Journal of Experimental Sciences 2012, 3(1): 27-30
ISSN: 2218-1768
Available Online: http://jexpsciences.com/

journal of experimental sciences

Quasi umbilical submanifold of KH-structure manifold

Shankar Lal*

*Department of Mathematics, HNB Garhwal University, SRT Campus, Badshahithaul Tehri Garhwal-249199 Uttarakhand, India

Abstract
The purpose of the present paper is to study the quasi umbilical submanifold of co-dimension-2 of KH-structure manifold. Hypersurfaces immersed in an almost hyperbolic Hermite manifolds studied by Dube [4]. On GH-structure manifold have been studied by Mishra, R.S. & Singh, S.D. [8]. We have obtained the conditions for this submanifold to be W-Quasi umbilical. The paper is organized as follows quasi umbilical. In section one, introductory part of GF-structure and KH-structure is defined some useful equations. In section two, we prove that the some theorems, definitions and corollaries in quasi umbilical submanifold as well as Nijenhuis tensor. In the end, we have discussed about manifold and quasi umbilical submanifold.

Keywords: GF-structure, KH-structure, Riemannian connexion, Riemannian metric, Nijenhuis tensor, W- quasi umbilical submanifold.

INTRODUCTION

Let us consider an n-dimensional differentiable manifold M^n of class C^∞ in which there exists a vector valued linear function F of differentiability class C^∞ satisfying

(1.1) $F^2 X = a^2 X$,

for an arbitrary vector field X, where a is any complex number, not equal to zero. Then F gives to M^n a GF- structure and the manifold is called a GF- structure manifold.
Let this structure be endowed with Riemannian metric G such that

(1.2) $G(FX, FY) = -a^2 G(X, Y)$,

then M^n is called a H-structure manifold.
If in M^n

(1.3) $(D_X F)Y = 0$,

where D be the Riemannian connexion in the GF- structure manifold, then M^n is called a KH- structure manifold [2]. Let M^{n-2} be a differentiable submanifold of co-dimension 2 of a KH- structure manifold such that

(1.4) $FBX = BfX + u \otimes P + v \otimes Q,$
(1.5) $FP = BU - \lambda Q,$
(1.6) $FQ = BV - \lambda P$,

Where P,Q are two unit normal vector fields to M^n, f is a tensor field of type (1,1) U,V are vector fields,u,v 1- forms and λ is a C^∞ function.
Operating equations (1.4),(1.5), and (1.6) by F and using equations (1.1),(1.2),(1.4),(1.5),(1.6) and taking the tangential and normal parts separately, we get

(1.7)a $a^2 X = f^2 X + u(X)U + v(X)V$
(1.7)b $u(fX) = (a^2 - \lambda^2)V(X),\ v(fX) = (a^2 - \lambda^2)U(X)$
(1.7)c $fU = (a^2 - \lambda^2)V,\ fV = (a^2 - \lambda^2)U$
(1.7)d $u(U) = -(a^2 + \lambda^2),\ u(V) = 0$
(1.7)e $v(V) = -(a^2 + \lambda^2),\ v(U) = 0$.

Let g be the induced Riemannian metric in M^n defined by

(1.8)a $g(X, Y) = -a^2 G(X, Y)$

Also, we have

(1.8)b $G(BX, P) = (BfX, BV) - G(BfX, \lambda Q) + G(uXf, BU)$
$- G(uXF, \lambda Q) + G(VXF, BV) - G(VXQ, \lambda Q),$
(1.8)c $G(P, P) = -(a^2 + \lambda) = G(Q, Q).$

Then using equations (1.4) and (1.8) in equation (1.2), we get

(1.9) $g(fX, fY) = a^2 g(X, Y) - u(X)u(Y) - v(X)v(Y).$
(1.10) $g(U, X) =) = -a^2 u(X), \qquad g(V, X) = -a^2 v(X).$

Thus the submanifold M^{n-2} is a generalized $\{f, g, u, v, \lambda\}$ structure manifold of a KH – structure manifold. Let D be an affine connection in M^n induced by the Riemannian connection of KH-structure manifold M^n then Gauss and Weingarten equations are given by

Received: Jan 2, 2012; Revised: Jan 12, 2012; Accepted: Jan 28, 2012.

*Corresponding Author

Shankar Lal*
Department of Mathematics, HNB Garhwal University, SRT Campus, Badshahithaul Tehri Garhwal-249199 Uttarakhand, India

Tel: 91-9411316082
Email: shankar_alm@yahoo.com

(1.11) $\overline{D}_{BX}BY = BD_X Y + H(X,Y)P + K(X,Y)Q,$
(1.12) $\overline{D}_{BX}P = BHX + \ell(X)Q,$
(1.13) $\overline{D}_{BX}Q = BKX + \ell(X)P$,

where *h* & *k* are second fundamental forms and ℓ is the third fundamental form defined by

(1.14) $g(HX,Y) \stackrel{def}{=\!=} h(X,Y),$

(1.15) $g(KX,Y) \stackrel{def}{=\!=} k(X,Y).$

H and *K* being the tensors of type (1,1).

Differentiating equations (1.4), (1.5), (1.6) covariant and using equations (1.1), (1.6), (1.11), (1.12) and (1.13), we get

(1.16) $(D_X f)Y = -a^4 h(X,Y)U - a^4 k(X,Y)V + a^2 u(Y)H(X) + a^2 v(Y)K(X),$
(1.17) $(D_X u)Y = a^4 h(X,fY)U + a^4 \lambda K(X,Y) + a^2 V(Y)\ell(X),$
(1.18) $(D_X v)Y = \lambda a^4 h(X,Y)U + a^4 K(X,fY) + a^2 v(Y)\ell(X),$
(1.19) $(D_X U)Y = -a^4 fHX - \lambda a^4 KX + a^2 \ell(X)V,$
(1.20) $(D_X V) = -a^4 fKX - \lambda a^4 HX + a^2 \ell(X)V,$
(1.21) $h(X,U) = a^2 u(H,X),$
(1.22) $K(X,V) = a^2 v(K,X).$

QUASI UMBILICAL SUBMANIFOLD OF CO-DIMENSION-2
Definition 2.1

Let in the submanifold

(2.1)a $h(X,Y) = \alpha g(X,Y) + \beta W(X).W(Y), \quad h(X) = \alpha(X) + \beta W(X)W,$
and
(2.1)b $k(X,Y) = \alpha' g(X,Y) + \beta' W(X)W(Y), k(X) = \alpha'(X) + \beta' W(X)W.$

be satisfied, where *α,β* and *α',β'* are scalar function and *W* is 1-form. Then M^{n-2} is called Quasi Umbilical submanifold of co-dimension-2.

Definition 2.2.

If in addition

(2.2) $W(X) = g(W,X)$,
where *W* is the vector field and g is Riemannian metric then M^{n-2} is called a *W*- Quasi submanifold of class C^∞ [1].

Definition 2.3.

Further if $\alpha = 0, \alpha' = 0, \beta \neq 0, \beta' \neq 0$ and $\ell = 0$, then the *W*- Quasi-Umbilical submanifold is called a cylindrical submanifold [1].

Theorem 2.1.

If the submanifold of co dimensions -2 of a KH- structure manifold M^n is quasi umbilical then,

(2.3)a $(D_X f)Y = -a^4 \alpha g(X,Y)U - a^4 \beta W(X)W(Y)U - a^4 \alpha' g(X,Y)V$
$- a^4 \beta' W(X)W(Y)V + a^2 u(Y)\{\alpha(X) + \beta W(X)W\}$

$+ a^2 v(Y)\{\alpha'(X) + \beta' W(X)W\},$
(2.3)b $(D_X u)Y = a^4 \alpha g(X,fY)U + a^4 \beta W(X)W(fY) + a^4 \lambda \alpha' g(X,Y)$
$+ a^4 \lambda \beta' W(X)W(Y) + a^2 V(Y)\ell(X),$
(2.3) $(D_X v)Y = \lambda a^2 \alpha g(X,Y) + \lambda a^4 \beta W(X)W(Y) + a^4 \alpha' g(X,fY)$
$+ a^4 \beta' W(X)W(Y) + a^2 V(Y)\ell(X),$
(2.3)d $(D_X U) = -a^4 \alpha(fX) - a^4 \beta W(fX)W - \lambda a^4 \alpha'(X)$
$- \lambda a^4 \beta' W(X)W + a^2 \ell(X)V,$
(2.3)e $(D_X V) = -a^4 \alpha'(fX) - a^4 \beta' W(fX)W - \lambda a^4 \alpha(X) - a^4 \beta W(X)W + a^2 \ell(X)V,$
(2.3)f $h(X,U) = a^2 \alpha u(X) + a^2 \beta W(X)u(W),$
(2.3)g $k(X,V) = a^2 \alpha' v(X) + a^2 \beta' W(X)u(W).$

Proof. In view of equation (2.1) *a, b* and equations (1.15), (1.16), (1.17), (1.18), (1.19), (1.20), (2.1) and (1.22), we get the required results.

Corollary 2.1.

If $\alpha \neq 0, \alpha' \neq 0, \beta = \beta' = 0$ and $\ell = 0$, then 1- forms *u,v* are projectively killing in M^n i.e.,

(2.4)a $(D_X u)(Y) + (D_Y u)(X) = 2a^4 \lambda \alpha' g(X,Y)$
And
(2.4)b $(D_X v)(Y) + (D_Y v)(X) = 2a^4 \lambda \alpha g(X,Y).$

Proof. By using the condition in equations (2.3) b and (2.3) c, we get equations (2.4) a and (2.4) b.

Corollary 2.2.

If $\alpha = \alpha' = 0, \beta \neq 0, \beta' \neq 0$ and $\ell = 0$ be satisfied in the submanifold M^{n-2} of co- dimension –2, then

(2.5)a $(D_X f)Y = -a^4 \beta W(X)W(Y)U - a^4 \beta' W(X)W(Y)V + a^2 \beta u(Y)W(X)W$
$+ a^2 \beta' v(Y)W(X)W,$
(2.5)b $(D_X u)Y = a^4 \beta W(X)W(fY) + a^4 \lambda \beta' W(X)W(Y),$
(2.5)c $(D_X V)Y = \lambda a^4 \beta W(X)W(Y) + a^4 \beta' W(X)W(fY).$

Proof. Using the conditions in equations (2.3) a, b, c, we get equations (2.5) a,b,c.

Corollary 2.3.

If the submanifold M^{n-2}, being W- Quasi submanifold of co-dimension-2 satisfies $\alpha = \alpha' = 0, \beta = \beta' = 0$ and $\ell = 0$. Then the 1-form *u,v* in M^n are killing and the vector field *U,V* are parallel fields in $M2^n$.
(2.5)d $(D_X f)Y = 0$,
(2.5)e $(D_X u)Y = 0$,
(2.5)f $(D_X v)Y = 0$,
(5.2.5)g $(D_X U) = 0$,
(2.5)h $(D_X V) = 0$,

(2.5)i $h(X,U)=0$,
(2.5)j $k(X,V)=0$.

PROOF. By using $\alpha=\alpha'=0,\beta=\beta'=0$ and $\ell=0$ in equations (2.3) a, b, c, d, e, f, g, we get (2.5) d, e, f, g, h, i, j.

Theorem 2.2.

In a generalized cylindrical submanifold M^{n-2} of co-dimensional-2, we have

(2.6)(a) $(D_X u)Y-(D_Y u)X=a^4\beta[W(fX)W(Y)-W(X)W(fX)]$
And
(2.6)(b) $(D_X v)Y-(D_Y v)X=a^4\beta'[W(fX)W(Y)-W(X)W(fY)]$.

Proof. From equation (2.5)b, we have

$$(D_X u)Y=a^4\beta W(X)W(fY)+a^4\lambda\beta' W(X)W(Y),$$
$$(D_X u)X=a^4\beta W(Y)W(fX)+a^4\lambda\beta' W(Y)W(X).$$

Now subtracting the above two equations, we get equation (2.6)a. The proof of equation (2.6)b follows in the same manner.

Theorem 2.3.

The Nijenhuis tensor of *W*- Quasi umbilical submanifold M^{n-2} of co-dimension-2 of a KH-structure manifold with generalized $\{f,g,u,v,\lambda\}$ structure is given by

(2.7) $N(X,Y)=-a^4\alpha g(fX,Y)U-a^4\beta W(fX)W(Y)U-a^4\alpha' g(fX,Y)V$
$-a^4\beta' W(fX)W(Y)V+a^2u(Y)\{\alpha(fX)+\beta W(fX)W\}$

$+a^2v(Y)\{\alpha'(fX)+\beta' W(fX)W\}+a^4\alpha g(fY,X)U$

$+a^4\beta W(fY)W(X)U+a^4\alpha' g(fY,X)V+a^4\beta' W(fY)W(X)V$

$-a^2u(X)\{\alpha(fY)+\beta W(fY)W\}-a^2V(X)\{\alpha'(fY)$

$+\beta' W(fY)W\}-f[-a^2\alpha g(X,Y)U$

$-a^4\beta W(X)W(Y)U-a^4\alpha' g(X,Y)V-a^4\beta' W(X)W(Y)V$

$+a^2V(Y)\{\alpha(X)+\beta(X)W\}+a^2V(Y)\{\alpha'(X)$
$+\beta' W(X)W\}]+f[-a^4\alpha g(X,Y)U-a^4\beta W(Y)W(X)U$

$-a^4\alpha' g(Y,X)V-a^4\beta' W(Y)W(X)V+a^2u(X)\{\alpha(Y)$

$+\beta W(Y)W\}+a^2V(X)\{\alpha'(Y)+\beta' W(Y)W\}]$.

Proof. Let *N* be the Nijenhuis tensor corresponding to the tensor field *f* in M^n, given by

(2.8) $N(X,Y)=(D_{fX}f)(Y)-(D_{fY}f)(X)-f(D_X f)(Y)+f(D_Y f)(X).$

In view of equation (2.3) a and (2.8), we get the required result.

Theorem 2.4.

The Nijenhuis tensor of a generalized cylindrical submanifold M^{n-2} of co-dimension-2 of a KH- structure manifold M^n is given by

(2.9)a $N(X,Y)=a^4\beta[W(fu)W(X)U-W(fX)W(Y)U+W(X)W(Y)(fU)$
$-W(Y)W(X)(fU)]+a^4\beta'[W(fu)W(X)V-W(fX)W(Y)V$

$-W(X)W(Y)V-W(Y)W(X)V]+a^2\beta[W(fX)Wu(Y)$

$-u(X)W(fu)W+u(Y)W(X)W+u(X)W(Y)W]$

$+a^2\beta'[V(Y)W(fX)W-V(X)W(fu)W+V(Y)W(X)W$

$+V(X)W(Y)W]$.

(2.9)b $N(X,Y)=a^2\alpha[-g(fX,Y)U+g(fY,X)U+g(X,Y)(fU)-g(Y,X)(fu)]$

$+a^2\alpha'[-g(fX,Y)V+g(fY,X)V+g(X,Y)(fV)+g(Y,X)(fV)]$

$+a^2[u(Y)\alpha(fX)+v(Y)\alpha'(fX)-u(X)\alpha(fY)-v(X)\alpha'(fY)$

$+\alpha g(X,Y)(fV)]+a^2f[u(Y)\alpha(X)+V(Y)+\alpha'(X)-u(X)\alpha f(Y)$

$-V(X)\alpha'(Y)]$.

(2.9)c $N(X,Y)=0.$

Proof. By putting $\alpha=\alpha'=0,\beta\neq0,\beta'\neq0$ and $\alpha\neq0,\alpha'\neq0,\beta=\beta'=0$ and also $\alpha=\alpha'=\beta=\beta'=0$ in equation (2.8) we get the required results (2.9) a,b.

DISCUSSION

Manifold are important role of dealing the extended of n-dimensional space of modeling heavenly body because it is construct the higher dimensional space and they allow more complicated structures. We can easily calculate all structure and spaces of manifold from quasi umbilical submanifold. A point of a quasi umbilical submanifold in the neighborhood of which all this lines of curvature indeterminate (not fixed).

REFERENCES

[1] Auslander, L., and MacKenzin, R.E.1963. Introduction to differentiable manifolds, McGraw-Hill, New York.

[2] Chen, S.S.1951. Topics in Differential Geometry, Inst. Advanced study lecture notes.

[3] Duggal, K.L.1971. On differentiable structure defined by Algebraic equation-1 Nijenhuis tensor, *Tensor* (N.S.) 22 pp.

238-242.

[4] Dube, K.K. 2004. On quasi umbilical submanifold of co-dimension-2 of an almosthyperbolic manifold, *Journal of Ultra Scientist of Physical Sciences,* Vol. 16(3) M, pp. 267- 270.

[5] Hick, N.I.1965. Notes on Differential Geometry, D. Van Nostrand Company, Inc. Princeton, New York.

[6] John, H. 1976. 3-Manifolds, Princeton University Press, ISBN 0-8218-3695-1.

[7] Mishra, R.S. 1967. On Almost Hermite Spaces-II, Nijenhuis tensor, *Indian J. of Math.,* Vol. 9 No. 1 pp. 161-168.

[8] Mishra, R.S. & Singh, S.D. 1975. On GF-structures, Indian *Journal Pure and Applied Mathematics,* 6(1), pp. 1317-1325.