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Relation between reversal dominance time and carrying capacities in multiplex ecological ammensalism - A numerical study

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Abstract
The paper aims to investigate a mathematical model of multiplex Ecological Ammensalism with the help of classical RK-method of fourth order in view of reversal dominance time. The mathematical model constitutes of Ammensal-enemy species pair with cover protection for Ammensal species, alternative resources for enemy species and both the species are immigrated. In addition to this, harvesting variable rates are also incorporated. The model is characterized by a couple of first order non linear ordinary differential equations. The relation between the carrying capacity of Ammensal /enemy species with the reversal dominance time is investigated numerically. Some conclusions are derived from the relationships. AMS Classification: 92 D 25, 92 D 40

Keywords: Non-linear system, Ammensal species, Enemy species, Carrying capacity, Reversal dominance time.

INTRODUCTION

Carrying capacity is defined as the maximum population of a given species that can survive indefinitely in a given environment. It reckons on the resources and conditions occurring in the particular environment and the consumption habits of the species considered. Carrying capacity is always changing from time to time, because both what is available in the area, and the consumption habits of the species which change according to the time. Thus carrying capacity is a measure of sustainability within these changing conditions.Kapur J.N [11,12] investigated analytically and numerically in various mathematical models in Biology and Medicine. Later N.C.Srinivas [14] discovered few competitive models to dissolve the complicated real life situations. Lakshmi Narayan with N.Ch.pattbahi Ramacharulu [13] enriched the competitive mathematical models with computational techniques. K.V.L.N.Acharyuly and N.Ch.Pattabhiramacharyulu [1-10] looked into multifarious mathematical models in Ecological Ammensalism.

The authors applied the classical RK method of fourth order to this model for tracing the relations among Carrying capacity of Ammensal/enemy species and reversal dominance time. The present paper is a numerical study to investigate a mathematical model of Ecological Ammensalism with multiplex conditions with the aid of classical RK- method of fourth order. The mathematical model consists of Ammensal-enemy species pair with cover protection for Ammensal, alternative resources for enemy and harvesting for both the species. The model is characterized by a couple of first order non linear ordinary differential equations. The relation between the carrying capacity of Ammensal/enemy species and the reversal dominance time is identified. The interactions are noticed by changing the value of one variable while fixing all other parameters. The figures are depicted with the help of Matlab wherever needed and useful. The results are mentioned along with the conclusions.

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Notations Adopted:

N_1 (t)	The population rate of the species S_1 at time t
N_2 (t)	The population rate of the species S_2 at time t
a_i	The natural growth rates of S_i, i = 1, 2.
a_{ii}	The rate of decrease of S_i; due to its own insufficient resources ,i=1,2.
a_{12}	The inhibition coefficient of S_1 due to S_2
a_{21}	The inhibition coefficient of S_2 due to S_1
$H_1(t)$	The replenishment or renewal of S_1 per unit time
$H_2(t)$	The replenishment or renewal of S_2 per unit time
K_i	The carrying capacity of N_i, i = 1, 2
α	The coefficient of Ammensalism
h_1	a_{11} H_1 is rate of harvest of the Ammensal
h_2	a_{22} H_2 is rate of harvest of the enemy.
m_1	Rate of decrease of the Ammensal due to harvesting.
m_2	Rate of decrease of the enemy due to harvesting
m	Cover protection constant for Ammensal Species

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{22}, K_1, K_2, h_1, h_2 are assumed to be non-negative constants.

The Basic Model Equations

The model equations for a two species ecological Ammensalism in this case is constucted by the following system of non-linear ordinary differential equations.

$$\frac{dN_1(t)}{dt} = a_1(1-m_1)N_1(t) - a_{11}N_1^2(t) - a_{12}(1-m)N_1(t)N_2(t) + h_1(t)$$

$$\frac{dN_2(t)}{dt} = a_2(1-m_2)N_2(t) - a_{22}N_2^2(t) + h_2(t) \;\; and \;\; N_i(0) = N_{i0} \geq 0, i = 1,2$$

The relation between Carrying capacity of Ammensal Species and reversal dominance time(t*)

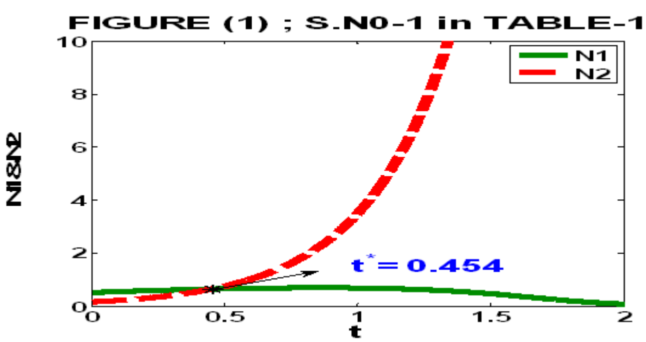
The fixed parameters are considered as, a_{11}=0.687956, a_{12}=0.314514, a_2=3.085399, a_{22}=1.679375, N_{10}=0.509887, N_{20}=0.159696, $m_1 = m_2 = h_1 = h_2 = m$=0.5.

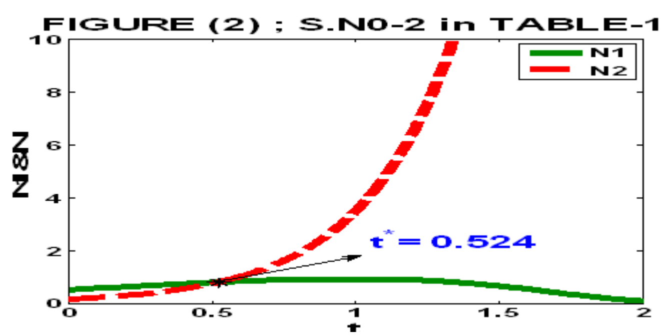
The varying variable is a_1 i.e a a_1=0.242465, 1.242465,2.242465,3.242465,4.242465, 5.242465, 6.242465, 7.242465 , 8.242465, 9.242465 , 10.242465 and then t* is obtained. The obtained solutions are tabulated as in Table-1.

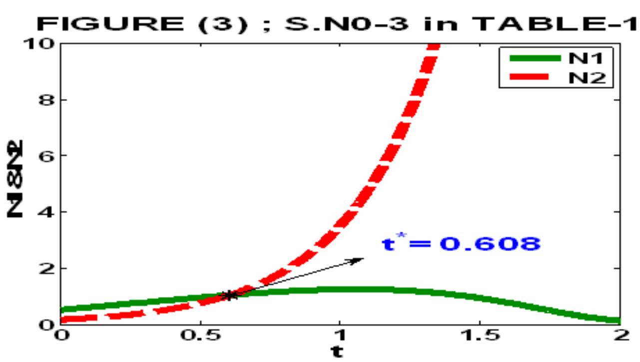
Table 1.

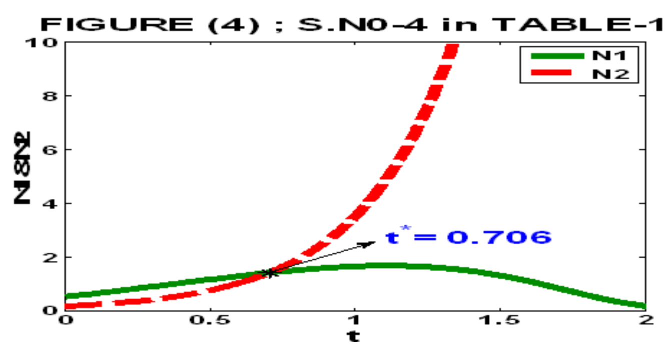
S.No	a_1	a_{11}	a_{12}	a_2	a_{22}	N_{10}	N_{20}	m_1=m_2= m =h_1=h_2	t^*
1	0.242465	0.687956	0.314514	3.085399	1.679375	0.509887	0.159696	0.5	0.454
2	1.242465	0.687956	0.314514	3.085399	1.679375	0.509887	0.159696	0.5	0.524
3	2.242465	0.687956	0.314514	3.085399	1.679375	0.509887	0.159696	0.5	0.608
4	3.242465	0.687956	0.314514	3.085399	1.679375	0.509887	0.159696	0.5	0.706
5	4.242465	0.687956	0.314514	3.085399	1.679375	0.509887	0.159696	0.5	0.807
6	5.242465	0.687956	0.314514	3.085399	1.679375	0.509887	0.159696	0.5	0.909
7	6.242465	0.687956	0.314514	3.085399	1.679375	0.509887	0.159696	0.5	0.996
8	7.242465	0.687956	0.314514	3.085399	1.679375	0.509887	0.159696	0.5	1.064
9	8.242465	0.687956	0.314514	3.085399	1.679375	0.509887	0.159696	0.5	1.119
10	9.242465	0.687956	0.314514	3.085399	1.679375	0.509887	0.159696	0.5	1.165
11	10.24247	0.687956	0.314514	3.085399	1.679375	0.509887	0.159696	0.5	1.203

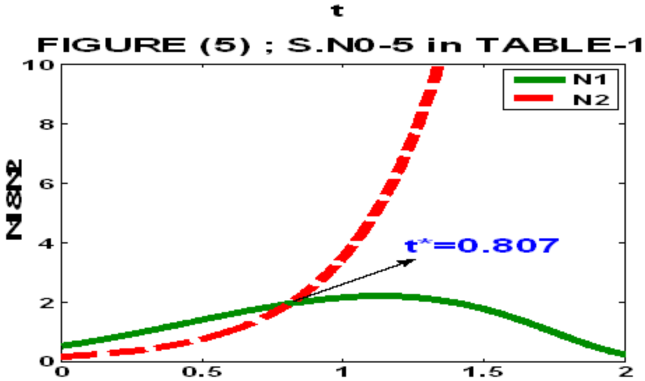
The solution curves are depicted as below.

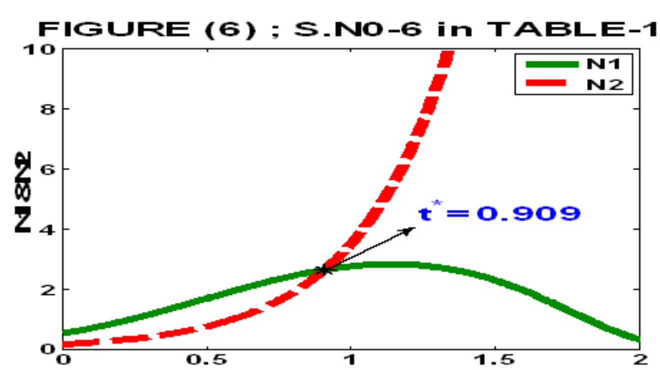


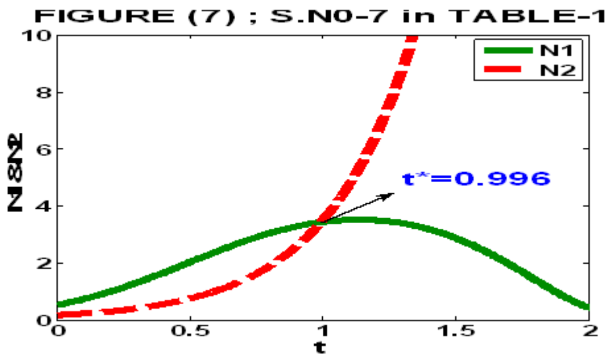


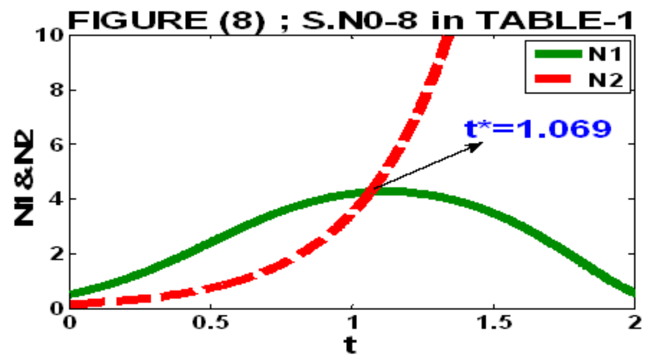


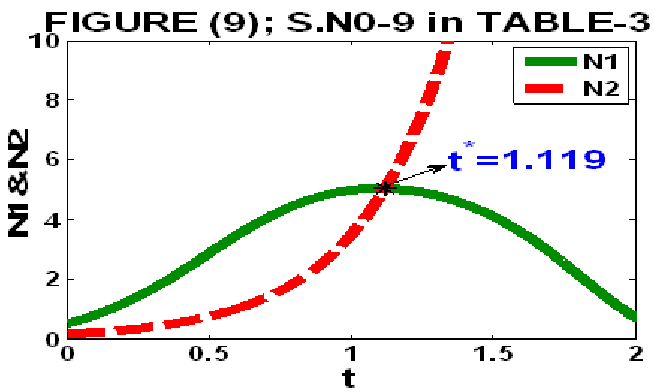


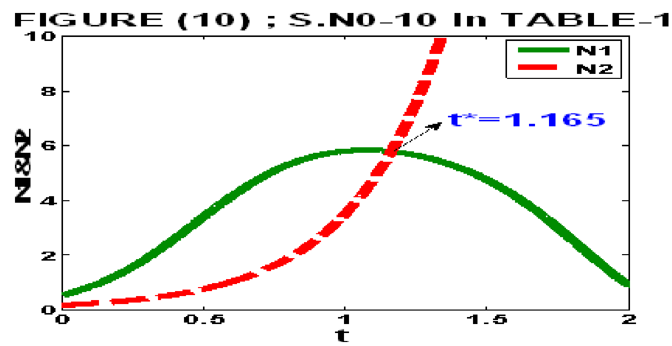


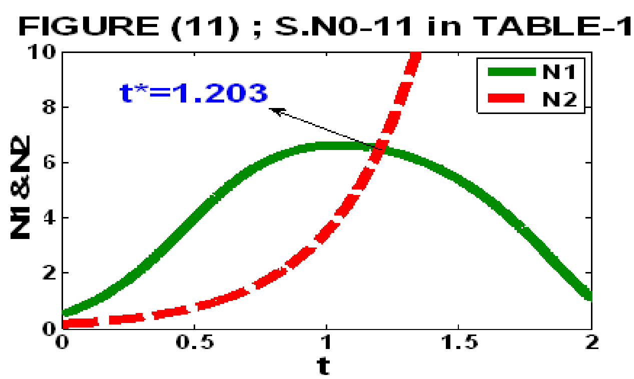












CONCLUSIONS

I. The Ammensal species dominates over the enemy species up to t* and then declines gradually throughout the interval .It appears to be almost extinct with negligible growth rate at the end of the interval.

II. The enemy species flourishes throughout the interval and eclipses the Ammensal species after dominance reversal time.

III. It is also observed that when the natural growth rate of Ammensal species increases, the dominance reversal time also increases.

The carrying capacity of Ammensal species is obtained by the ratio of the natural growth rate of Ammensal species and the rate of decrease of Ammensal spices due to its own insufficient resources. The values of Carrying capacity of Ammensal species with respect to the derived numerical solutions are tabulated in Table-2 along with the corresponding values of dominance reversal time(t*).

Table 2.

S.No	K_1	t*
1	0.352442	0.454
2	1.806023	0.524
3	3.259605	0.608
4	4.713186	0.706
5	6.166767	0.807
6	7.620349	0.909
7	9.073930	0.996
8	10.527511	1.064
9	11.981093	1.119
10	13.434674	1.165
11	14.888255	1.203

Conclusions: It is identified that when the Carrying capacity of Ammensal species gradually increases, the dominance reversal time also increases step by step.

The relation between Carrying capacity of Ammensal Species and reversal dominance time(t*)

The fixed parameters are considered as a_1=1.669844, a_{11}=2.578302, a_{12}=0.652708, a_{22}=0.24536, N_{10}=4.795719,

N_{20}=0.651831, m_1 = m_2 = h_1 = h_2 = 0.5. The varying variable is a_2 i.e a_2=0.021215, 0.121215, 0.221215, 0.321215, 0.421215, 0.521215, 0.621215, 0.721215, 0.821215,0.921215,1.021215, 1.121215, 1.221215, 1.321215, 1.421215,1.521215 and then t* is traced. The obtained solutions are tabulated as in Table-3.

Table 3.

S.No	a_1	a_{11}	a_{12}	a_2	a_{22}	N_{10}	N_{20}	$m_1=m_2$ $m=h_1=h_2$	t^*
1	1.669844	2.578302	0.652708	0.021215	0.24536	4.795719	0.651831	0.5	0.943
2	1.669844	2.578302	0.652708	0.121215	0.24536	4.795719	0.651831	0.5	0.768
3	1.669844	2.578302	0.652708	0.221215	0.24536	4.795719	0.651831	0.5	0.673
4	1.669844	2.578302	0.652708	0.321215	0.24536	4.795719	0.651831	0.5	0.608
5	1.669844	2.578302	0.652708	0.421215	0.24536	4.795719	0.651831	0.5	0.56
6	1.669844	2.578302	0.652708	0.521215	0.24536	4.795719	0.651831	0.5	0.522
7	1.669844	2.578302	0.652708	0.621215	0.24536	4.795719	0.651831	0.5	0.491
8	1.669844	2.578302	0.652708	0.721215	0.24536	4.795719	0.651831	0.5	0.465
9	1.669844	2.578302	0.652708	0.821215	0.24536	4.795719	0.651831	0.5	0.442
10	1.669844	2.578302	0.652708	0.921215	0.24536	4.795719	0.651831	0.5	0.415
11	1.669844	2.578302	0.652708	1.021215	0.24536	4.795719	0.651831	0.5	0.405
12	1.669844	2.578302	0.652708	1.121215	0.24536	4.795719	0.651831	0.5	0.389
13	1.669844	2.578302	0.652708	1.221215	0.24536	4.795719	0.651831	0.5	0.375
14	1.669844	2.578302	0.652708	1.321215	0.24536	4.795719	0.651831	0.5	0.355
15	1.669844	2.578302	0.652708	1.421215	0.24536	4.795719	0.651831	0.5	0.347
16	1.669844	2.578302	0.652708	1.521215	0.24536	4.795719	0.651831	0.5	0.34

The solution curves are depicted as below.

FIGURE (12) ; S.N0-1 in TABLE-3

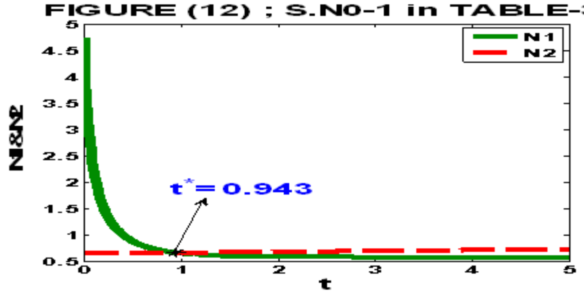


FIGURE (13) ; S.N0-2 in TABLE-3

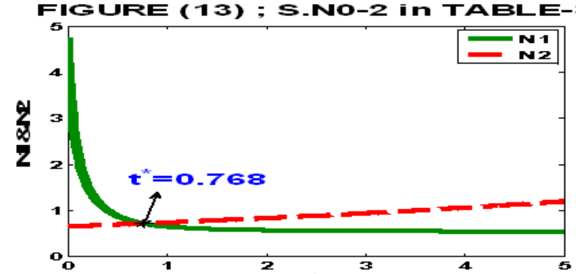


FIGURE (14) ; S.N0-3 In TABLE-3

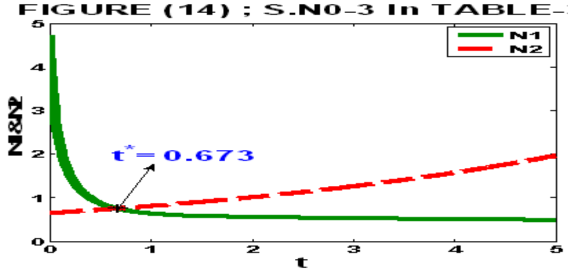


FIGURE (15) ; S.N0-4 in TABLE-3

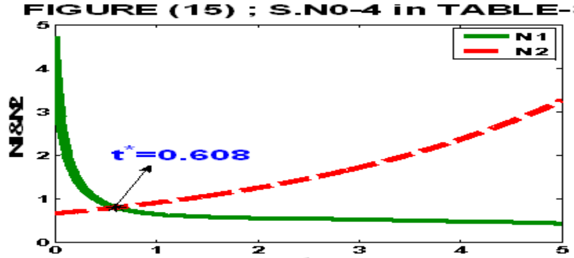


FIGURE (16) ; S.N0-5 in TABLE-3

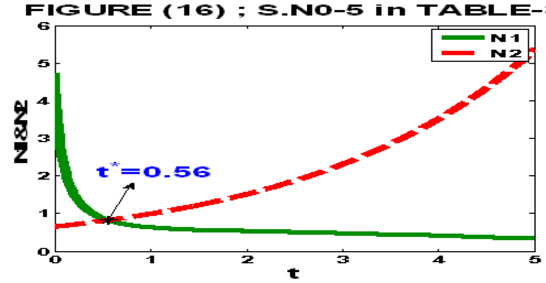


FIGURE (17) ; S.N0-6 in TABLE-3

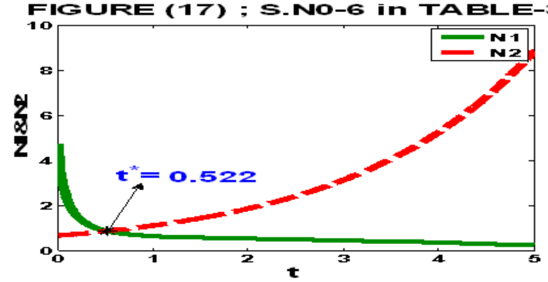


FIGURE (18) ; S.N0-7 in TABLE-3

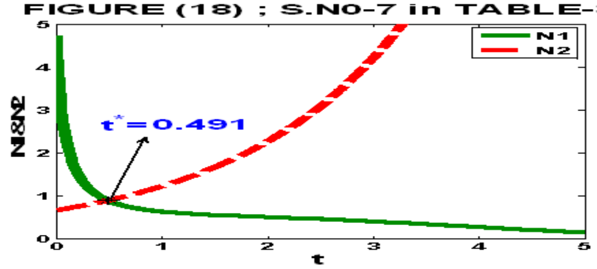


FIGURE (19) ; S.N0-8 in TABLE-3

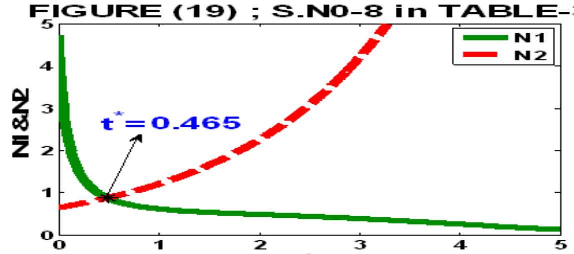


FIGURE (20) ; S.N0-9 in TABLE-3

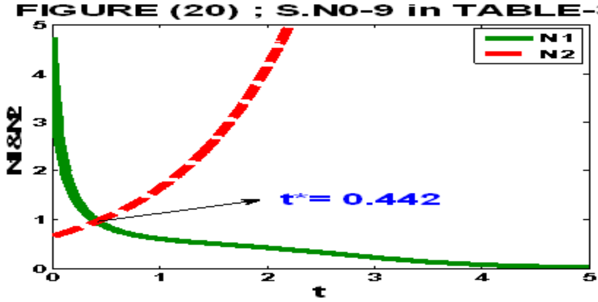


FIGURE (21) ; S.N0-10 In TABLE-3

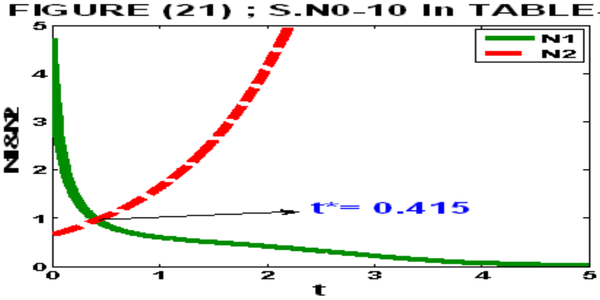


FIGURE (22) ; S.N0-11 in TABLE-3

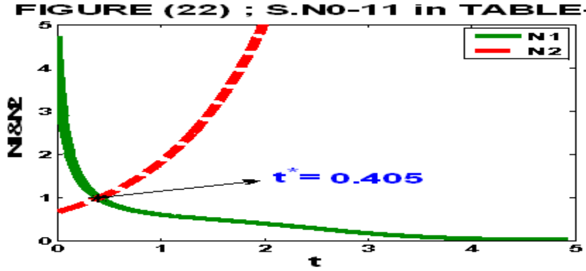


FIGURE (23) ; S.N0-12 in TABLE-3

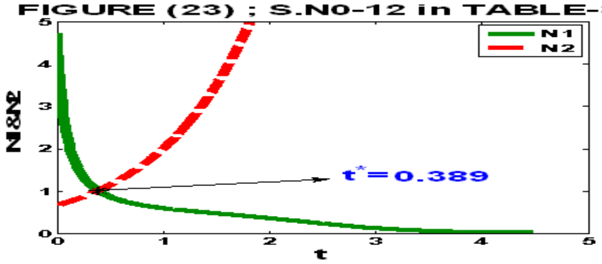


FIGURE (24) ; S.N0-13 in TABLE-3

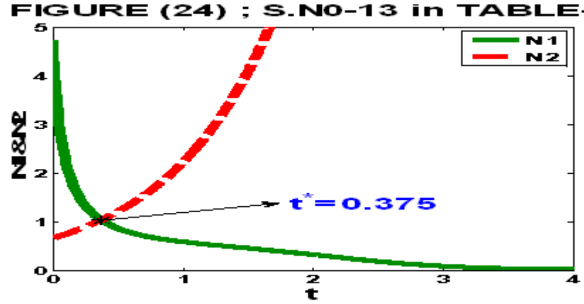


FIGURE (25) ; S.N0-14 in TABLE-3

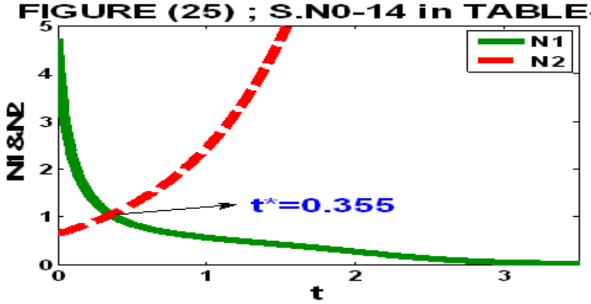


FIGURE (26) ; S.N0-15 in TABLE-3

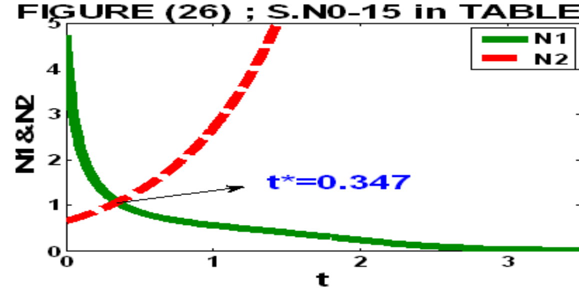
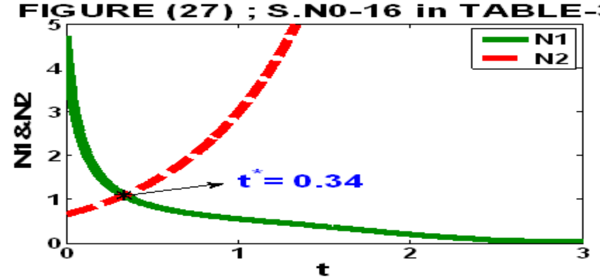


FIGURE (27) ; S.N0-16 in TABLE-3



I. Ammensal species outnumbers the enemy species till the dominance reversal time.

II. The enemy outnumbers the Ammensal after dominance reversal time (t*).

III. Further the Ammensal species declines and is asymptotic to the equilibrium point.

IV. The enemy species has exponential growth rate throughout the interval.

V. It is also observed that when the natural growth rate of Enemy species increases, the dominance reversal time decreases.

The carrying capacity of enemy species is obtained by the ratio of the natural growth rate of enemy species and the rate of decrease of enemy spices due to its own insufficient resources. The values of Carrying capacity of enemy species with respect to the derived numerical solutions are tabulated in Table-4 along with the corresponding values of dominance reversal time(t*).

Table 4.

S.NO	K_2	t*
1	0.086464	0.943
2	0.494039	0.768
3	0.901593	0.673
4	1.309159	0.608
5	1.717756	0.56
6	2.124286	0.522
7	2.531851	0.491
8	2.939415	0.465
9	3.346979	0.442
10	3.754544	0.415
11	4.162108	0.405
12	4.569673	0.389
13	4.977237	0.375
14	5.384801	0.355
15	5.792366	0.347
16	6.199931	0.340

Conclusions

It is noticed that the dominance reversal time gradually decreases even though the carrying capacity of enemy species increases.

The identified relationships can be classified as in Table-5 given below.

Table 5.

S.No	criterion	Conclusion
1	The natural growth rate(a1) of Ammensal species increases	The dominance reversal time (t*) gradually increases
2	The carrying capacity(K1) of Ammensal species increases	
3	The natural growth rate(a2) of Enemy species increases	The dominance reversal time (t*) gradually decreases
4	The carrying capacity(K2) of Enemy species increases	

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REFERENCES

[1]. Acharyulu.K.V.L.N. &Pattabhi Ramacharyulu. N.Ch. 2011. "On An Ammensal-Enemy Ecological Model With Variable Ammensal Coefficient"- *International Journal of Computational Cognition (IJCC),* Vol.9,No.2,pp.9-14.

[2]. Acharyulu.K.V.L.N. & Pattabhi Ramacharyulu.N.Ch.2011. "Multiple Constraints in Ecological Ammensalism- A Numerical Approach-"*International Journal of Advances in Soft Computing and Its Applications(IJASCA),* Vol. 3, No.2, pp.1-15.

[3]. Acharyulu.K.V.L.N.&Pattabhi Ramacharyulu. N.Ch. 2010. "An Ammensal-Enemy Specie Pair With Limited And Unlimited Resources Respectively-A Numerical Approach"- *Int. J. Open Problems Compt. Math (IJOPCM).,* Vol. 3, No. 1,pp.73-91.

[4]. Acharyulu.K.V.L.N. &Pattabhi Ramacharyulu. N.Ch.2010. Some Threshold Results For An Ammensal- Enemy Species Pair With Limited Resources"-*International Journal Of Scientific Computing(IJSC),*Vol.4,No.1,pp.33-36..

[5]. Acharyulu. K.V.L.N.&Pattabhi Ramacharyulu. N.Ch.2010. "In View Of The Reversal Time Of Dominance In An Enemy-Ammensal Species Pair With Unlimited And Limited Resources Respectively For Stability By Numerical Technique"- *International journal of Mathematical Sciences and Engineering Applications(IJMSEA);* Vol.4, No. II, pp.109-131.

[6]. Acharyulu. K.V.L.N. &Pattabhi Ramacharyulu. N.Ch.2010. "On The Stability Of Harvested Ammensal - Enemy Species Pair With Limited Resources" -*International Journal of Logic Based Intelligent Systems*, Vol. 4, No. 1,pp.1-16.

[7]. Acharyulu.K.V.L.N. & Pattabhi Ramacharyulu. N.Ch. 2010."Liapunov's Function For Global Stability Of Harvested Ammensal And Enemy Species Pair With Limited Resources"-*International Review of pure and applied mathematics,*Vol. 6, No. 2,pp.263-271.

[8]. Acharyulu.K.V.L.N.&Pattabhi Ramacharyulu. N.Ch. 2010. "On the Stability of an Ammensal- Harvested Enemy Species Pair with Limited Resources"- *International Journal of computational Intelligence Research (IJCIR),* Vol. 6, No.3, pp.343-358.

[9]. Acharyulu.K.V.L.N. &Pattabhi Ramacharyulu. N.Ch. 2010. "On The Stability Of An Ammensal- Enemy Harvested Species Pair With Limited Resources"- *Int. J. Open Problems Compt. Math (IJOPCM).,* Vol. 3, No. 2., pp.241-266.

[10]. Acharyulu.K.V.L.N. &Pattabhi Ramacharyulu. N.Ch. 2009. "On The Stability Of An Enemy – Ammensal Species Pair With Limited Resources- *International Journal of Applied Mathematical Analysis and Applications*, vol 4, No.2,pp. 149-161.

[11]. Kapur J.N. 1985.Mathematical modeling in biology and Medicine, affiliated east west.

[12]. Kapur J.N. 1985. Mathematical modeling, wiley, easter.

[13]. Lakshmi Narayan K & Pattabhi Ramacharyulu N.Ch.2007. "A prey-predator model with cover for prey lternate food for the predator with time delay". *International journal of scientific computing* Vol1,pp-7-14.

[14]. Srinivas N.C. 1991."Some Mathematical aspects of modeling in Bio-medical sciences "Ph.D Thesis, Kakatiya University.