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A Mathematical model of four species syn-ecosymbiosis comprising of prey-predation, mutualism and commensalisms-V(the co-existent state)

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Abstract

This investigation deals with a mathematical model of a four species (S_1, S_2, S_3 and S_4) Syn-Ecological system (The Co-existent State). S_2 is a predator surviving on the prey S_1: the prey is a commensal to the host S_3 which itself is in mutualism with the fourth species S_4; S_2 and S_4 are neutral. The model equations of the system constitute a set of four first order non-linear ordinary differential coupled equations. In all, there are sixteen equilibrium points. Criteria for the asymptotic stability of one of the sixteen equilibrium points: The Co-existent State only is established in this paper. The Co-existent State is found to be stable. The linearized equations for the perturbations over the equilibrium points are analyzed to establish the criteria for stability and the trajectories illustrated. Further the global stability is discussed using Liapunov's method.

Keywords: Equilibrium state, stability, Mutualism, Co-Existent State

INTRODUCTION

Mathematical modelling in eco-system was initiated in 1925 by Lotka [6] and in 1931 by Volterra [12]. The general concepts of modelling have been presented in the treatises of Meyer [7], Paul Colinvaux [8], Freedman [2], Kapur [3, 4]. The ecological interactions can be broadly classified as prey-predation, competition, mutualism and so on. N.C. Srinivas [11] studied the competitive eco-systems of two species and three species with regard to limited and unlimited resources. Later, Lakshmi Narayan [5] has investigated the two species prey-predator models. Recently stability analysis of competitive species was investigated by Archana Reddy [1]. Local stability analysis for a two-species ecological mutualism model has been investigated by B. Ravindra Reddy et. al [9, 10].

BASIC EQUATIONS

Notation Adopted

$N_1(t)$: The Population of the Prey (S_1)
$N_2(t)$: The Population of the Predator (S_2)
$N_3(t)$: The Population of the Host (S_3) of the Prey (S_1) and mutual to S_4
$N_4(t)$:The Population of S_4 mutual to S_3
t : Time instant
a_1, a_2, a_3, a_4 : Natural growth rates of S_1, S_2, S_3, S_4
$a_{11}, a_{22}, a_{33}, a_{44}$: Self inhibition coefficients of S_1, S_2, S_3, S_4
a_{12}, a_{21} : Interaction (Prey-Predator) coefficients of S_1 due to S_2 and S_2 due to S_1
a_{13} : Coefficient for commensal for S_1 due to the Host S_3
a_{34}, a_{43} : Mutually interaction between S_3 and S_4
$\frac{a_1}{a_{11}}, \frac{a_2}{a_{22}}, \frac{a_3}{a_{33}}, \frac{a_4}{a_{44}}$: Carrying capacities of S_1, S_2, S_3, S_4

Further the variables N_1, N_2, N_3, N_4 are non-negative and the model parameters a_1, a_2, a_3, a_4; a_{11}, a_{22}, a_{33}, a_{44}; a_{12}, a_{21}, a_{13}, a_{24} are assumed to be non-negative constants.

The model equations for the growth rates of S_1, S_2, S_3, S_4 are

$$\frac{dN_1}{dt} = a_1N_1 - a_{11}N_1^2 - a_{12}N_1N_2 + a_{13}N_1N_3 \quad \text{....(2.1)}$$

$$\frac{dN_2}{dt} = a_2N_2 - a_{22}N_2^2 + a_{21}N_2N_1 \quad \text{....(2.2)}$$

$$\frac{dN_3}{dt} = a_3N_3 - a_{33}N_3^2 + a_{34}N_3N_4 \quad \text{.... (2.3)}$$

$$\frac{dN_4}{dt} = a_4N_4 - a_{44}N_4^2 + a_{43}N_4N_3 \quad \text{.... (2.4)}$$

EQUILIBRIUM STATES

The system under investigation has sixteen equilibrium states are given by

$$\frac{dN_i}{dt} = 0,\ i = 1,2,3,4 \quad \text{...... (3.1)}$$

I. Fully washed out state:

(1) $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$

II. States in which three of the four species are washed out and fourth is surviving

(2) $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$

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(3) $\overline{N_1}=0, \overline{N_2}=0, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=0$

(4) $\overline{N_1}=0, \overline{N_2}=\frac{a_2}{a_{22}}, \overline{N_3}=0, \overline{N_4}=0$

(5) $\overline{N_1}=\frac{a_1}{a_{11}}, \overline{N_2}=0, \overline{N_3}=0, \overline{N_4}=0$

III. States in which two of the four species are washed out while the other two are surviving

(6) $\overline{N_1}=0, \overline{N_2}=0, \overline{N_3}=\frac{a_4a_{34}+a_3a_{44}}{a_{33}a_{44}-a_{34}a_{43}}, \overline{N_4}=\frac{a_3a_{43}+a_4a_{33}}{a_{33}a_{44}-a_{34}a_{43}}$

(7) $\overline{N_1}=0, \overline{N_2}=\frac{a_2}{a_{22}}, \overline{N_3}=0, \overline{N_4}=\frac{a_4}{a_{44}}$

(8) $\overline{N_1}=0, \overline{N_2}=\frac{a_2}{a_{22}}, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=0$

(9) $\overline{N_1}=\frac{a_1}{a_{11}}, \overline{N_2}=0, \overline{N_3}=0, \overline{N_4}=\frac{a_4}{a_{44}}$

(10) $\overline{N_1}=\frac{a_1a_{33}+a_3a_{13}}{a_{11}a_{33}}, \overline{N_2}=0, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=0$

(11) $\overline{N_1}=\frac{a_1a_{22}-a_2a_{12}}{a_{11}a_{22}+a_{12}a_{21}}, \overline{N_2}=\frac{a_1a_{21}+a_2a_{11}}{a_{11}a_{22}+a_{12}a_{21}}, \overline{N_3}=0, \overline{N_4}=0$

IV. States in which one of the four species is washed out while the other three are surviving

(12) $\overline{N_1}=0, \overline{N_2}=\frac{a_2}{a_{22}}, \overline{N_3}=\frac{a_4a_{34}+a_3a_{44}}{a_{33}a_{44}-a_{34}a_{43}}, \overline{N_4}=\frac{a_4a_{33}+a_3a_{43}}{a_{33}a_{44}-a_{34}a_{43}}$

(13) $\overline{N_1}=\frac{\alpha_1}{\alpha_2}, \overline{N_2}=0, \overline{N_3}=\frac{a_4a_{34}+a_3a_{44}}{a_{33}a_{44}-a_{34}a_{43}}, \overline{N_4}=\frac{a_4a_{33}+a_3a_{43}}{a_{33}a_{44}-a_{34}a_{43}}$

Where

$\alpha_1=a_{13}(a_4a_{34}+a_3a_{44})+a_1(a_{33}a_{44}-a_{34}a_{43}),\ \alpha_2=a_{11}(a_{33}a_{44}-a_{34}a_{43})$

(14) $\overline{N_1}=\frac{a_1a_{22}-a_2a_{12}}{a_{11}a_{22}+a_{12}a_{21}}, \overline{N_2}=\frac{a_1a_{21}+a_2a_{11}}{a_{11}a_{22}+a_{12}a_{21}}, \overline{N_3}=0, \overline{N_4}=\frac{a_4}{a_{44}}$

(15) $\overline{N_1}=\frac{\beta_2}{\beta_1}, \overline{N_2}=\frac{\beta_3}{\beta_1}, \overline{N_3}=\frac{a_3}{a_{33}}, \overline{N_4}=0$

Where

$\beta_1=a_{33}(a_{11}a_{22}+a_{12}a_{21}),\ \beta_2=a_{22}(a_1a_{33}+a_3a_{13})-a_2a_{12}a_{33}$

$\beta_3=a_{21}(a_1a_{33}+a_3a_{13})+a_2a_{11}a_{33}$

V. The co-existent state (or) Normal steady state

(16) $\overline{N_1}=\frac{\gamma_1+a_{13}a_{22}\gamma_2}{\gamma_3}, \overline{N_2}=\frac{\gamma_4+a_{13}a_{21}\gamma_2}{\gamma_3},$

$\overline{N_3}=\frac{a_4a_{34}+a_3a_{44}}{a_{33}a_{44}-a_{34}a_{43}}, \overline{N_4}=\frac{a_4a_{33}+a_3a_{43}}{a_{33}a_{44}-a_{34}a_{43}}$

Where

$\gamma_1=(a_1a_{22}+a_2a_{12})(a_{33}a_{44}-a_{34}a_{43}),\ \gamma_2=a_3a_{44}+a_4a_{34}$

$\gamma_3=(a_{11}a_{22}+a_{12}a_{21})(a_{33}a_{44}-a_{34}a_{43}),\ \gamma_4=(a_1a_{21}-a_2a_{11})(a_{33}a_{44}-a_{34}a_{43})$

This can exist only when

$(a_1a_{21}-a_2a_{11})>0$ and $(a_{33}a_{44}-a_{34}a_{43})>0$

The present paper deals with the Co-existent State only. The stability of the other equilibrium states will be presented in the forth coming communications.

Stability of the Equilibrium State 16:
(The co-existent state (or) Normal steady State)

To discuss the stability of equilibrium point

$$\overline{N_1}=\frac{\gamma_1+a_{13}a_{22}\gamma_2}{\gamma_3}, \overline{N_2}=\frac{\gamma_4+a_{13}a_{21}\gamma_2}{\gamma_3}, \overline{N_3}=\frac{a_4a_{34}+a_3a_{44}}{a_{33}a_{44}-a_{34}a_{43}}, \overline{N_4}=\frac{a_4a_{33}+a_3a_{43}}{a_{33}a_{44}-a_{34}a_{43}} \quad (4.1)$$

Let us consider small deviations $u_1(t), u_2(t), u_3(t), u_4(t)$ from the steady state

i.e. $N_i(t)=\bar{N}_i+u_i(t),\ i=1,2,3,4$ --- (4.2)

Substituting (4.1) in (2.1), (2.2), (2.3), (2.4) and neglecting products and higher powers of u_1, u_2, u_3, u_4, we get

$$\frac{du_1}{dt}=-a_{11}\bar{N}_1u_1-a_{12}\bar{N}_1u_2+a_{13}\bar{N}_1u_3 \quad \text{---- (4.3)}$$

$$\frac{du_2}{dt}=-a_{22}\bar{N}_2u_2+a_{21}\bar{N}_2u_1 \quad \text{---- (4.4)}$$

$$\frac{du_3}{dt}=-a_{33}\bar{N}_3u_3+a_{34}\bar{N}_3u_4 \quad \text{---- (4.5)}$$

$$\frac{du_4}{dt}=-a_{44}\bar{N}_4u_4+a_{43}\bar{N}_4u_3 \quad \text{---- (4.6)}$$

The characteristic equation of which is

$$\left[\lambda^2+(a_{11}\bar{N}_1+a_{22}\bar{N}_2)\lambda+(a_{11}a_{22}+a_{12}a_{21})\bar{N}_1\bar{N}_2\right]\times \left[\lambda^2+(a_{33}\bar{N}_3+a_{44}\bar{N}_4)\lambda+(a_{33}a_{44}-a_{34}a_{43})\bar{N}_3\bar{N}_4\right]=0 \quad \text{--- (4.7)}$$

The characteristic roots of (4.7) are

$$\lambda=\frac{-(a_{11}\bar{N}_1+a_{22}\bar{N}_2)\pm\sqrt{(a_{11}\bar{N}_1-a_{22}\bar{N}_2)^2-4a_{12}a_{21}\bar{N}_1\bar{N}_2}}{2},$$

$$\lambda=\frac{-(a_{33}\bar{N}_3+a_{44}\bar{N}_4)\pm\sqrt{(a_{33}\bar{N}_3-a_{44}\bar{N}_4)^2+4a_{34}a_{43}\bar{N}_3\bar{N}_4}}{2} \quad (4.8)$$

$$\Rightarrow \lambda=\frac{-(a_{11}\bar{N}_1+a_{22}\bar{N}_2)\pm\sqrt{\Delta_1}}{2},\ \lambda=\frac{-(a_{33}\bar{N}_3+a_{44}\bar{N}_4)\pm\sqrt{\Delta_2}}{2} \quad (4.9)$$

Where

$\Delta_1=(a_{11}\bar{N}_1-a_{22}\bar{N}_2)^2-4a_{12}a_{21}\bar{N}_1\bar{N}_2$,

$\Delta_2=(a_{33}\bar{N}_3-a_{44}\bar{N}_4)^2+4a_{34}a_{43}\bar{N}_3\bar{N}_4$

Case (i): When $\Delta_1>0$ *and* $\Delta_2>0$

In this case the roots are real and negative. Hence the equilibrium state is **stable.**

Case (ii): When $\Delta_1<0$ *and* $\Delta_2<0$

In this case the roots are complex with negative real parts. Hence the equilibrium state is **stable.**

Case (iii): When $\Delta_1 = 0 \; and \; \Delta_2 = 0$

In this case the roots are repeated, which are negative. Hence the equilibrium state is stable.The trajectories are given by

$$u_1 = \left[\frac{a_{12}\bar{N}_1(u_{10}-u_{20})+\mu_1\lambda_2+\mu_2 a_{13}\bar{N}_1-\mu_3}{\lambda_1-\lambda_2}\right]e^{\lambda_1 t} + \left[\frac{(\mu_1-u_{10})\lambda_1+u_{10}\lambda_2+(a_{12}u_{10}-a_{12}u_{20}+\mu_2 a_{13})\bar{N}_1-\mu_3}{\lambda_2-\lambda_1}\right]e^{\lambda_2 t}+\sigma_1 e^{\lambda_3 t}+\sigma_2 e^{\lambda_4 t} \quad \text{--- (4.10)}$$

$$u_2 = \left[\frac{a_{12}N_1(u_{10}-u_{20})+\mu_1\lambda_2+\mu_2 a_{13}N_1-\mu_3}{\lambda_1-\lambda_2}\right]\delta_1 e^{\lambda_1 t} + \left[\frac{(\mu_1-u_{10})\lambda_1+u_{10}\lambda_2+(a_{12}u_{10}-a_{12}u_{20}+\mu_2 a_{13})\bar{N}_1-\mu_3}{\lambda_2-\lambda_1}\right]\delta_2 e^{\lambda_2 t}+\sigma_3 e^{\lambda_3 t}+\sigma_4 e^{\lambda_4 t} \quad \text{--- (4.11)}$$

$$u_3 = \left[\frac{u_{30}(\lambda_3+a_{44}\bar{N}_4)+u_{40}a_{34}\bar{N}_3}{\lambda_3-\lambda_4}\right]e^{\lambda_3 t} + \left[\frac{u_{30}(\lambda_4+a_{44}\bar{N}_4)+u_{40}a_{34}\bar{N}_3}{\lambda_4-\lambda_3}\right]e^{\lambda_4 t} \quad \text{--- (4.12)}$$

$$u_4 = \left[\frac{u_{40}(\lambda_3+a_{33}\bar{N}_3)+u_{30}a_{43}\bar{N}_4}{\lambda_3-\lambda_4}\right]e^{\lambda_3 t} + \left[\frac{u_{40}(\lambda_4+a_{33}\bar{N}_3)+u_{30}a_{43}\bar{N}_4}{\lambda_4-\lambda_3}\right]e^{\lambda_4 t} \quad \text{--- (4.13)}$$

Here

$$\mu_1 = \sigma_1+\sigma_2 ; \mu_2 = p_1+p_2 ; \; \mu_3 = \sigma_1\lambda_3+\sigma_2\lambda_4 ;$$

$$\sigma_1 = \frac{\alpha_2}{\lambda_3^{\,2}+(a_{11}\bar{N}_1+a_{22}\bar{N}_2)\lambda_3+\alpha_1} ; \; \sigma_2 = \frac{\alpha_3}{\lambda_4^{\,2}+(a_{11}\bar{N}_1+a_{22}\bar{N}_2)\lambda_4+\alpha_1} ;$$

$$\alpha_1 = (a_{11}a_{22}+a_{12}a_{21})\bar{N}_1\bar{N}_2 ; \; \alpha_2 = p_1 a_{13}\bar{N}_1(\lambda_3+a_{22}\bar{N}_2) ; \; \alpha_3 = p_2 a_{13}\bar{N}_3(\lambda_4+a_{22}\bar{N}_2) ;$$

$$p_1 = \frac{u_{30}(\lambda_3+a_{44}\bar{N}_4)+u_{40}a_{34}\bar{N}_3}{\lambda_3-\lambda_4} ; \; p_2 = \frac{u_{30}(\lambda_4+a_{44}\bar{N}_4)+u_{40}a_{34}\bar{N}_3}{\lambda_4-\lambda_3} ; \; \delta_1 = \frac{a_{11}}{a_{12}}-\frac{\lambda_1}{a_{12}\bar{N}_1} ;$$

$$\delta_2 = \frac{a_{11}}{a_{12}}-\frac{\lambda_2}{a_{12}\bar{N}_1} ; \; \sigma_3 = \frac{a_{13}p_1}{a_{12}}+\left(a_{11}-\frac{\lambda_3}{\bar{N}_1}\right)\frac{\sigma_1}{a_{12}} ; \sigma_4 = \frac{a_{13}p_2}{a_{12}}+\left(a_{11}-\frac{\lambda_4}{\bar{N}_1}\right)\frac{\sigma_2}{a_{12}}$$

and $u_{10}, u_{20}, u_{30}, u_{40}$ are the initial values of u_1, u_2, u_3, u_4 respectively.

There would arise in all 576 cases depending upon the ordering of the magnitudes of the growth rates a_1, a_2, a_3, a_4 and the initial values of the perturbations $u_{10}(t), u_{20}(t), u_{30}(t), u_{40}(t)$ of the species S_1, S_2, S_3, S_4. Of these 576 situations some typical variations are illustrated through respective solution curves that would facilitate to make some reasonable observations.

The solution curves are exhibited in figures 1 & 2.

Case (i): If $u_{10} < u_{40} < u_{30} < u_{20}$ and $a_3 < a_1 < a_2 < a_4$

In this case initially the Host (S_3) of S_1 dominates S_4 and the Prey (S_1) till the time instant t^*_{43}, t^*_{13} respectively and the dominance gets reversed there after. Also S_4 dominates over the Prey (S_1) till the time instant t^*_{14} and there after the dominance is reversed. Also u_1, u_2, u_3, u_4 are converging asymptotically to the equilibrium point. Hence the equilibrium point is stable.

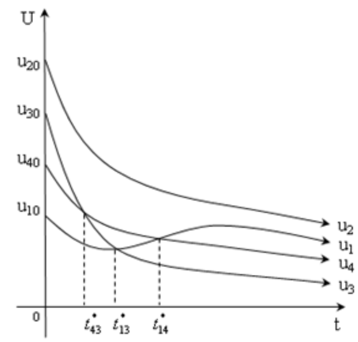

Fig 1.

Case (ii): If $u_{40} < u_{10} < u_{30} < u_{20}$ and $a_3 < a_2 < a_4 < a_1$

In this case initially the Host (S_3) of S_1 dominates S_4 till the time instant t^*_{43} and there after the dominance is reversed. Also the Prey (S_1) dominates over S_4 till the time instant t^*_{41} and the dominance gets reversed there after. As $t \to \infty$, all the four species approach to the equilibrium point. Hence the equilibrium state is stable.

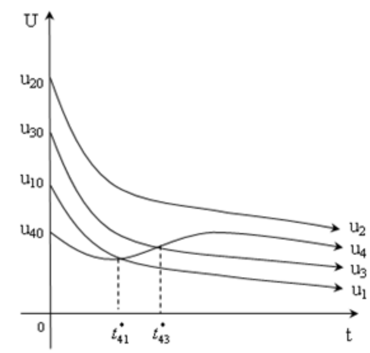

Fig 2.

Liapunov's Function for Global Stability

We discussed the local stability of the state of co-existence. We now examine the global stability of the dynamical system (2.1), (2.2), (2.3) and (2.3). We have already noted that this system has a unique, stable non-trivial co-existent equilibrium state at

$$\overline{N_1} = \frac{\gamma_1+a_{13}a_{22}\gamma_2}{\gamma_3}, \overline{N_2} = \frac{\gamma_4+a_{13}a_{21}\gamma_2}{\gamma_3}, \overline{N_3} = \frac{a_4 a_{34}+a_3 a_{44}}{a_{33}a_{44}-a_{34}a_{43}}, \overline{N_4} = \frac{a_4 a_{33}+a_3 a_{43}}{a_{33}a_{44}-a_{34}a_{43}}$$

We define a Liapunov function

$$V(N_1,N_2,N_3,N_4) = N_1-\bar{N}_1-\bar{N}_1\log\left(\frac{N_1}{\bar{N}_1}\right)+l_1\left\{N_2-\bar{N}_2-\bar{N}_2\log\frac{N_2}{\bar{N}_2}\right\} + l_2\left\{N_3-\bar{N}_3-\bar{N}_3\log\left(\frac{N_3}{\bar{N}_3}\right)\right\}+l_3\left\{N_4-\bar{N}_4-\bar{N}_4\log\left(\frac{N_4}{\bar{N}_4}\right)\right\} \quad \text{--- (5.1)}$$

where l_1, l_2 and l_3 are suitable constants to be determined in the subsequent steps.

Now, the time derivative of V along the solution of (2.1), (2.2), (2.3) and (2.4) is

$$\frac{dV}{dt}=\left(\frac{N_1-\overline{N}_1}{N_1}\right)\frac{dN_1}{dt}+l_1\left(\frac{N_2-\overline{N}_2}{N_2}\right)\frac{dN_2}{dt}+l_2\left(\frac{N_3-\overline{N}_3}{N_3}\right)\frac{dN_3}{dt}+l_3\left(\frac{N_4-\overline{N}_4}{N_4}\right)\frac{dN_4}{dt} \quad \text{- (5.2)}$$

$$\frac{dV}{dt}=\left(\frac{N_1-\overline{N}_1}{N_1}\right) N_1\{a_1-a_{11}N_1-a_{12}N_2+a_{13}N_3\}$$
$$+l_1\left(\frac{N_2-\overline{N}_2}{N_2}\right) N_2\{a_2-a_{22}N_2+a_{21}N_1\}+$$
$$l_2\left(\frac{N_3-\overline{N}_3}{N_3}\right) N_3\{a_3-a_{33}N_3+a_{34}N_4\}$$
$$+l_3\left(\frac{N_4-\overline{N}_4}{N_4}\right) N_4\{a_4-a_{44}N_4+a_{43}N_3\} \quad \text{--- (5.3)}$$

$$=\left(N_1-\overline{N}_1\right)\{a_1-a_{11}N_1-a_{12}N_2+a_{13}N_3\}$$
$$+\ l_1\left(N_2-\overline{N}_2\right)\{a_2-a_{22}N_2+a_{21}N_1\}+$$
$$l_2\left(N_3-\overline{N}_3\right)\{a_3-a_{33}N_3+a_{34}N_4\}$$
$$+l_3\left(N_4-\overline{N}_4\right)\{a_4-a_{44}N_4+a_{43}N_3\} \quad \text{--- (5.4)}$$

$$\frac{dV}{dt}=\left(N_1-\overline{N}_1\right)\left\{a_{11}\overline{N}_1+a_{12}\overline{N}_2-a_{13}\overline{N}_3-a_{11}N_1-a_{12}N_2+a_{13}N_3\right\}$$
$$+\ l_1\left(N_2-\overline{N}_2\right)\left\{a_{22}\overline{N}_2-a_{21}\overline{N}_1-a_{22}N_2+a_{21}N_1\right\}$$
$$+\ l_2\left(N_3-\overline{N}_3\right)\left\{a_{33}\overline{N}_3-a_{34}\overline{N}_4-a_{33}N_3+a_{34}N_4\right\}$$
$$+l_3\left(N_4-\overline{N}_4\right)\left\{a_{44}\overline{N}_4-a_{43}\overline{N}_3-a_{44}N_4+a_{43}N_3\right\} \quad \text{--- (5.5)}$$

$$=\left(N_1-\overline{N}_1\right)\left\{-a_{11}\left(N_1-\overline{N}_1\right)-a_{12}\left(N_2-\overline{N}_2\right)-a_{13}\left(N_3-\overline{N}_3\right)\right\}$$
$$+\ l_1\left(N_2-\overline{N}_2\right)\left\{-a_{22}\left(N_2-\overline{N}_2\right)+a_{21}\left(N_1-\overline{N}_1\right)\right\}$$
$$+\ l_2\left(N_3-\overline{N}_3\right)\left\{-a_{33}\left(N_3-\overline{N}_3\right)+a_{34}\left(N_4-\overline{N}_4\right)\right\}$$
$$+\ l_3\left(N_4-\overline{N}_4\right)\left\{-a_{44}\left(N_4-\overline{N}_4\right)+a_{43}\left(N_3-\overline{N}_3\right)\right\} \quad \text{--- (5.6)}$$

$$\frac{dV}{dt}=-a_{11}\left(N_1-\overline{N}_1\right)^2-a_{12}\left(N_1-\overline{N}_1\right)\left(N_2-\overline{N}_2\right)-a_{13}\left(N_1-\overline{N}_1\right)\left(N_3-\overline{N}_3\right)$$
$$+\ l_1\left\{(-a_{22})\left(N_2-\overline{N}_2\right)^2+a_{21}\left(N_1-\overline{N}_1\right)\left(N_2-\overline{N}_2\right)\right\}$$
$$+\ l_2\left\{(-a_{33})\left(N_3-\overline{N}_3\right)^2+(a_{34})\left(N_3-\overline{N}_3\right)\left(N_4-\overline{N}_4\right)\right\}$$
$$+\ l_3\left\{(-a_{44})\left(N_4-\overline{N}_4\right)^2+(a_{43})\left(N_3-\overline{N}_3\right)\left(N_4-\overline{N}_4\right)\right\} \quad \text{--- (5.7)}$$

Choosing $l_1=\frac{a_{12}}{a_{21}}$, l_2 and l_3 are any positive constants, (5.7) becomes,

$$\frac{dV}{dt}=-a_{11}\left(N_1-\overline{N}_1\right)^2-a_{13}\left(N_1-\overline{N}_1\right)\left(N_3-\overline{N}_3\right)-\frac{a_{12}a_{22}}{a_{21}}\left(N_2-\overline{N}_2\right)^2-l_2a_{33}\left(N_3-\overline{N}_3\right)^2$$
$$+l_2a_{34}\left(N_3-\overline{N}_3\right)\left(N_4-\overline{N}_4\right)-l_3a_{44}\left(N_4-\overline{N}_4\right)^2+l_3a_{43}\left(N_3-\overline{N}_3\right)\left(N_4-\overline{N}_4\right) \quad \text{--- (5.8)}$$

$$<-a_{11}\left(N_1-\overline{N}_1\right)^2-\frac{a_{13}}{2}\left\{\left(N_1-\overline{N}_1\right)^2+\left(N_3-\overline{N}_3\right)^2\right\}-\frac{a_{12}a_{22}}{a_{21}}\left(N_2-\overline{N}_2\right)^2$$
$$-a_{33}l_2\left(N_3-\overline{N}_3\right)^2-a_{44}l_3\left(N_4-\overline{N}_4\right)^2+\frac{(a_{34}l_2+a_{43}l_3)}{2}\left\{\left(N_3-\overline{N}_3\right)^2+\left(N_4-\overline{N}_4\right)^2\right\} \quad \text{--- (5.9)}$$

$$<(-a_{11}-\frac{a_{13}}{2})\left(N_1-\overline{N}_1\right)^2-\frac{a_{12}a_{22}}{a_{21}}\left(N_2-\overline{N}_2\right)^2$$
$$\left[\frac{(a_{34}l_2+a_{43}l_3)}{2}-\frac{a_{13}}{2}-a_{33}l_2\right]\left(N_3-\overline{N}_3\right)^2+\left[\frac{(a_{34}l_2+a_{43}l_3)}{2}-a_{44}l_3\right]\left(N_4-\overline{N}_4\right)^2 \quad \text{--- (5.10)}$$

<0, Provided

$$\frac{(a_{34}l_2+a_{43}l_3)}{2}<\frac{a_{13}}{2}+a_{33}l_2 \ and \ \frac{(a_{34}l_2+a_{43}l_3)}{2}<a_{44}l_3$$

Hence the co-existent is globally asymptotically stable.

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REFERENCES

[1] R. Archana Reddy. 2009. On the stability of some mathematical models in biosciences- interacting species, Ph.D thesis,JNTU.

[2] H.I. 1980. Freedman, Deterministic Mathematical Models in Population Ecology, Marcel – Decker, New York.

[3] J.N. Kapur. 1988. Mathematical Modeling, Wiley – Eastern.

[4] J.N. Kapur. 1985. Mathematical Models in Biology and Medicine Affiliated East -West.

[5] K. Lakshmi Narayan. 2004. A Mathematical study of Prey-Predator Ecological Models with a partial covers for the prey and alternative food for the predator, Ph.D thesis,J.N.T.University.

[6] A.j. Lotka. 1925. Elements of Physical biology, Williams and Wilkins, Baltimore.

[7] W.J. Meyer. 1985. Concepts of Mathematical Modeling, Mc Graw – Hill.

[8] Paul Colinvaux. 1986. Ecology, John Wiley and Sons Inc., New York.

[9] B.Ravindra Reddy, K. Lakshmi Narayan and N.Ch. Pattabhiramacharyulu. 2010. A model of two mutually interacting species with limited resources and a time delay, Advances in Theoretical and Applied Mathematics, Vol.5, No.2 pp. 121-132.

[10] B.Ravindra Reddy, K. Lakshmi Narayan and N.Ch. Pattabhiramacharyulu. 2010. A model of two mutually interacting species with limited resources and harvesting of both the species at a constant rate, International J. of Math. Sci & Engg. Appls. (IJMSEA), Vol. 4, No. III . pp. 97-106.

[11] N.C. Srinivas. 1991. Some Mathematical aspects of modeling in Bio Medical Sciences, Ph.D thesis. Kakatiya University.

[12] V. Volterra. 1931. Leconssen la theorie mathematique de la leitte pou lavie, Gauthier –Villars, Paris.