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Stability of a three species ecological system consisting of prey- predator species and a third species which is a host to the prey and enemy to the predator

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Abstract

Prey-Predator ecological system was presented by Lotka and Volterra in their classical model. Inspired by that, several researchers made significant contributions in this area by considering various special types of interactions between the prey and the predator. This has been the motivation for others in bringing a third species into the system thus forming a three species ecological system. Recently, some researchers worked on this three species system by considering interactions like Prey-Predator, Commensal -Host, Ammensal-Enemy, between the three species, which motivated the present paper. In this paper we discussed a three species ecological system consisting of a Prey (S1), a Predator (S2) and a third species (S3) which is a host to the prey and enemy to the predator. Hence the prey plays Commensal for the third species and the predator plays Ammensal for the same. The mathematical model consists of three equations which constitute a set of three first order non-linear simultaneous differential equations in N1, N2 and N3, which are respective populations of the species S1, S2 and S3.The Equation for the third species is non-linear but decoupled with the prey-predator pair. Totally, eight equilibrium points of the model are identified and the criteria for their local stability are discussed. Solutions for the linearized perturbed equations are found and the results are illustrated.

Keywords: Prey, Predator, Commensal,Ammensal,Host,Enemy

INTRODUCTION

Ecology is a branch of evolutionary biology- a science that explains how different kinds of living beings can live together in the same environment for generations, sharing the same habitat, interact with each other in diverse ways. Some typical interactions between the species are given as Neutralism, Commesalism, Mutualism, Syntrophism, Competition, Ammensalism, Parasitism, Predation.

Mathematical modeling of ecosystems was initiated by Lotka[9] and Volterra[16]. The general concepts of modeling have been presented in the treatises of Meyer[10],Cushing[3],Paul Colinvaux,Freedman[4],Kapur[5,6] and several others. As models in any branch of science and technology, mathematical models in theoretical ecology are of great importance and utility because they both answer and raise questions related to natural phenomena. This is the primary reason for many researchers to pursue the path of Ecological models. Some of those people who made significant contributions in the recent years are listed here. N.C. Srinivas [15] studied the competitive ecosystems of two species and three species with limited and unlimited recourses. Lakshminarayana and Pattabhi Ramacharyulu [7,8] investigated prey-predator ecological models with a partial cover for the prey and alternative food for the predator and prey predator model with cover for prey and alternate food for the predator and to me delay. Recently, stability analysis of competitive species was carried out by Archana Reddy, Pattabhi Ramacharyulu and Gandhi [1], by Bhaskara Rama Sarma and Pattabhi Ramacharyulu [2] , While the mutualism between two species was examined by Ravindra Reddy [13]. Recently Phanikumar, Seshagiri Rao and Pattabhi Ramacharyuly [14] investigated on the stability of a host-A decaying commensal species pair with limited resources.

The present investigation is an analytical study of three species system: Prey-commensal-Predator and Host/Enemy system. In all Eight equilibrium points are identified based on the model equations and these are spread over three distinct classes: (i)Fully washed out (ii)Semi / partially washed out and (iii) Co-existent states. Criteria for the asymptotic stability of the states have been derived.

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NOTATION

N_1: The population of the Prey / Commensal.

N_2: The population of the Predator/ Ammensal.(Predator to prey N_1 & Ammensal to N_3)

N_3: The population of Host to N_1 / Enemy to N_2

a_i : The Natural growth rate of N_i, i=1,2,3.

a_{ii}: The rate of decrease of N_i due to insufficient resources of N_i, i=1,2,3

a_{12}: The rate of decrease of prey (N_1) due to inhibition by Predator(N_2)

a_{13}: The rate of increase of the Commensal (N_1) due to its successful promotion by the host (N_3).

a_{21} : The rate of increase of the predator (N_2) due to its successful attacks on the prey (N_1)

a_{23} : The rate of decrease of the Ammensal (N_2) due to the harm caused by its enemy (N_3).

K_i (= a_i/ a_{ii}) : Carrying Capacity of N_i, i=1,2,3.
P (= a_{12}/ a_{11}) : Coefficient of Prey / Commensal inhibition of the predator.
q (= a_{13}/ a_{11}) : Coefficient of commensalism.
r(= a_{21}/ a_{22}) : Coefficient of predator consumption of the prey
s(= a_{23}/ a_{22}) : Coefficient of Ammensalism.

BASIC BALANCE EQUATIONS OF THE MODEL

The model equations for a three species multi reactive ecosystem are given by the following system of non-linear ordinary differential Equations.

1. Equation for the growth rate of the Prey/ Commensal species (N_1).

$$\frac{dN_1}{dt} = a_1 N_1 - a_{11} N_1^2 + a_{13} N_1 N_3 - a_{12} N_1 N_2$$

2. Equation for the growth rate of predator /Ammensal species (N_2).

$$\frac{dN_2}{dt} = a_2 N_2 - a_{22} N_2^2 + a_{21} N_1 N_2 - a_{23} N_2 N_3$$

3. Equation for the growth rate of Host to prey / Enemy to predator (N_3).

$$\frac{dN_3}{dt} = a_3 N_3 - a_{33} N_3^2$$

Further, the Variables N_1, N_2, N_3 are non negative and the model parameters a_1, a_2, a_3, a_{11}, a_{22}, a_{33}, a_{13}, a_{12}, a_{21}, a_{23} are all non-negative and assumed to be constants.

In terms of the notation adopted in (2) equations 1, 2 & 3 can be rewritten as

$$\frac{dN_1}{dt} = a_{11} N_1 [K_1 - N_1 - p N_2 + q N_3] \qquad (1)$$

$$\frac{dN_2}{dt} = a_{22} N_2 [K_2 - N_2 + r N_1 - s N_3] \qquad (2)$$

$$\frac{dN_3}{dt} = a_{33} N_3 [K_3 - N_3] \qquad (3)$$

EQUILIBRIUM STATES

The Equilibrium states ($\overline{N_1}$, $\overline{N_2}$, $\overline{N_3}$) are obtained by considering $\frac{dN_i}{dt} = 0$, i=1,2,3.

The System under investigation has *Eight Equilibrium states* given by:

A. Fully washed out state

$$\overline{N_1} = 0,\quad \overline{N_2} = 0,\quad \overline{N_3} = 0$$

B. States in which two of the three species are washed out and third is not.

$$\overline{N_1} = 0,\quad \overline{N_2} = 0,\quad \overline{N_3} = K_3$$
$$\overline{N_1} = K_1,\quad \overline{N_2} = 0,\quad \overline{N_3} = 0$$
$$\overline{N_1} = 0,\quad \overline{N_2} = K_2,\quad \overline{N_3} = 0$$

C. States in which only one of the three species is washed out while the other two are not.

$$\overline{N_1} = 0,\quad \overline{N_2} = K_2 - sK_3\ (K_2 > sK_3),\quad \overline{N_3} = K_3$$
$$\overline{N_1} = K_1 + qK_3,\quad \overline{N_2} = 0,\quad \overline{N_3} = K_3$$
$$\overline{N_1} = \frac{K_1 - pK_2}{1+rp}\ (K_1 > pK_2),\quad \overline{N_2} = \frac{rK_1 + K_2}{1+rp},\quad \overline{N_3} = 0$$

D. The Co-existent state or normal steady state.

$$\overline{N_1} = \frac{K_1 + pK_2 + (q+ps)K_3}{1+rp},$$
$$\overline{N_2} = \frac{rK_1 + K_2 + (rq-s)K_3}{1+rp}\ (rK_1+K_2+rqK_3>sK_3),\quad \overline{N_3} = K_3$$

The Stability of the Equilibrium States

We consider slight deviations U_1 (t), U_2 (t) and U_3 (t) over the steady state ($\overline{N_1}$ $\overline{N_2}$ $\overline{N_3}$):

i.e. $N_1 = \overline{N_1} + U_1$ (t), $N_2 = \overline{N_2} + U_2$ (t), $N_3 = \overline{N_3} + U_3$ (t)
Where U_1 (t), U_2 (t) and U_3 (t) are so small so that their second and higher powers and products are neglected.

Fully Washed out Equilibrium State

$\overline{N_1} = 0$, $\overline{N_2} = 0$, $\overline{N_3} = 0$.
In this case, we get from (1), (2) and (3),

$$\frac{dU_1}{dt} = K_1 a_{11} U_1;\quad \frac{dU_2}{dt} = K_2 a_{22} U_2;\quad \frac{dU_3}{dt} = K_3 a_{33} U_3. \qquad (5.1.1)$$

The characteristic roots of the system (5.1.1) are $K_1 a_{11}$, $K_2 a_{22}$, $K_3 a_{33}$, which are all positive. So the state is *Unstable*.
The Equations (5.1.1) yield the solution curves

$$U_1=U_{10}e^{K_1 a_{11} t};\ U_2=U_{20}e^{K_2 a_{22} t};\ U_3=U_{30}e^{K_3 a_{33} t} \qquad (5.1.2)$$

Where U_{10}, U_{20} and U_{30} are initial values of U_1, U_2 and U_3 respectively.

Several different solution curves have been observed of which a few of them are discussed in the following figures and the conclusions are presented.

Case: 5.1(i) $U_{10} > U_{20} > U_{30}$ & $K_1 a_{11} > K_2 a_{22} > K_3 a_{33}$.

In this case the initial strength of prey as well as its growth rate is greater than those of others. The prey out numbers the host and the predator till the end, as shown in Fig.1.

Case: 5.1(ii) $U_{10} > U_{30} > U_{20}$ & $K_3 a_{33} > K_1 a_{11} > K_2 a_{22}$

In this case, the prey dominates the predator till the time instant t_{13}^* as shown in Fig. 2.

Case: 5.1(iii) $U_{20} > U_{10} > U_{30}$ & $K_3 a_{33} > K_1 a_{11} > K_2 a_{22}$

Here the population of the host remains less than that of prey until the time instant t_{31}^*. It remains less than that of the predator until the time instant t_{32}^* . The prey out numbers the predator from the time instant t_{12}^*.

Where $t_{13}^* = t_{31}^* = \frac{1}{(a_1 - a_3)} \log \left(\frac{U_{30}}{U_{10}}\right)$, $t_{32}^* = \frac{1}{(a_2 - a_3)} \log \frac{U_{30}}{U_{20}}$

$t_{12}^* = \frac{1}{(a_2 - a_1)} \log \frac{U_{10}}{U_{20}}$ The results are shown in Fig. 3.

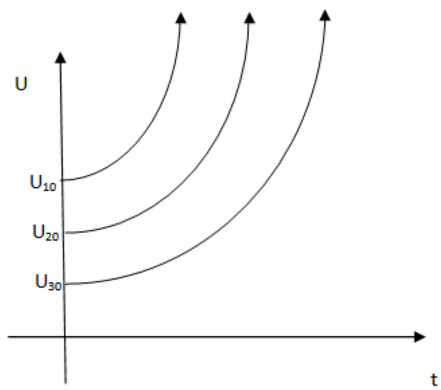

Fig. 1. $U_{10} > U_{20} > U_{30}$ & $K_1 a_{11} > K_2 a_{22} > K_3 a_{33}$

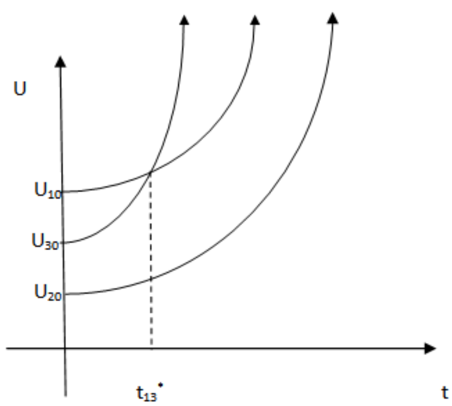

Fig. 2. $U_{10} > U_{30} > U_{20}$ & $K_3 a_{33} > K_1 a_{11} > K_2 a_{22}$

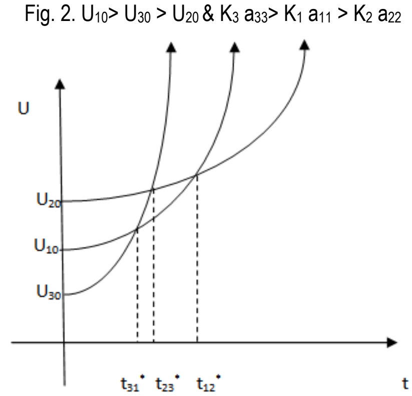

Fig 3. $U_{20} > U_{10} > U_{30}$ & $K_3 a_{33} > K_1 a_{11} > K_2 a_{22}$

Prey and Predator Washed out State:

$\overline{N_1} = 0$, $\overline{N_2} = 0$, $\overline{N_3} = K_3$

In this case, we get from (1),(2) and (3)

$\frac{dU_1}{dt} = a_{11}(K_1 + K_3 q) U_1$; $\frac{dU_2}{dt} = a_{22}(K_2 - sK_3) U_2$; $\frac{dU_3}{dt} = -K_3 a_{33} U_3$ (5.2.1)

The Characteristic roots of (5.2.1) are $\lambda_1 = a_{11}(K_1+K_3q)$, $\lambda_2 = a_{22}(K_2-sK_3)$ and $\lambda_3 = -K_3 a_{33}$

Of these, λ_1 is always positive. So the state is *unstable* .The equations (5.2.1) yield the solution curves

$U_1 = U_{10} e^{a_{11}(K_1+K_3q)t}$; $U_2 = U_{20} e^{a_{22}(K_2-sK_3)t}$; $U_3 = U_{30} e^{-K_3 a_{33} t}$ (5.2.2)

Where U_{10}, U_{20} , U_{30} are initial values of U_1, U_2 , U_3 respectively.

Case A: $K_2 - sK_3 > 0$

When $K_2 - sK_3 > 0$, the first two roots are positive and the third is negative.

Case 5.2 A(i): $U_{10} > U_{20} > U_{30}$ and $a_{22}(K_2-sK_3) > a_{11}(K_1+K_3q)$

Here, the prey dominates the predator until the time instant $t_{21}^* = \frac{1}{a_{22}(K_2 - sK_3) - a_{11}(K_1 + K_3 q)} \log \left(\frac{U_{10}}{U_{20}}\right)$ and there after the predator outnumbers the prey, as shown in Fig. 4.

Case 5.2 A(ii): $U_{10} > U_{30} > U_{20}$ and $a_{22}(K_2-sK_3) > a_{11}(K_1+K_3q)$

Here the enemy dominates the ammensal till the time instant $t_{23}^* = \frac{1}{a_{22}(K_2 - sK_3) + k_3 a_{33}} \log \left(\frac{U_{30}}{U_{20}}\right)$ and the predator is dominated by the prey until the time instant $t_{21}^* = \frac{1}{a_{22}(K_2 - sK_3) - a_{11}(K_1 + K_3 q)} \log \left(\frac{U_{10}}{U_{20}}\right)$.The results are illustrated in Fig. 5.4.

Case 5.2 A(iii): $U_{30} > U_{20} > U_{10}$ and $a_{11}(K_1 + K_3 q) > a_{22}(K_2-sK_3)$

In this case, the ammensal is dominated by the enemy till the time instant $t_{23}^* = \frac{1}{a_{22}(K_2 - sK_3) + k_3 a_{33}} \log \left(\frac{U_{30}}{U_{20}}\right)$

Also the host dominates the commensal till the time instant $t_{13}^* = \frac{1}{a_{11}(K_1 + K_3 q) + K_3 a_{33}} \log \left(\frac{U_{30}}{U_{10}}\right)$, which is shown in Fig. 6

Case B: $K_2 - sK_3 < 0$

In this case, λ_1 is positive and the remaining two roots are negative.

Case 5.2 B(i): $U_{10} > U_{20} > U_{30}$ and $a_{22}(sK_3-K_2) > K_3 a_{33}$

From the time instant $t_{23}^* = \frac{1}{a_{22}(K_2 - sK_3) + k_3 a_{33}} \log \left(\frac{U_{30}}{U_{20}}\right)$, the Ammensal and the enemy species move towards the equilibrium, which is shown in Fig. 7.

Case C: $K_2 = sK_3$

Case 5.2 C(i) : $U_{10} > U_{20} > U_{30}$

In this, the third species approaches the equilibrium point asymptotically, as shown in Fig.8.

Case 5.2 C(ii): $U_{20} > U_{10} > U_{30}$

In this case, the prey out numbers the predator from the time instant $t_{12}^{*}= \dfrac{1}{-a_{11}(K_1+K_3q)}$ log ($\dfrac{U_{10}}{U_{20}}$),as shown in Fig.9 .

Case 5.2 C(iii) : $U_{30}>U_{10}>U_{20}$

In this case, the prey dominates its host from the time instant $t_{13}^{*}= \dfrac{1}{a_{11}(K_1+K_3q)+K_3a_{33}}$ log ($\dfrac{U_{30}}{U_{10}}$), and the predator and its enemy move towards the equilibrium point asymptotically from the time instant $t_{32}^{*}= \dfrac{1}{k_3a_{33}}$ log ($\dfrac{U_{30}}{U_{20}}$).

These results are shown in Fig. 10.

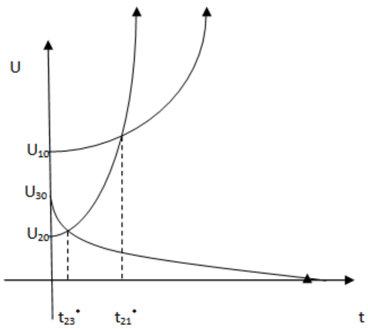

Fig 4. $U_{10}> U_{20} > U_{30}$ and $a_{22}(K_2\text{-}sK_3) > a_{11}(K_1\text{+}K_3q)$

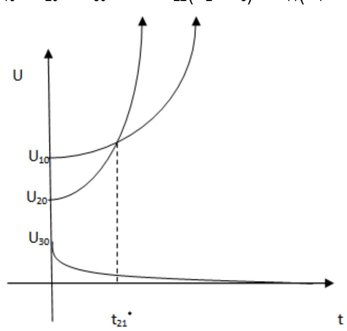

Fig 5. $U_{10}> U_{30} > U_{20}$ and $a_{22}(K_2\text{-}sK_3)>a_{11}(K_1\text{+}K_3q)$

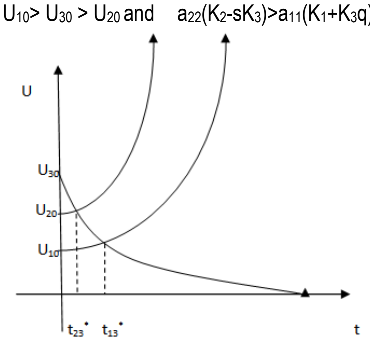

Fig.6. $U_{30}> U_{20} > U_{10}$ and $a_{11}(K_1\text{+} K_3q) > a_{22}(K_2\text{-}sK_3)$

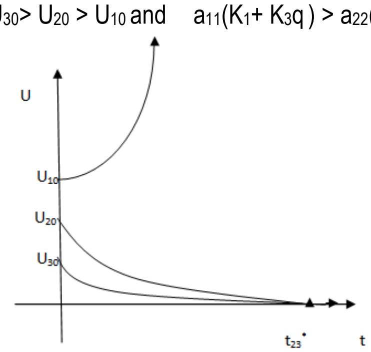

Fig 7. $U_{10}> U_{20} > U_{30}$ and $a_{22}(sK_3\text{-}K_2) > K_3a_{33}$

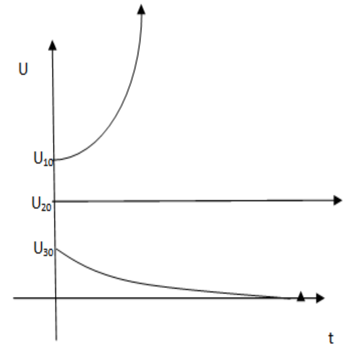

Fig 8. $K_2\text{=}sK_3$ and $U_{10}>U_{20}>U_{30}$

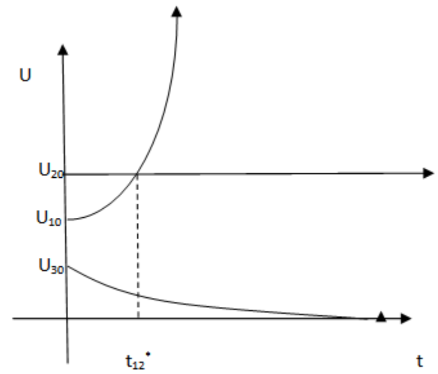

Fig 9. $K_2\text{=}sK_3$ and $U_{20}>U_{10}>U_{30}$

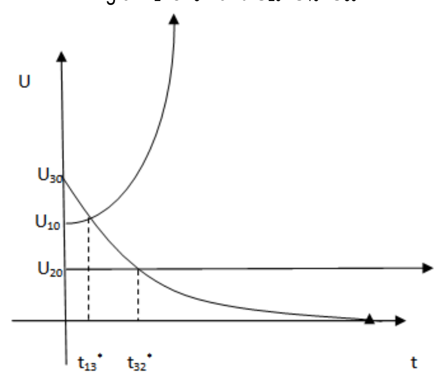

Fig10. $K_2\text{=}sK_3$ and $U_{30}>U_{10}>U_{20}$

Prey/Commensal washed out state

$\overline{N}_1 =0,\quad \overline{N}_2 = K_2\text{-}sK_3 > 0,\quad \overline{N}_3 =K_3$

In this case, we get from (1),(2) and (3):

$\dfrac{dU_1}{dt}$ =$a_{11}[K_1 - p(K_2\text{-}sK_3)\text{+}qK3]U_1$; $\dfrac{dU_2}{dt}$ = a_{22} $(K_2\text{-}sK_3$)$[rU_1\text{-} U_2 - sU_3]$; $\dfrac{dU_3}{dt}$ = $-a_{33}K_3 U_3$ (5.3.1)

The characteristic roots of (5.3.1) are $\lambda_1 = a_{11}[K_1 - p(K_2\text{-}sK_3)\text{+}qK3]$; $\lambda_2\text{=-}a_{22} (K_2\text{-}sK_3)$; $\lambda_3 = -a_{33}K_3$

The equations (5.3.1.) yield the solutions

$U_1\text{=} U_{10}e^{\lambda_1 t}$; $U_2\text{=-}\lambda_2\left(\dfrac{rU_{10}e^{\lambda_1 t}}{\lambda_1-\lambda_2} - \dfrac{sU_{30}e^{\lambda_3 t}}{\lambda_3-\lambda_2}\right) + [U_{20} + \lambda_2\left(\dfrac{rU_{10}}{\lambda_1-\lambda_2} - \dfrac{sU_{30}}{\lambda_3-\lambda_2}\right)]e^{\lambda_2 t}$; $U_3\text{=}U_{30}e^{\lambda_3 t}$ (5.3.2)

Where U_{10}, U_{20}, U_{30} are the initial values of U_1, U_2, U_3 respectively. When $U_{20}=-\lambda_2(rU_{10}/(\lambda_1-\lambda_2) - sU_{30}/(\lambda_3-\lambda_2))$ the equations (5.3.2.) become

$U_1= U_{10}e^{\lambda_1 t}$; $U_2=-\lambda_2(rU_{10}e^{\lambda_1 t}/(\lambda_1-\lambda_2) - sU_{30}e^{\lambda_3 t}/(\lambda_3-\lambda_2))$; $U_3=U_{30}e^{\lambda_3 t}$
Let $\gamma_1= -\lambda_2 rU10/(\lambda_1-\lambda_2)$, $\gamma_2=-\lambda_2 sU_{30}/(\lambda_3-\lambda_2)$.
Then we have

$$U_1= U_{10}e^{\lambda_1 t};U_2 = U_{20}e^{\lambda_1 t} + \gamma_2(e^{\lambda_1 t}- e^{\lambda_3 t});U_3=U_{30}e^{\lambda_3 t} \quad (5.3.3)$$

Case A: $K_1+qK_3 > p(K_2-sK_3)$

In this case $\lambda_1>0$, so this state is *unstable*.

Case A(i): $U_{30} > U_{20} > U_{10}$

In this case, the prey and predator go away from the equilibrium point while, the third species moves towards the equilibrium point asymptotically. Further, the third species is dominated by the prey from the time instant $t_{13}^* = \frac{1}{\lambda_3-\lambda_1}\log(\frac{U_{10}}{U_{30}})$ and by the predator from the time instant $t_{23}^* = \frac{1}{\lambda_1-\lambda_3}\log(\frac{U_{30}+\gamma_2}{U_{30}+\gamma_2})$. This is shown in Fig. 11

Case A(ii) : $U_{10} > U_{30} > U_{20}$

In this case, the third species is dominated by the predator from the time instant $t_{23}^* = \frac{1}{\lambda_1-\lambda_3}\log(\frac{U_{30}+\gamma_2}{U_{30}+\gamma_2})$,
As shown in Fig. 12.

Case B: $K_1+qK_3 < p(K_2-sK_3)$

In this case, all the three characteristic roots are negative, so the system is *stable*.

Case B(i): $U_{10} > U_{30} > U_{20}$

In this case all the three species move towards the equilibrium point asymptotically, as shown in Fig. 13

Case B(ii): $U_{20} > U_{30} > U_{10}$

In this case too , all the three species move towards the equilibrium state asymptotically, as shown in Fig. 14.

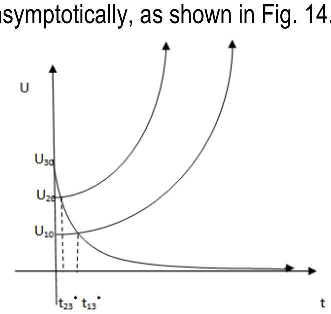

Fig 11. $K_1+qK_3 > p(K_2-sK_3)$ and $U_{30} > U_{20} > U_{10}$

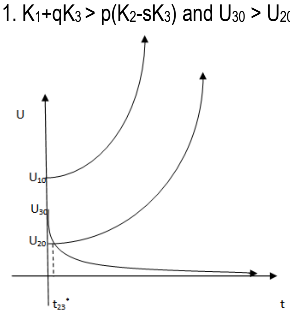

Fig 12. $K_1+qK_3 > p(K_2-sK_3)$ and $U_{10} > U_{30} > U_{20}$

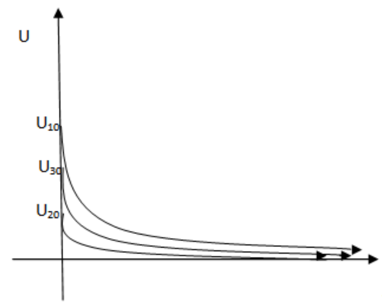

Fig 13. $K_1+qK_3 < p(K_2-sK_3)$ and $U_{10} > U_{30} > U_{20}$

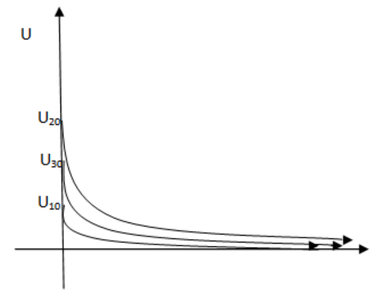

Fig 14. $K_1+qK_3 < p(K_2-sK_3)$ and $U_{20} > U_{30} > U_{10}$

Predator/Ammensal washed out State:

$\overline{N_1} = K_1+qK_3$, $\overline{N_2} = 0$, $\overline{N_3} = K_3$.

In this case we get from (1), (2) and (3):

$$\frac{dU_1}{dt} = a_{11}(K_1+qK_3)[-U_1-pU_2+qU_3]; \quad \frac{dU_2}{dt} = a_{22}[K_2+r(K_1+qK_3)-sK_3]U_2; \frac{dU_3}{dt} = -a_{33}K_3U_3 \quad (5.4.1)$$

The characteristic roots of the equations (5.6.1) are $\lambda_1 = -a_{11}(K_1+qK_3)$; $\lambda_2=a_{22}[K_2+r(K_1+qK_3)-sK_3]$; $\lambda_3 = -a_{33}K_3$
The equations (5.4.1) yield the solutions curves

$$U_1=\lambda_1\left(\frac{pU_{20}e^{\lambda_2 t}}{\lambda_2-\lambda_1}-\frac{qU_{30}e^{\lambda_3 t}}{\lambda_3-\lambda_1}\right)+\left[U_{10}-\lambda_1\left(\frac{pU_{20}}{\lambda_2-\lambda_1}-\frac{qU_{30}}{\lambda_3-\lambda_1}\right)\right]e^{\lambda_1 t};$$
$$U_2= U_{20}e^{\lambda_2 t}; U_3=U_{30}e^{\lambda_3 t} \quad (5.4.2)$$

Let $\gamma_1= -\lambda_1 pU_{20}/(\lambda_2-\lambda_1)$, $\gamma_2=-\lambda_1 qU_{30}/(\lambda_3-\lambda_1)$ .When $U_{10}=\lambda_1(pU_{20}/\lambda_2-\lambda_1 - qU_{30}/\lambda_3-\lambda_1)=\gamma_1-\gamma_2$ the equations (5.4.2.) become

$$U_1(t)= U_{10}e^{\lambda_3 t} + \gamma_2(e^{\lambda_2 t}- e^{\lambda_3 t}); U_2(t)=U_{20}e^{\lambda_2 t};U_3(t)=U_{30}e^{\lambda_3 t} \quad (5.4.3)$$

Case A: $K_2+r(K_1+qK_3)> sK_3$,

In this case, two of the Characteristic roots are negative and one is positive. So this state is ***unstable.***

Case 5.4A(i) : $U_{30}>U_{20}>U_{10}$

In this case, the predator goes away from the equilibrium point while the other two species move towards the same.This is shown in Fig. 15

Case 5.4A(ii) : $U_{20}>U_{10}>U_{30}$

In this, the prey species and the third species move towards the equilibrium point while, the predator species move away from the equilibrium point, as shown in Fig. 16.

Case B: $K_2+r(K_1+qK_3) < sK_3$

In this case, all the three characteristic roots are negative. So this state is *stable.*

Case 5.4 B(i): $U_{20} > U_{10} > U_{30}$

In this case, all the three species move towards the equilibrium point asymptotically as shown in Fig. 17.

Case 5.4 B(ii) : $U_{30} > U_{10} > U_{20}$

In this case too, all the three species move towards the equilibrium point asymptotically as shown in Fig. 18.

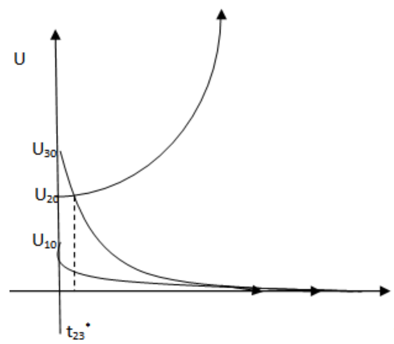

Fig 15. $K_2+r(K_1+qK_3) > sK_3$ and $U_{30}>U_{20}>U_{10}$

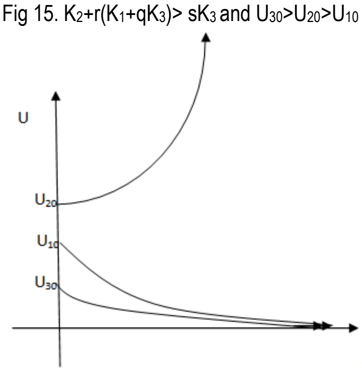

Fig16. $K_2+r(K_1+qK_3) > sK_3$ and $U_{20}>U_{10}>U_{30}$

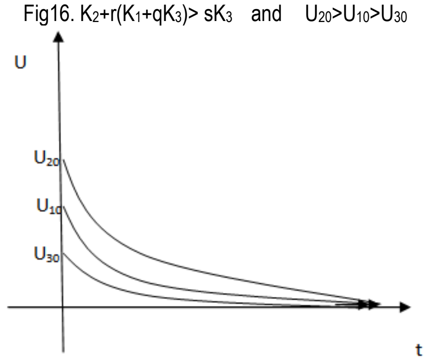

Fig 17. $K_2+r(K_1+qK_3) < sK_3$ and $U_{20} > U_{10} > U_{30}$

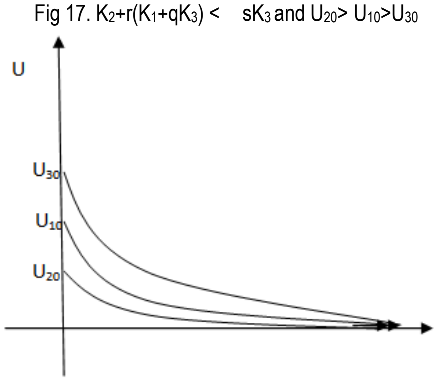

Fig18. $K_2+r(K_1+qK_3) < sK_3$ and $U_{30}>U_{10}>U_{20}$

DISCUSSION AND CONCLUSION

In this paper, a three species eco system is considered. The interactions that are considered are Prey-Predator, Ammensalism and Commensalism. In all, eight equilibrium states are identified and the local stability criteria for four of the equilibrium states are established. Of the states that are examined, some cases of Prey washed out state and some cases of predator washed out state are found to be stable.

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