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η-Duals of Some Double Sequence Spaces

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Abstract
P. Chandra and B.C. Tripathy [13] have generalized the notion of the köthe-toeplitz dual of sequence spaces on introducting the concept of η-dual of order r, for $r \geq 1$ of sequence spaces. B.C. Tripathy and B. Sharma [3] have introduced the notion of η-dual of order r, for $r \geq 1$ of double sequence spaces. Ansari and Gupta [1] have generalized the notion of the köthe-Toeplitz dual of sequence spaces on introducing the concept of η-dual of order r, for $0 < r \leq 1$ of sequence spaces. In this paper, we have defined and determined the η-dual of some double sequence spaces for $0 < r \leq 1$ and have establised their perfectness in relation to η-dual for $0 < r \leq 1$.

Keywords: α-dual, η-dual, perfect space, Double sequence, Bounded variation, Regular convergent, ${}_2l_r$-space.

INTRODUCTION

A sequence space is defined to be a linear space of sequences as its element with respect to the coordinate wise addition and scalar multiplication. It is a scalar sequence space or a vector sequence space according as the sequences consists of scalar (real or complex) or vectors taken from a vector space. A sequence of the form $(a_k)_{k=1}^{\infty}$ will be called a single sequence and a sequence of the form $(a_{mn})_{m,n=1}^{\infty}$ will be called a double sequence or a matrix.

Köthe and Toeplitz [8] introduced the idea of dual sequence space, whose main results concerned with -duals. Later on it was studied by P. Chandra and B.C. Tripathy [13] Cook [5], Kamtham and Gupta [7], Maddox [10], Lascarides [9], Okutoyi [12] and many others. P. Chandra and B.C. Tripathy [13] have generalized the notion of α-duals on introducing the notion of η-duals of order r, for r ≥ 1 of sequence spaces and Ansari & Gupta [1], have generalized the notion of α-duals on introducing the notion of η-duals of order r, for $0 < r \leq 1$ of sequence spaces.

Browmich [04] introduced the notion of double sequence spaces and Hardy [6] introduced the notion of bounded variation double sequences spaces. Later on it was studied by B.C. Tripathy and B. Sarma [3], Basarir and Sonalean [2], Tripathy, Choudhary and Sharma [14], Moricz [11] and many others.

In this paper, the space of all, bounded, convergent in Pringsheim's sense, regularily null, absolutely summable, p-absolutely summable, finite, bounded variation regularily convergence, Null in Pringsheim's sense, eventually alternating and strongly p-cesaro summable double sequence spaces are denoted by ${}^2\omega$, ${}^2l^{\infty}$, ${}^2c$, ${}^2c^R$ ${}^2c^0$, ${}^2c^{0^R}$, ${}^2l^1$, ${}^2l^p$, ${}^2\phi$, ${}^2bv$, ${}^2\sigma$, ${}^2\omega^p$ respectively and a double sequence is denoted by (a^{mn}) or (x^{mn}) according as elements of η-dual and element of given spaces respectively.

Throughout the article, the sums without limit means that summation is from m = 1 to ∞and n = 1 to∞.

List of some double sequence spaces, whose -dual will be obtained in this paper are :

1. ${}^2c^0$ $\{< a^{mn} > \in {}^2\omega : a^{mn} \to 0$ as min (m, n) $\to \infty\}$
2. ${}^2c^{0^R} = \{ < a^{mn} > \in {}^2\omega : a^{mn} \to 0$ as max (m, n) $\to \infty\}$
3. a. ${}^2c^R = \{ < a^{mn} > \in {}^2\omega$: (a) $\lim\limits_{n\to\infty} a^{mn} = L^m$, where $L^m \in c$ for each $m \in N\}$

   b. $\lim\limits_{m\to\infty} a^{mn} = J^n$, where $J^n \in c$, for each $n \in N\}$
4. ${}^2c = \{ < a^{mn} > \in {}^2\omega : a^{mn} \to L$ as min (m, n) $\to \infty$ for some $L \in c\}$
5. ${}^2l^{\infty} = \{ < a^{mn} > \in {}^2\omega : \sup\limits_{m,n} | a^{mn} | < \infty\}$
6. ${}^2l^r = \{ < a^{mn} > \in {}^2\omega : \sum\limits_m \sum\limits_n | a^{mn} |^r < \infty$, where r is a real no. such that $0 < r < \infty\}$
7. ${}^2bv = \{ < a^{mn} > \in {}^2\omega : \Sigma |\Delta^m a^{m,n} | < \infty, \Sigma | \Delta^n a^{m,n} | < \infty$ and $\Sigma\Sigma | \Delta^{m,n} a^{m,n} | < \infty\}$

where $\Delta^m a^{m,n} = a^{m,n} - a^{m+1,n}$, $\Delta^n a^{m,n} = a^{m,n} - a^{m,n+1}$ and $\Delta^{m,n} a^{m,n} = \Delta^n a^{m,n} - \Delta^n a^{m+1,n}$8. ${}^2bv^0 = {}^2bv \cap {}^2c^0$.

8. ${}^2\omega^p = \{< a^{ij} > \in {}^2\omega : \lim\limits_{\substack{m\to\infty \\ n\to\infty}} \frac{1}{mn} \sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n} | a_{ij} - L |^p = 0 \; \frac{1}{mn}$ for some $L \in c$, and p is a real no.$\}$
9. The space $({}^2\sigma)$ of all eventually alternating double sequences is defined by

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${_2\sigma} = \{\langle a^{mn} \rangle \in {_2\omega} : a^{mn} = -a^{m,\, n+1}$ for all $n \ge n^0$ and $a^{mn} = -a^{m+1,n}$ for all $m \ge m^0\}$

The spaces ${_2c}^R$, ${_2co}^R$, ${_2c}^R$, ${_2c} \cap {_2l^\infty}$, ${_2c^0} \cap {_2l^\infty}$ and ${_2l^\infty}$ are normed linear spaces with the norm given by

$$\| \langle a^{nk} \rangle \| = \sup_{m,n} | a^{mn} |$$

from the above definition, It is clear that

$${_2c}^R \subset {_2c} \cap {_2l^\infty} \subset {_2l^\infty}\, ({_2c})$$

and $${_2c^0}^R \subset {_2c}^0 \cap {_2l^\infty} \subset {_2l^\infty}\, ({_2c}^0)$$

Definition 1. The α-dual of a subset E of ${_2\omega}$ is defined as $E^\alpha = \{(a^{mn}) \in {_2\omega} : \sum_m \sum_n | a_{mn} x_{mn} | < \infty$ for all $(x^{mn}) \in E\}$

Definition 2. Let E be a non-empty subset of ${_2\omega}$ and $r \ge 1$. Then η-dual of order r of E is denoted by E^η and defined by B.C. Tripathy and B. Sarma [3] as $E^\eta = \{\langle a^{nk} \rangle \in {_2\omega} : \sum \sum | a_{mn} x_{mn} |^r < \infty$ for all $\langle x^{mn} \rangle \in E\}$

Taking r = 1 in above definition, we also get the α-dual of E.

A non-empty subset E of ω is said to be perfect or η-reflexive with respect to η-dual if $E^{\eta\eta} = (E^\eta)^\eta = E$.

Definition 3. Let E be a non-empty subset of ${_2\omega}$ and $0 < r \le 1$; then we [1] define the η-dual of order r of E as

$E^\eta = \{(a^{mn}) \in {_2\omega} : \sum_m \sum_n | a_{mn} x_{mn} |^r < \infty$, for all $(x^{mn}) \in E\}$

Taking r = 1 in above definition, we also get the α-dual of E. Also, a non-empty susbset E of ω is said to be perfect or η-reflexive with respect to η-dual if $E^{\eta\eta} = E$. In this paper, the sum without limit means that the summation is from m = 1 to ∞ and n = 1 to ∞.

MAIN RESULTS

The proof of the following Lemma is obvious in view of the definition of η-dual of double sequences.

Lemma (1) :

(i) E^η is a linear subspace of ${_2\omega}$ for every $E \subset {_2\omega}$.

(ii) $E \subset F$ implies $E^\eta \supset F^\eta$

(iii) $E \subset E^{\eta\eta}$ for every $E \subset {_2\omega}$.

Theorem 1. $({_2l^r})^\eta = {_2l^\infty}$ and $({_2l^\infty})^\eta = {_2l^r}$. The spaces ${_2l^r}$ and ${_2l^\infty}$ are perfect spaces. where $0 < r \le 1$.

Proof : First, we shall show that $({_2l^r})^\eta = {_2l^\infty}$.

where, $({_2l^r})^\eta = \{ (a^{mn}) \in {_2\omega} : \sum_m \sum_n | a_{mn} x_{mn} |^r < \infty$ for all $\langle x^{mn} \rangle \in ({_2l^r})\}$

Let $\langle a^{mn} \rangle \in {_2l^\infty}$ and $(x^{mn}) \in {_2l^r}$.

$$\Rightarrow \sup_{m,n} | a^{mn} | < \infty \text{ and } \sum_m \sum_n | x_{mn} |^r < \infty$$

$$\Rightarrow \sup_{m,n} | a^{mn} |^r < \infty \text{ and } \sum_m \sum_n | x_{mn} |^r < \infty$$

$$\because \sum_m \sum_n | a_{mn} x_{mn} |^r = \sum_m \sum_n | a_{mn} |^r . | x_{mn} |^r$$

$$\le \left(\sup_{m,n} | a_{mn} |^r \right) \left(\sum_m \sum_n | x_{mn} |^r \right) < \infty$$

$$\Rightarrow \sum \sum | a_{mn} x_{mn} |^r \text{ converges for every } (x^{mn}) \in {_2l^r}.$$

which shows that $(a^{mn}) \in ({_2l^r})^\eta$, Therefore, ${_2l^\infty} \subset ({_2l^r})^\eta$

For the converse, Let $\langle a^{mn} \rangle \notin {_2l^\infty}$

Then there exists a single sequence $\langle a^{i,\, ni} \rangle$ such that $a^{i,\, ni} \ge i^s$ for some fixed real number $s > \frac{1}{r}$. where i is a positive integer. Consider a double sequence (x^{mn}) which is defined as

$$x^{mn} = \begin{cases} \frac{1}{i^s} \text{ if } m = i, n = n_i, i \in N \\ 0, \text{otherwise} \end{cases}$$

$$\text{Then } \sum_m \sum_n | x_{mn} |^r = \sum_{m-i} \sum_{n=n_i} \left| \frac{1}{i^s} \right|^r$$

$$= \sum_{i=1}^{\infty} \frac{1}{i^{rs}} < \infty \text{ since } r\, s > 1.$$

$$\Rightarrow (x^{mn}) \in {_2l^r}$$

$$\text{But, } \sum_m \sum_n | a_{mn} x_{mn} |^r \ge \sum_{i=1}^{\infty} \left| \frac{1}{i^s} . i^s \right|^r = \infty$$

$$\Rightarrow (a^{mn}\, x^{mn}) \notin {_2l^r}$$

Then, $(a^{mn}) \notin ({_2l^r})^\eta$

Hence $({_2l^r})^\eta \subset {_2l^\infty}$

Thus, $({_2l^r})^\eta = {_2l^\infty}$.

Similarly, we can prove that $({_2l^\infty})^\eta = {_2l^r}$

Furthermore,

Since $({_2l^\infty})^{\eta\eta} = \left| ({_2l_\infty})^\eta \right|^\eta = [{_2l_r}]^\eta = {_2l^\infty}$

and $({_2l^r})^{\eta\eta} = \left| ({_2l_r})^\eta \right|^\eta = [{_2l_\infty}]^\eta = {_2l^r}$

Therefore, the spaces ${_2l^\infty}$ and ${_2lr}$ are perfect.

Theorem 2. $({_2c}^R)^\eta = ({_2co}^R) = {_2l^r}$. The spaces ${_2c}^R$ and ${_2co}^R$ are not perfect, where $0 < r \le 1$.

Proof : Since ${_2c^0}^R \subset {_2c}^R \subset {_2l}^R{}_\infty \subset {_2l^\infty}$

By Lemma 1 (ii) and thorem (1),

$2l^r = (2l^\infty)^\eta \subset (2l^R_\infty)^\eta \subset (2c^R)^\eta \subset (2c_0^R)^\eta$

Hence, In order to prove the theorem, It is sufficient to show that $(2c_0^R)^\eta \subset 2l^r$.

Let $< a^{mn} > \notin 2l^r$.

Then we can find sequences $< m^i >$ and $< n^i >$ of natural numbers with $m^0 = 1$, $n^0 = 1$ such that

$$\sum_{m=1}^{m_i}\sum_{n=1}^{n_i}|a_{mn}|^r - \sum_{m=1}^{m_{i-1}}\sum_{n=1}^{n_{i-1}}|a_{mn}|^r > \frac{1}{(i+1)^{r/2}}, \ i = 0, 1, 2, 3, ...$$

Define a sequence (x^{mn}) by

$x^{mn} = (i+1)^{-\frac{1}{3}}$ for $m^{i-1} < m \geq m^i$ and $n^{i-1} < n \leq n^i$, for all $i \in N$.

Then $(x^{mn}) \in 2c_0^R$

Now,

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}|a_{mn}x_{mn}|^r = \sum_{i-0}^{\infty}\left(\sum_{m=1}^{m_i}\sum_{n=1}^{m_i}|a_{mn}x_{mn}|^r - \sum_{m=1}^{m_{i-1}}\sum_{n=1}^{n_{i-1}}|a_{mn}x_{mn}|^r\right)$$

$$= \sum_{i=0}^{\infty}\frac{1}{(1+i)^{\frac{r}{3}}}\left(\sum_{m=1}^{m_i}\sum_{n=1}^{n_i}|a_{mn}|^r - \sum_{m=1}^{m_{i-1}}\sum_{n=1}^{n_{i-1}}|a_{mn}|^r\right)$$

$$> \sum_{i=0}^{\infty}\frac{1}{(1+i)^{\frac{r}{3}}(1+i)^{\frac{r}{2}}} = \sum_{i=0}^{\infty}\frac{1}{(1+i)^{\frac{5r}{6}}} = \infty \text{ (because } 0 < \frac{5r}{6} < 1\}$$

Thus $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}|a_{mn}x_{mn}|^r = \infty$

Then $(a^{mn}) \notin (_2c_0^R)^\eta$

Hence we have $(_2c_0^R)^\eta \subset 2l^r$

Thus, It is proved that

$$(_2c^r)^\eta = (_2c_0^R)^\eta = 2l^r$$

Further more,

since $(_2c^R)^{\eta\eta} = \left[(_2c^R)^\eta\right]^\eta = \left[_2l_r{}^\eta = {}_2l_\infty \neq {}_2c^R\right]$

and $(_2c^R)^{\eta\eta} = \left[(_2c^R)^\eta\right]^\eta = \left[_2l_r{}^\eta = {}_2l_\infty \neq {}_2c^R\right]$

Therefore, the spaces $2c^R$ and $_2c_0^R$ are not perfect.

Theorem 3. $(2bv)^\eta = (2bv^0)^\eta = 2l^r$. The spaces $2bv$ and $2bv^0$ are not perfect. where $0 < r \leq 1$

Proof : Since, $2bv^0 \subset 2bv \subset 2l^\infty$

By Lemma 1 (ii) and theorem 1,

$2l^r = (2l^\infty)^\eta \subset (2bv)^\eta \subset (2bv0)^\eta$

Then, In order to prove the theorem, It is sufficient to show that $(2bvo)^\eta \subset 2l^r$

Let $(a^{mn}) \notin 2l^r$

Then we can find a sequence (n^k) of natural numbers such that $n^1 = 1$ such that

$$\sum_{m=1}^{\infty}\sum_{n=n_k}^{n_{k+1}-1}|a_{mn}|^r > k^r \text{ for all } k = 1, 2, 3, 4, ...$$

Define a sequence (x^{mn}) by

$x^{mn} = k^{-1}$ if $n^k \leq n \leq n^{k+1}$ for all $k = 1, 2, 3, ...$

Then

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}|\Delta x_{mn}| = \sum_{m=1}^{\infty}\sum_{k=1}^{\infty}\left(\sum_{n=n_k}^{n_{k+1}-1}|\Delta x_{mn}|\right)$$

$$= \sum_{m=1}^{\infty}\sum_{k=1}^{\infty}\left(\sum_{n=n_k}^{n_{k+1}-1}|x_{mn} - x_{m,n+1} - x_{m+1,n} + x_{m+1,n+1}|\right)$$

$$= \sum_{m=1}^{\infty}\sum_{k=1}^{\infty}\left(\sum_{n=n_k}^{n_{k+1}-1}\left|\frac{1}{k} - \frac{1}{k+1} - \frac{1}{k} + \frac{1}{k+1}\right|\right) = 0$$

Hence $(x^{mn}) \in 2bv^0$.

Now,

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}|a_{mn}x_{mn}|^r = \sum_{m=1}^{\infty}\sum_{k=1}^{\infty}\left(\sum_{n=n_k}^{n_{k+1}-1}|a_{mn}x_{mn}|^r\right)$$

$$= \sum_{m=1}^{\infty}\sum_{k=1}^{\infty}\left(\sum_{n=n_k}^{n_{k+1}-1}|a_{mn}|^r . |x_{mn}|^r\right)$$

$$= \sum_{m=1}^{\infty}\sum_{k=1}^{\infty}\left(\sum_{n=n_k}^{n_{k+1}-1}|a_{mn}|^r . \frac{1}{k^r}\right)$$

$$= \sum_{k=1}^{\infty}\frac{1}{k^r}\left(\sum_{m=1}^{\infty}\sum_{n=n_k}^{n_{k+1}-1}|a_{mn}|^r\right)$$

$$> \sum_{k=1}^{\infty}\frac{1}{k^r}.k^r = \infty$$

$$\Rightarrow \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}|a_{mn}x_{mn}|^r = \infty$$

$\Rightarrow (a^{mn}) \notin (_2bv_0)^\eta$

Hence $(_2bv_0)^\eta \subset 2l^r$

Therefore, we get $(_2bv)^\eta = (_2bv_0)^\eta = 2l^r$

Furthermore,

Since $\left[(_2bv)^{\eta\eta} = (_2bv)^\eta\right]^\eta = (_2lr)^\eta = {}_2l_\infty \neq {}_2bv$

and $(_2bv_0)^{\eta\eta} = \left[(_2bv_0)^\eta\right]^\eta = \left[_2lr\right]^\eta = {}_2l_\infty \neq {}_2bv_0$

Therefore, the spaces $2bv$ and $2bv^0$ are not perfect.

Theorem 4. $(2\sigma)^\eta = 2l^r$. The space 2σ is not pefect.

Proof : Since, $2\sigma \subset 2l^\infty$

By Lemma 1 (ii) and theorem 1,

$2l^r = (2l^\infty)^\eta \subset (2\sigma)^\eta$.

conversely let $(a^{mn}) \in (2\sigma)^\eta$

Then $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}| a_{mn}x_{mn} |^{r} < \infty$ for all $(a^{mn}) \in {}^{2}\sigma$.

Define a sequence (x^{mn}) as

$x^{mn} = 1 = -x^{m+1,n} = -x^{m,n+1}$ for all m, n $\in$ N

Then $(x^{mn}) \in {}^{2}\sigma$.

and hence $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}| a_{mn}x_{mn} |^{r} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}| a_{mn} |^{r} < \infty$

$\Rightarrow (a^{mn}) \in {}^{2}l^{r}$

Therefore, $({}^{2}\sigma)^{\eta} \subset {}^{2}l^{r}$.

Thus, we get $({}^{2}\sigma)^{\eta} = {}^{2}l^{r}$

Furthermore,

$\because$ since, $({}^{2}\sigma)^{\eta\eta} = \left[({}_{2}\sigma)^{\eta}\right]^{\eta} = [{}^{2}l^{r}]^{\eta} = {}^{2}l^{\infty} \neq {}^{2}\sigma$

Then, the space ${}^{2}\sigma$ is not perfect.

Theorem 5. $({}^{2}\omega p \cap {}^{2}l^{\infty})^{\eta} = {}^{2}l^{r}$. The space ${}^{2}\omega p \cap {}^{2}l^{\infty}$ is not perfect where $0 < r \leq 1$.

Proof . Since $({}^{2}\omega p \cap {}^{2}l^{\infty}) \subset {}^{2}l^{\infty}$

By Lemma 1 (ii) and theorem (1),

${}^{2}l^{r} = ({}^{2}l^{\infty})^{\eta} \subset ({}^{2}\omega p \cap {}^{2}l^{\infty})^{\eta}$

conversely, Let $(a^{mn}) \notin {}^{2}l^{r}$

$\Rightarrow \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}| a_{mn} |^{r} = \infty$

Define a sequence (x^{mn}) as

$x^{mn} = 1$, for all m, n $\in$ N

Then, $(x^{mn}) \in {}^{2}\omega p \cap {}^{2}l^{\infty}$

But $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}| a_{mn}x_{mn} |^{r} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}| a_{mn} |^{r} = \infty$

$\Rightarrow (a^{mn}) \notin ({}^{2}\omega p \cap {}^{2}l^{\infty})^{\eta}$

Hence $({}^{2}\omega p \cap {}^{2}l^{\infty})^{\eta} \subset {}^{2}l^{r}$

Thus, we get $({}^{2}\omega p \cap {}^{2}l^{\infty})^{\eta} = {}^{2}l^{r}$

Furthermore,

$({}_{2}\omega p \cap_{2} l_{\infty})^{\eta\eta} = \left[({}_{2}\omega p \cap_{2} l_{\infty})^{\eta}\right]^{\eta} = [{}_{2}lr]^{\eta} = {}^{2}l^{\infty} \neq {}^{2}\omega p \cap {}^{2}l^{\infty}$

$\Rightarrow$The space ${}^{2}\omega p \cap {}^{2}l^{\infty}$ is not perfect.

Remark: From theorem (1), (2), (3), (4), (6). It is obvious that

$({}_{2}l_{\infty})^{\eta} = ({}_{2}c^{R})^{\eta} = ({}_{2}c_{0}^{R})^{\eta} = ({}_{2}bv)^{\eta} = ({}_{2}bv_{0})^{\eta} = ({}_{2}\rho)^{\eta} = ({}_{2}\omega p \cap {}_{2}l_{\infty})^{\eta} = {}_{2}\, lr$

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