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Some threshold results of ecological competition model with reserve for one species and harvesting at fixed rates.

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Abstract
In the present investigation the stability analysis of a two-species competing ecological model with reserve for one species and harvesting at fixed rates is highlighted in view of principle of competitive exclusion due to Gause (1934). The model is characterized by a pair of non-linear system of ordinary differential equations. The equilibrium states are identified and a threshold theorem is derived to establish the stability of the co-existent equilibrium state.

Keywords: Competitive exclusion, Threshold results, Stability, Equilibrium states
AMS Classification: 92 D 25, 92 D 40

INTRODUCTION

Competition between two or more species or individuals would occur when they are to strive together in a habitat if the resources for their growth or existence are in a short supply. It arises essentially during the struggle for existence. Popular histories of competition have been dealt by the researchers such as Gause [5], Paul Colinvaux [9], Kapur.J.N. [6] Lotka.A.J. [7], Meyer[8]. Bhaskara Rama Sarma .B & N. Ch. Pattabhiramacharyulu [2,3,4] have extensively studied the Ecological Competition models under various conditions. Archana Reddy. R and N.Ch. Pattabhiramacharyulu [1] have studied the stability analysis of competition model with reserve for one species and harvesting both the species at fixed rates.

In the present investigation some threshold results are established following the Principle of Competitive Exclusion (Gause, 1934) [5] and results are illustrated. In Section 1.1 basic equations of two species competition model incorporating i) Reserve for one Species. ii) Both the Species are harvested at constant rates are presented along with the required notations. In Section 1.2 the Locus of the co-existent equilibrium point is obtained and a particular equilibrium point which corresponds to half the carrying capacities of the species is identified .In Section1.3 the local stability analysis of the equilibrium state is carried out. In Section1.4, Threshold theorem on the lines of Principle of competitive exclusion is derived and the phase-portrait diagram is presented to explain the global stability of the equilibrium point under consideration. In Section1.5 the conclusions of the work are recorded.

1.1 Basic Equations

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The model equations for a two Species competing system is given by the following system of non-linear ordinary differential equations

Equation for the growth rate of S_1 Species

$$\frac{dN_1}{dt} = a_1 N_1 - a_{11} N_1^{\,2} - a_{12}(1-k)N_1 N_2 - h_1 \tag{1.1}$$

Equation for the growth rate of Species S_2

$$\frac{dN_2}{dt} = a_2 N_2 - a_{22} N_2^{\,2} - a_{21}(1-k)N_1 N_2 - h_2 \tag{1.2}$$

Where N_1, N_2 : Population strengths of species S_1 ,S_2 respectively at time t

a_1, a_2 : Natural growth rates of the two species;
a_{11}, a_{22} : Self-limiting co-efficients(i.e ,the carrying capacities are limited)
a_{12} ,a_{21} : Inhibition co-efficients of each species on the other .
k : Reserve for species S_1
h_1, h_2 :fixed harvesting rates of respective species

1.2 Equilibrium states

The equilibrium states are given by $\frac{dN_1}{dt} = 0$ and $\frac{dN_2}{dt} = 0$

i.e. $$N_1\{a_1 - a_{11}N_1 - a_{12}(1-k)N_2\} = h_1 \tag{1.3}$$

$$N_2\{a_2 - a_{22}N_2 - a_{21}(1-k)N_1\} = h_2 \tag{1.4}$$

Equation $(1.3)\times\{-\alpha_{21}(1-k)\} + \text{Equation } (1.4)\times\alpha_{12}(1-k)$ we get

$$-h_1a_{21}(1-k)+h_2a_{12}(1-k)=$$

$$-N_1a_{21}(1-k)\{a_1-a_{11}N_1-a_{12}(1-k)N_2\}+N_2a_{12}(1-k)\{a_2-\alpha_{22}N_2-a_{21}(1-k)N_1\}$$

$$=-a_{21}(1-k)a_1N_1+a_{11}a_{21}(1-k)N_1^2+a_{12}(1-k)N_2a_2-a_{22}a_{12}(1-k)N_2^2 \tag{1.5}$$

This on rearranging terms can brought to the form

$$\left\{\begin{array}{l} \left(-a_{21}(1-k)\alpha_{11}(N_1-\frac{\alpha_1}{2\alpha_{11}})^2+\alpha_{12}(1-k)a_{22}(N_2-\frac{\alpha_2}{2\alpha_{22}})^2\right) \\ \\ +\left(\alpha_{12}(1-k)(h_2-\frac{\alpha_2^2}{4a_{22}})-a_{21}(1-k)(h_1-\frac{\alpha_1^2}{4a_{11}})\right) \end{array}\right\}=0 \tag{1.6}$$

This equation connects the harvesting rates and the normal steady state. From this equation two cases can be drawn.

(i) Case of exclusive harvesting i.e. the harvesting rates of S_1 and S_2 are independent of each other

i.e. $h_1=\frac{a_1^2}{4a_{11}}$ and $h_2=\frac{a_2^2}{4a_{22}}$ (1.7)

(ii) Case of mixed or gross harvesting characterized by

$$a_{12}(1-k)(h_2-\frac{a_2^2}{4a_{22}})-a_{21}(1-k)(h_1-\frac{a_1^2}{4a_{11}})=0 \tag{1.8}$$

In either of the cases, the equilibrium values of N_1 and N_2 are related by

$$-a_{21}(1-k)a_{11}(N_1-\frac{a_1}{2a_{11}})^2+a_{12}(1-k)a_{22}(N_2-\frac{a_2}{2a_{22}})^2=0 \tag{1.9}$$

The equilibrium point lies on the line

$$\frac{N_1-\dfrac{a_1}{2a_{11}}}{\sqrt{a_{12}a_{22}}}=\frac{N_2-\dfrac{a_2}{2a_{22}}}{\sqrt{a_{21}a_{11}}} \tag{1.10}$$

Which passes through the point $(\overline{N}_1,\overline{N}_2)=\left(\frac{a_1}{2a_{11}},\frac{a_2}{2a_{22}}\right)$ (1.11)

corresponding to half of the carrying capacities of the two species S_1 and S_2

Put $N_1=u_1+\overline{N_1}$ and $N_2=u_2+\overline{N_2}$ (1.12)
where u_1 and u_2 are small perturbations from the equilibrium state.

1.3 Stability of Equilibrium points:

Our basic Equations are:

$$\frac{dN_1}{dt}=N_1f_1(N_1N_2)-h_1=N_1\{a_1-a_{11}N_1-a_{12}(1-k)N_2\}-h_1 \tag{1.13}$$

$$\frac{dN_2}{dt}=N_2f_2(N_1N_2)-h_2=N_2\{a_2-a_{22}N_2-a_{21}(1-k)N_1\}-h_2 \tag{1.14}$$

The linearised basic equations are

$$\frac{du_1}{dt}=-a_{11}(1-k)\left(\overline{N_1}u_1+\overline{N_1}u_2\right) \tag{1.15}$$

$$\frac{du_2}{dt}=-a_{21}(1-k)\overline{N_2}u_1+\overline{N_1}u_2 \tag{1.16}$$

The characteristic equation is

$$\lambda^2+(1-k)(a_{12}\overline{N_2}+a_{21}\overline{N_1})\lambda=0 \tag{1.17}$$

one root of which can be noted to be negative and the other is zero.

∴ The co-existent equilibrium state is neutrally stable.
The trajectories are

$$u_1=u_{10}-\frac{a_{12}(1-k)}{\lambda_1}\left[u_{10}\overline{N}_2+u_{20}\overline{N}_1\right]\left[e^{\lambda_1 t}-1\right] \tag{1.18}$$

$$u_2=u_{20}-\frac{a_{21}(1-k)}{\lambda_1}\left[u_{10}\overline{N}_2+u_{20}\overline{N}_1\right]\left[e^{\lambda_1 t}-1\right] \tag{1.19}$$

where λ_1 is negative characteristic root of (1.17). The curves are illustrated in fig.1.1 &1.2 .

Case 1: Initially S_1 dominates S_2 and both Species attain asymptotic limits u_1^* and u_2^* i.e. $u_{10} < u_{20}$, $a_{12}>a_{21}$. In this case S_2 always out numbers S_1.

It is evident that both the Species converging asymptotic to the equilibrium limits (u_1*, u_2*)
Where

$u_1^*=$ $u_{10}+\frac{a_{12}(1-k)}{\lambda_1}\left[u_{10}\overline{N}_2+u_{20}\overline{N}_1\right]$ &

$u_2^*=$ $u_{20}+\frac{a_{21}(1-k)}{\lambda_1}\left[u_{10}\overline{N}_2+u_{20}\overline{N}_1\right]$ (1.21)

Hence this state is neutrally stable.

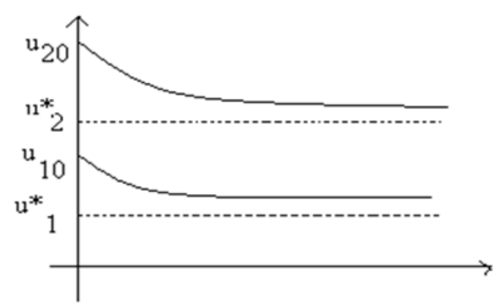

Fig 1.1.

Case 2: The Species S_1 dominates the Species S_2 in natural growth rate but its initial strength is less than that of Species S_2 i.e. $u_{10} > u_{20}$, $a_{12}>a_{21}$:

Initially S_1 dominates over S_2 up to the time instant

$$t^*=\frac{1}{\lambda_1}\ln\left[1+\frac{(u_{10}-u_{20})\lambda_1}{(a_{12}-a_{21})(1-k)}\right] \tag{1.22}$$

and there after Species S_2 out numbers Species S_1 and both the Species converge asymptotically to the equilibrium limits u_1^*,u_2^*. Hence this state is neutrally stable.

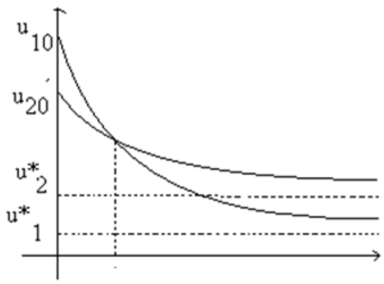

Fig 1.2

Trajectories of perturbed Species:

The trajectories in the u_1-u_2 plane are

$u_1 = \frac{a_{12}}{a_{21}} u_2 + u_{10} - \frac{a_{12}}{a_{21}} u_{20}$

The trajectory denotes a straight line.

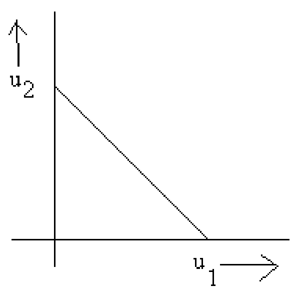

Fig 1.3

1.4 Threshold Theorem

In consonance with the principle of competitive exclusion Gauss {1934} we deduce a threshold theorem on the basic equations (1.13)&(1.14) are written for competitive Species converging to asymptotic stable equilibrium point $(\overline{N}_1, \overline{N}_2) = \left(\frac{a_1}{2a_{11}}, \frac{a_2}{2a_{22}}\right)$

Now the basic equations can be written as

$$\left.\begin{aligned} \frac{dN_1}{dt} &= \frac{a_1 N_1}{k_1}\{K_1 - N_1 - \beta_1 N_2\} - h_1 \\ \frac{dN_2}{dt} &= \frac{a_2 N_2}{k_2}\{K_2 - N_2 - \beta_2 N_1\} - h_2 \end{aligned}\right\} \qquad (1.23)$$

where

$$K_1 = \frac{a_1}{a_{11}}; K_2 = \frac{a_2}{a_{22}}; \beta_1 = \frac{a_{12}(1-k)}{a_{11}} \text{ and } \beta_2 = \frac{a_2(1-k)}{a_{22}}$$

Theorem : Principle of competitive exclusion for co-existent equilibrium state:

$$\left(\overline{N}_1, \overline{N}_2\right) = \left(\frac{a_1}{2a_{11}}, \frac{a_2}{2a_{22}}\right)$$

When $\frac{k_1}{\beta_1} > k_2$ and $\frac{k_2}{\beta_2} > k_1$ then every solution of ($N_1(t), N_2(t)$) of (1.23) approach the equilibrium solution $\left(N_1(t), \overline{N_2(t)}\right) = \left(\overline{N}_1, \overline{N}_2\right) \neq (0,0)$ as t approaches infinity. In other words, if Species 1 and 2 are nearly identical and the microcosm can support both the members of Species 1 and 2 depending up on the initial conditions.

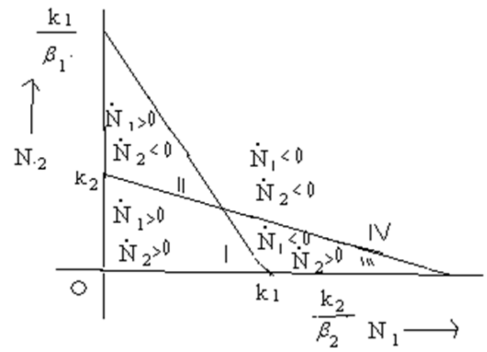

Fig 1.4

Proof: The first step in our proof is to show that $N_1(t)$ and $N_2(t)$ can never become negative. To this end, observe that $N_1(t) = \overline{N_1} = \frac{a_1}{2a_{11}}$ and $N_2(t) = \overline{N_2} = \frac{a_2}{2a_{22}}$ is a solution of (1.23) for any choice of $N_1(t)$.

The orbit of this solution in the $N_1 - N_2$ plane is –

- the point (0, 0) for $N_1(t) = 0$;
- the line $0 < N_1 < K_1$, $N_2=0$ for $0 < N_1(0) < K_1$;
- the point (k_1 , 0) for $N_1(t) = k_1$; and
- the line $K_1 < N_1 < \infty$, $N_2 = 0$ for $N_1(0) < k_1$.

Thus the N_1 axis, for N1 ≥ 0 is the union for four distinct orbits of (1.23).Similarly the N_2 axis , for $N_2 \geq 0$, is the union of four distinct orbits of. This implies that all solutions ($N_1(t), N_2(t)$) of (1.23) which start in the first quadrant $\left(N_1(t) > 0, N_2(0)\right)$ of the $N_1 - N_2$ plane must remain there for all future time.

The second step in our proof is to split the first quadrant into regions in which both $\frac{dN_1}{dt}$ and $\frac{dN_2}{dt}$ have fixed signs. This is accomplished in the following manner.

Let l_1 and l_2 be the lines

$k_1 - N_1 - \beta_1 N_2 = 0$, $k_2 - N_2 - \beta_2 N_1 = 0_2$

respectively and the point of their intersection ,is $(\overline{N_1}, \overline{N_2})$ .observe that $\frac{dN_1}{dt}$ is negative if (N_1, N_2) lies above the line l_1 and positive if (N_1, N_2) lies below l_1. Similarly , $\frac{dN_2}{dt}$ is negative if (N_1, N_2) lies below l_2. Thus the two lines l_1 and l_2 split the first quadrant of the $(N_1 - N_2)$ plane into four regions in which both

$\frac{dN_1}{dt}$ and $\frac{dN_2}{dt}$ have fixed signs.

- $N_1(t)$, $N_1(t)$ both increases with the time (along any solution of (1.23) in region I
- $N_1(t)$ increases and $N_2(t)$ decrease with time in region II
- $N_1(t)$ decreases and $N_2(t)$ increases with time region III and
- Both $N_1(t)$ and $N_2(t)$ decrease with time in region IV

In this region both the Species S_1 and Species S_2 compete with each other but do not flourish and at the same time do not get extinct. Finally we require the following four lemmas.

Lemma 1: Any solution of (N_2 (t), N_2(t)) of (1.23) which starts in region I at time t=t_0 will remain in this region for all future time t > t_0 and ultimately approach the equilibrium solution N_1(t)= $\overline{N_1}$, N_2 (t) = $\overline{N_2}$

Proof : Suppose that a solution (N_1(t), N_2(t)) of (1.23) leaves region I at time t=t*. Then either $\frac{dN_1}{dt}(t^*)$ or $\frac{dN_2}{dt}(t^*)$ is zero, since the only way a solution of (1.23) can leave region I is by crossing l_1 or l_2 . Assume that $\frac{dN_1}{dt}(t^*)$ =0.

Differentiating both sides of the first equation of (1.23) with respect to t and setting t= t*

gives
$$\frac{d^2N_1(t^*)}{dt^2}=\frac{-a_1\beta_1N_1(t^*)}{k_1}\frac{d\ \ N_1(t^*)}{dt}<0 \qquad (1.24)$$

Hence N_1 (t) is monotonically increasing and it has maximum whenever a solution of N_1 (t), N_2 (t) of (1.23) is in region I.

Similarly, if $\frac{dN_2}{dt}(t^*)$ =0 then
$$\frac{d^2N_2(t^*)}{dt^2}=\frac{-a_2\beta_2N_2(t^*)}{k_1}\frac{dN_1}{dt}(t^*)<0 \qquad (1.25)$$
implies that N_2 *(t)* is monotonic increasing and it has maximum whenever a solution (*N_1(t), N_2(t)*) of (1.23) is in region I . If a solution (*N_1 (t), N_2 (t)*) of 1.23) remains in region I for $t \geq t_0$, then both *N_1(t)* and *N_2(t)* are monotonic increasing function of time for $t \geq t_0$ with *N_1(t)* < k_1 and *N_2(t)* < k_2 , consequently, both *N_1(t)* and *N_2(t)* have limits ε, n respectively, as *t* approach infinity. This, in turn implies that (ε, n) is an equilibrium point of (1.23). Now, (ε, n) obviously cannot equal (0, 0); (*k_1,0)* or (0,k $_2$). Consequently (ε, n) = ($\overline{N_1}$,$\overline{N_2}$).

Lemma 2: Any solution of (*N_1 (t), N_2 (t)*) of (1.23) which starts in region II at time *t* = *t_0* will remain in this region for all future time *t* ≥ *t_0* and ultimately approach the equilibrium solution *N_1 (t)* = $\overline{N_1}$, *N_2 (t)* = $\overline{N_2}$

Proof: Suppose that a solution (*N_1 (t), N_2 (t)*) of (1.23) leaves region *II* at time *t* = *t**.

Then either $\frac{dN_1}{dt}(t^*)$ or $\frac{dN_2}{dt}(t^*)$ is zero, since the only way a solution of (1.23) can leave region II is by crossing *l_1* or *l_2*. Assume that $\frac{dN_1(t^*)}{dt}$ = 0.

Differentiating both sides of the first equation of (1.23) with respect to *t* and setting
t = *t** gives
$$\frac{d^2N_1(t^*)}{dt^2}=\frac{-a_2\beta_2N_2(t^*)}{k_1}\frac{dN_2(t^*)}{dt}>0 \qquad (1.26)$$

This quantity is positive. Hence *N_1 (t)* has minimum at *t* = *t**. However, this is impossible, since *N_1 (t)* is increasing whenever a solution of *N_1 (t), N_2 (t)* of (1.23) is in region II.

Similarly, if $\frac{dN_2(t^*)}{dt}=0$, then
$$\frac{d^2N_2(t^*)}{dt^2}=\frac{-a_2\beta_2N_2(t^*)}{k_2}\frac{dN_1}{dt}(t^*)<0 \qquad (1.27)$$

This quantity is negative, implying that *N_2 (t)* has maximum at *t*=*t* *, but this is impossible, since *N_2 (t)* is decreasing whenever a solution (*N_1 (t), N_2 (t)) of* (1.23) is in region II. The previous argument shows that any solution *N_1 (t), N_2 (t)* of (1.23) which starts in region *II* at time *t* = *t_0* while remain in region *II* for all future time *t* ≥ t_0. This implies that *N_1 (t)* is monotonic increasing and *N_2 (t)* is monotonic decreasing for *t* ≥ t_0 with *N_1 (t)* < *K_1* and *N_2 (t)* < *K_2*. Consequently, both *N_1 (t)* and *N_2 (t)* have limits $\in$, *n* respectively, as $t \to \infty$. This in turn, implies that ($\in$, *n*) is an equilibrium point of (1.23). Now ($\in$, *n*) obviously cannot be equal to (0, 0); (*K1, 0*) or (0, *K_2).* Consequently, ($\in$, *n*) = ($\overline{N}_1$, $\overline{N}_2$) and this proves Lemma 2.

Lemma 3: Any solution of (*N_1 (t), N_2 (t)*) of (1.23) which starts in region III at time *t* = *t_0* will remain in this region for all future time *t* ≥ t_0, and ultimately approach the equilibrium solution *N_1 (t)* = $\overline{N}_1$, *N_2 (t)* = $\overline{N}_2$ (Fig1.4)

Proof: Suppose that a solution (*N_1 (t), N_2 (t)*) of (1.23) leaves region *III* at time *t* = *t**. Then either $\frac{dN_1(t^*)}{dt}$ or $\frac{dN_2(t^*)}{dt}$ is zero, since the only way a solution of (1.23) can leave region II is by crossing *l_1* or *l_2*. Assume that $\frac{dN_1(t^*)}{dt}$ = 0.

Differentiating both sides of first equation of (1.23) with respect to *t* and setting *t* = *t** gives
$$\frac{d^2N_1(t^*)}{dt^2}=\frac{-a_1\beta_1N_1(t^*)}{k_1}\frac{dN_2(t^*)}{dt}. \qquad (1.28)$$

This quantity is negative. Hence $N_1(t)$ has a maximum at t = t^*. However, this is impossible, since $N_1(t)$ is decreasing whenever a solution of ($N_1(t), N_2(t)$) of (1.23) is in region III.

Similarly, if $\frac{dN_2(t^*)}{dt}$ =0

then $$\frac{d^2N_1(t^*)}{dt^2} = \frac{-a_2\beta_2 N_2(t^*)}{k_2}\frac{dN_1}{dt}(t^*). \quad (1.29)$$

This quantity is positive, implying that $N_2(t)$ has a minimum at $t = t^*$ but this is impossible, since $N_2(t)$ is increasing whenever a solution $N_1(t), N_2(t)$ of (1.23) is in region III.

The pervious argument shows that any solution $N_1(t), N_2(t)$ of (1.23) which starts in region III at time $t = t^*$ will remain in region III for all future time $t \geq t_0$. This implies that $N_1(t)$ is monotonic increasing and $N_2(t)$ is monotonic decreasing for $t \geq t_0$; with $N_1(t) > k_1$ and $N_2(t) < k_2$. Consequently, both $N_1(t)$ and $N_2(t)$ have limits $\in$,n respectively, as t approaches infinity. This in turn, implies that ($\in$, n) is an equilibrium point of (1.23) .Now ($\in$,n) obviously cannot equal (0,0); (k_1,0) or (0,k_2). Consequently, ($\in$, n) = $(\overline{N}_1, \overline{N}_2)$ and this proves Lemma 3.

Lemma 4: Any solution of ($N_1(t), N_2(t)$) of (1.23) which starts in region IV at time t = t_0 will remain in this region for all future time $t \geq t_0$, and ultimately approach the equilibrium solution $N_1(t) = \overline{N}_1$, $N_2(t) = \overline{N}_2$. (Fig.1.4)

Proof: Suppose that a solution ($N_1(t), N_2(t)$) of (1.23) leave region IV at time $t = t^*$.

Then either $\frac{dN_1(t^*)}{dt}$ or $\frac{dN_2(t^*)}{dt}$ is zero, since the only way a solution of (1.23) can leave region I is by crossing l_1 or l_2. Assume that $\frac{dN_1(t^*)}{dt}$ = 0.

Differentiating both sides of first equation of (1.23) with respect to t and setting t= t^* gives

$$\frac{d^2N_2(t^*)}{dt^2} = \frac{-a_1\beta_1 N_1(t^*)}{k_1}\frac{dN_2(t^*)}{dt}. \quad (1.30)$$

This quantity is positive, hence $N_1(t)$ is monotonic decreasing and it has minimum whenever a solution ($N_1(t), N_2(t)$) of (1.23) is in region IV.

Similarly, if $\frac{dN_2(t^*)}{dt}$ = 0,

then $$\frac{d^2N_1(t^*)}{dt^2} = \frac{-a_2\beta_2 N_2(t^*)}{k_2}\frac{dN_1}{dt}(t^*). \quad (1.31)$$

This quantity is positive, implying that $N_2(t)$ is monotonic decreasing and it has minimum whenever a solution $N_1(t), N_2(t)$ of (1.23) is in region IV.

If a solution ($N_1(t), N_2(t)$) of (1.23) remains in region IV for $t \geq t_0$, then both $N_1(t)$ and $N_2(t)$ are monotonic decreasing functions of time for $t \geq t_0$, with $N_1(t) > k_1$ and $N_2(t) > k_2$, consequently, both $N_1(t)$ and $N_2(t)$ have limits $\in$, n respectively, as $t \to \infty$. This, in turn implies that ($\in$, n) is an equilibrium point of (1.23). Now, ($\in$, n) obviously cannot be equal to (0,0); (k_1,0) or (0,k_2) and consequently ($\in$, n) = ($\overline{N}_1$, $\overline{N}_2$).

Proof of Theorem: Lemmas 1, 2, 3and 4 state that every solution ($N_1(t), N_2(t)$) of (1.23) which starts in region I, II, III, or IV at time $t = t_0$ and remains there for all future time must also approach equilibrium solution $N_1(t) = \overline{N}_1$, $N_2(t) = \overline{N}_2$ as $t \to \infty$. Next, observe that any solution ($N_1(t), N_2(t)$) of (1) which starts on l_1 or l_2 must immediately afterwards enter regions I, II, III, or IV. Finally the solution approaches the equilibrium solution $N_1(t) = \overline{N}_1$, $N_2(t) = \overline{N}_2$ .This is illustrated in Fig.1.5 Fig.1. 5

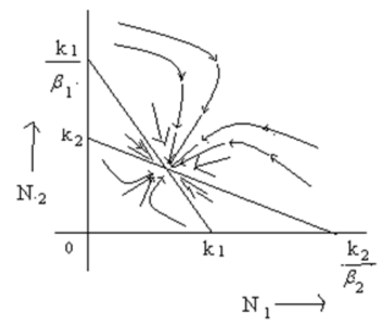

Fig 1.5

CONCLUSIONS

Washed-out equilibrium states are quite common in competition models.Co-existent state ,if any,is of practical utility.In this paper ,a particular co-existent equilibrium point is obtained at half the carrying capacities of the two species in competition.This equilibrium point is found to be neutrally stable.This is a remarkable observation.In general, the locus of equilibrium point is also obtained.The phase portrait analysis explained through the above threshold theorem clearly establishes the global stability of the co-existent equilirium point of the under lying model . The Principle of Competitive Exclusion plays the key role in this study.

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