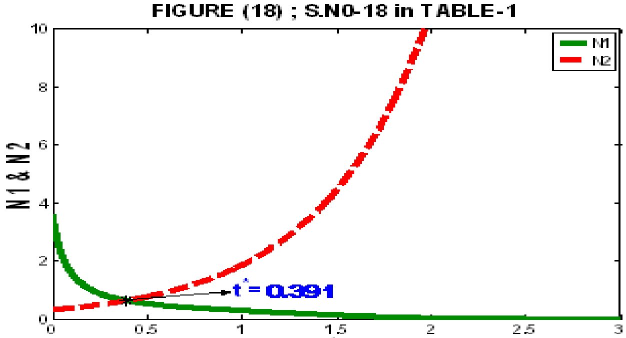


FIGURE (19) ; S.N0-19 in TABLE-1

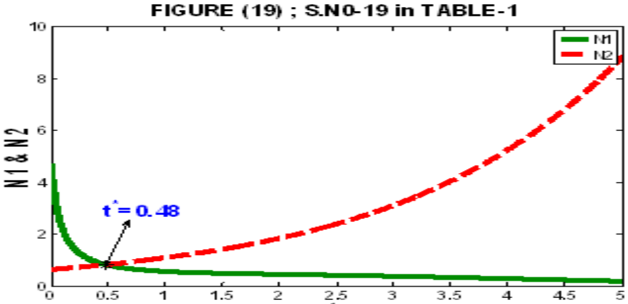


FIGURE (20) ; S.N0-20 in TABLE-1

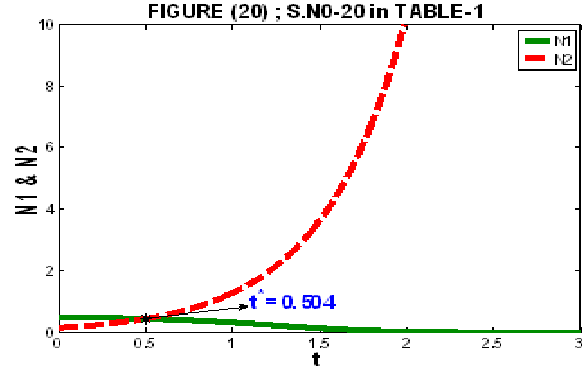


FIGURE (21) ; S.N0-21 in TABLE-1

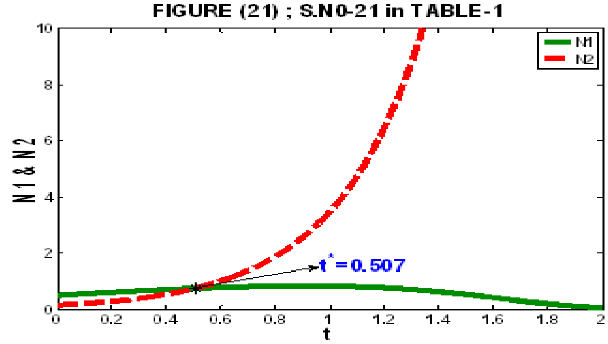
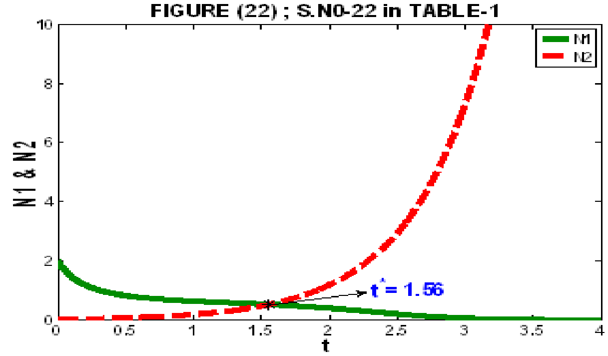


FIGURE (22) ; S.N0-22 in TABLE-1



CONCLUSIONS

1. The Ammensal species prevails over the enemy species up to dominance reversal time. The dominance is altered after dominance reversal time (t*).Ammensal species comes down and exists at a very negligible growth rate after dominance reversal time.

2. The enemy species has steadfast gain throughout the interval. In the course of time, it is noticed that the enemy species has a steep raise as the Ammensal species becomes extinct after dominance reversal time(t*).

The relation between Cover protected constant (m) for Ammensal Species & Dominance reversal time(t*)

The fixed parameters are considered as a_1=1.679163,a_{11}=2.276151, a_{12}=2.681616,a_2=1.817272, a_{22}= 3.093791, N_{10}=2.030574,N_{20}=0.031174, $m_1 = m_2 = h_1 = h_2 = 0.5$.
The varying variable is m i.e
m=0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0,1.1,1.2,1.3,1.4,1.5.
and then t* is traced. The obtained solutions are tabulated as in

Table 2

S.No	a_1	a_{11}	a_{12}	a_2	a_{22}	N_{10}	N_{20}	m1=m2 =h1=h2	m	t^*
1	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	0.1	1.547
2	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	0.2	1.562
3	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	0.3	1.577
4	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	0.4	1.593
5	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	0.5	1.61
6	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	0.6	1.627
7	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	0.7	1.646
8	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	0.8	1.666
9	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	0.9	1.688
10	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	1	1.712
11	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	1.1	1.737
12	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	1.2	1.765
13	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	1.3	1.796
14	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	1.4	1.831
15	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	1.5	1.871
16	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	1.6	1.919
17	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	1.7	1.977
18	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	1.8	2.056
19	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	1.9	2.184
20	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	2	*
21	1.679163	2.276151	2.681616	1.817272	3.093791	2.030574	0.031174	0.5	2.1	*

The solution curves are depicted as below.

FIGURE (23) ; S.N0-1 in TABLE-2

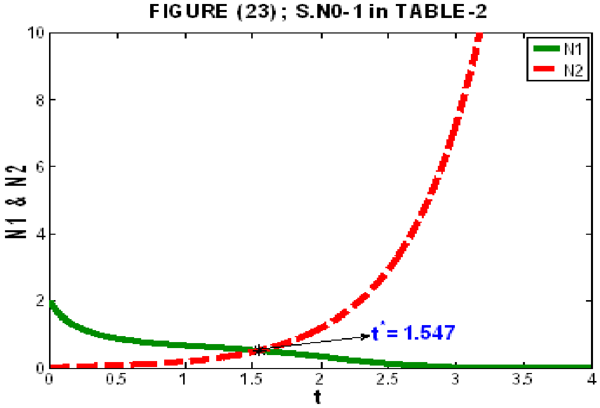


FIGURE (24) ; S.N0-2 in TABLE-2

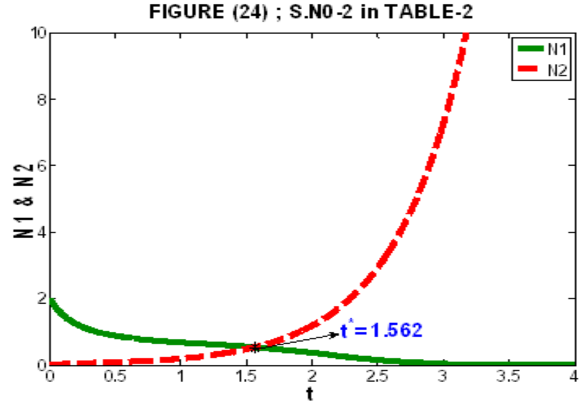


FIGURE (25) ; S.N0-3 in TABLE-2

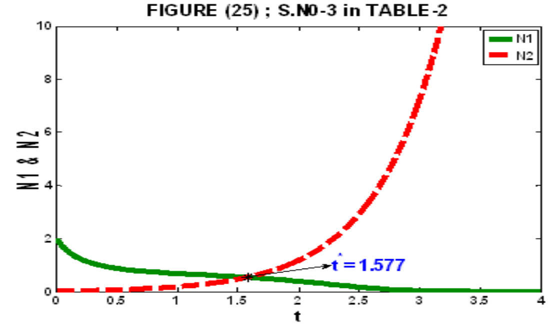


FIGURE (26) ; S.N0-4 in TABLE-2

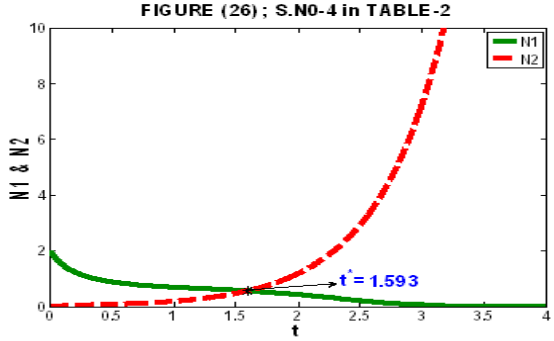


FIGURE (27) ; S.N0-5 in TABLE-2

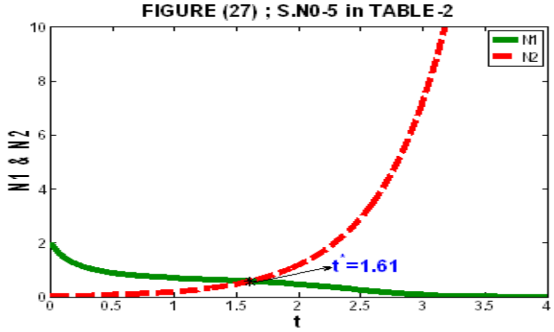


FIGURE (28) ; S.N0-6 in TABLE-2

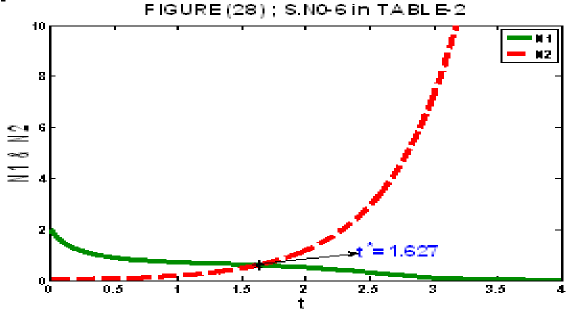


FIGURE (29) ; S.N0-7 in TABLE-2

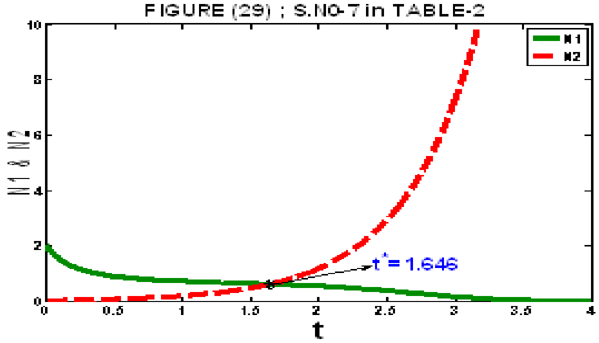


FIGURE (30) ; S.N0-8 in TABLE-2

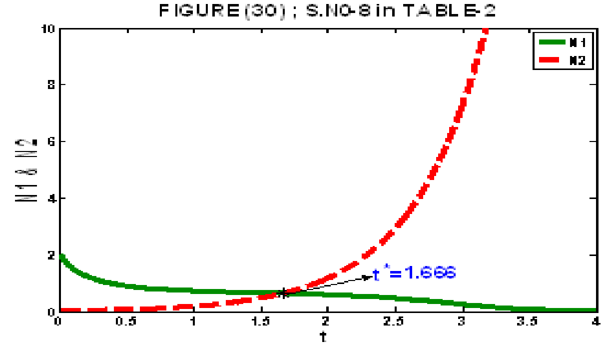


FIGURE (31) ; S.N0-9 in TABLE-2

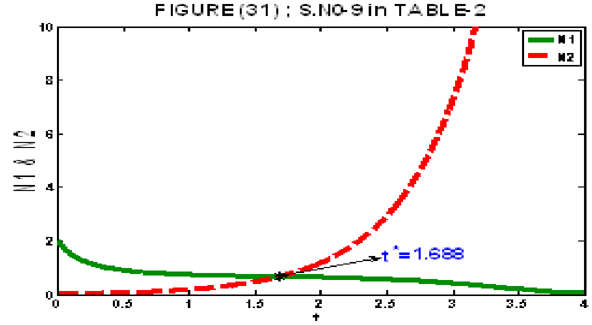


FIGURE (32) ; S.N0-10in TABLE-2

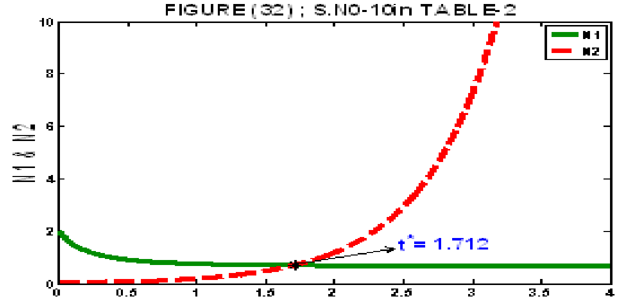


FIGURE (33) ; S.N0-11 in TABLE-2

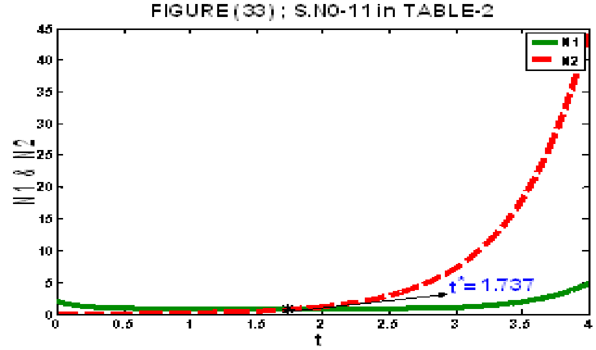


FIGURE (34) ; S.N0-12 in TABLE-2

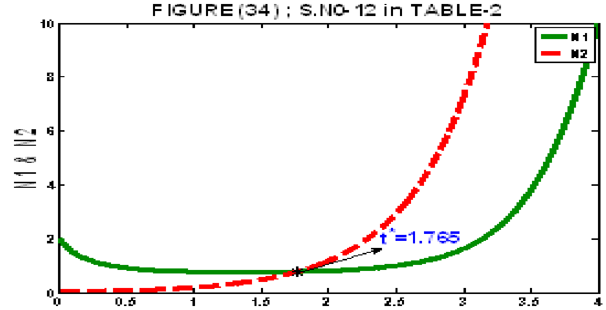


FIGURE (35) ; S.NO-13 in TABLE-2

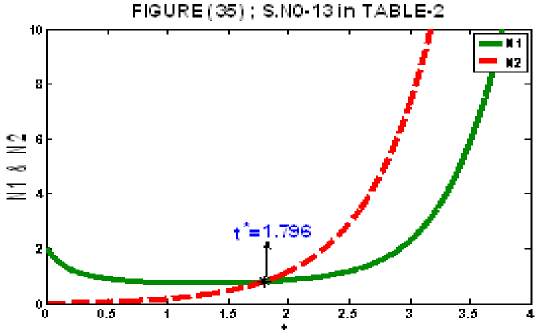


FIGURE (36) ; S.NO-14 in TABLE-2

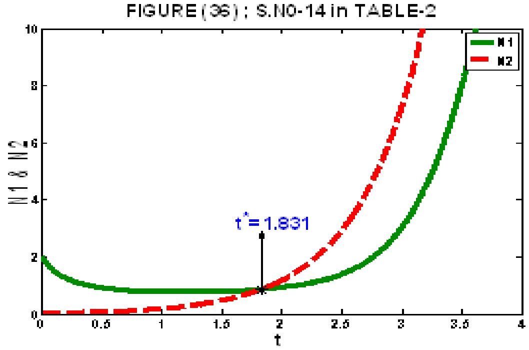


FIGURE (37) ; S.NO-15 in TABLE-2

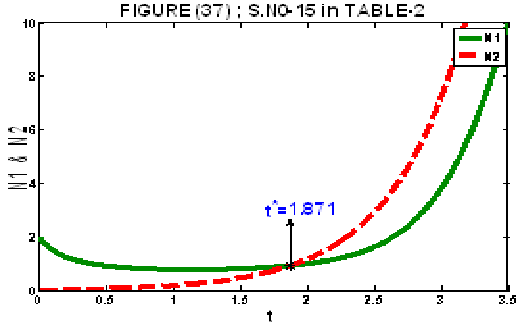


FIGURE (38) ; S.NO-16 in TABLE-2

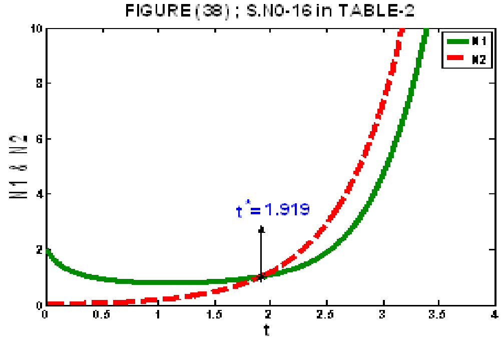


FIGURE (39) ; S.NO-17 in TABLE-2

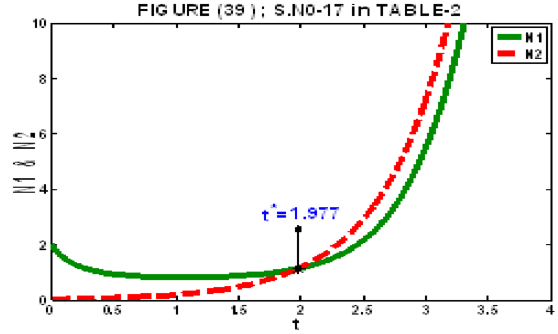


FIGURE (40) ; S.NO-18 in TABLE-2

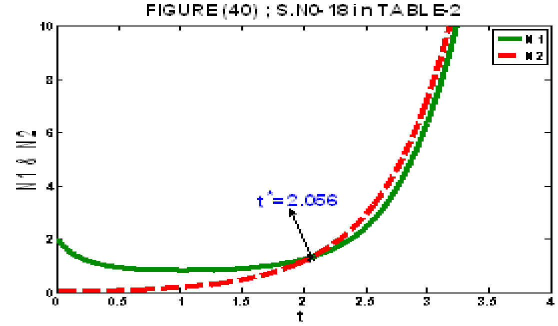


FIGURE (41) ; S.NO-19 in TABLE-2

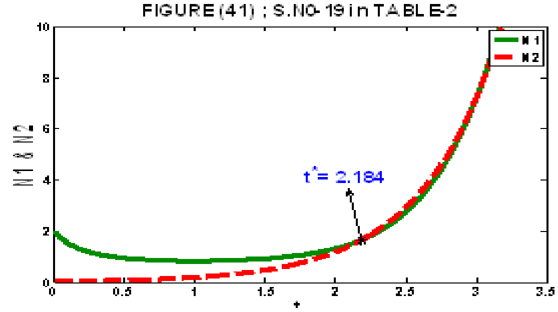


FIGURE (42) ; S.NO-20 in TABLE-2

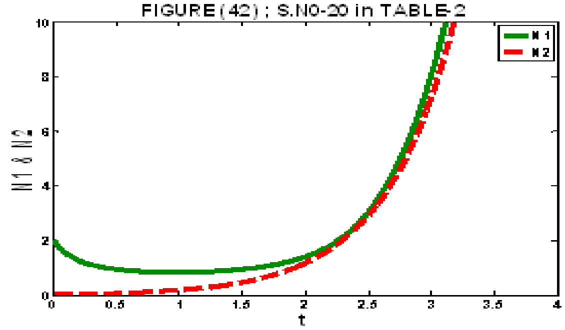
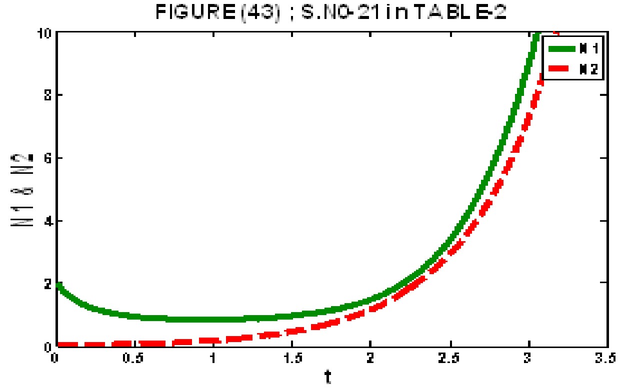


FIGURE (43) ; S.NO-21 in TABLE-2



CONCLUSIONS

Case (i): Fig (23) to Fig (32): Initially the Ammensal species prevails over the enemy species. The dominance is reversed after dominance reversal time (t*).Ammensal species diminishes steadily and subsists at a very low growth rate .The enemy species has a steady increase through out the interval.

Case (ii): Fig (33) to Fig (34): The Ammensal species dominates over the enemy species till dominance reversal time (t*).Further it is noticed that both the species bloom and then have a nominal variation with substantial growth rates.

Case (iii): Fig (35) to Fig (42): The Ammensal species persists to reign over the enemy species up to the dominance reversal time (t*) after which the enemy species eclipses Ammensal species for a while. Further it is identified that both the species survive adjacent with significant exponential growth rates.

Case(iv):Fig(43) : In this case, both the species flourish with exponential growth rates and Ammensal species is the dominant one as it reigns over the other species throughout the interval .It is identified that both the species exert with a gradual deviation with flourishing growth rates . Further the Ammensal species will not be influenced by enemy species in any manner, because of its ample cover protection.

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