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# Fourier descriptors under rotation, scaling, translation and various distortion for hand drawn planar curves

**Vinay Saxena***

*Department of Mathematics, Kisan Postgraduate College, Bahraich 271801 (U.P.), India

**Abstract**

The ordinary Fourier coefficients are difficult to use as input to categorizers because they contain factors dependent upon size and rotation as well as an arbitrary phase angle. From these Fourier coefficients, however, other more useful features are derived. By using these derived property constants, a distinction is made between genuine shape constants and constants representing size, location and orientation. In the present work, we extended the method of Fourier descriptors to produce a set of normalized coefficients, which are invariant under RST (Rotation, Scaling and Translation) for hand drawn planar curves. We have used these shapes for study of the behavior of Fourier descriptors under various distortions. For such planar curves, the optimal curve matching technique is used.

**Keywords:** Fourier descriptors, Hand Drawn Planar Curves, Rotation, Scaling and Translation

## INTRODUCTION

Boundary representation is important for analysis and synthesis of shape. Shape analysis is often required for detection and recognition of objects in a scene. Shape analysis is useful in computer-aided design (CAD) of parts and assemblies, image simulation applications such as video games, cartoon movies, environmental modeling of aircraft-landing testing & training and other computer graphics problems.

Some popular techniques for Boundary Representation are-

I. Fourier descriptors proposed by Cosgriff
II. Moment representation proposed by M.K. Hu
III. Chain encoding proposed by Freeman
IV. Polygonal approximation proposed by Pavlidis

Fourier descriptors first suggested by Cosgriff [1] for extracting a finite set of numerical features from a closed curve, features that will treat to separate the shape of different classes relative to the interclass dispersion. Affine invariant Fourier descriptors were introduced by Arbter et al [2] for binary images and have been generalized to cope with grey level images [3]. Interest in polynomial representation of boundaries is considerable because of its usefulness and simplicity. Typical work being done on Fourier descriptors is illustrated in the papers by Zahn and Roskies [4] and Persoon and Fu [5].

In the next section, we shall discuss Fourier descriptors to describing the boundary for hand drawn planar curves.

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*Corresponding Author

Vinay Saxena
Department of Mathematics, Kisan Postgraduate College, Bahraich 271801 (U.P.), India

Tel: +91-9415178901
Email: dr.vinaysaxena@gmail.com

## FOURIER DESCRIPTOR FOR HAND DRAWN PLANAR CURVE

A computer program using the Visual C++ is developed that picks up randomly the vertices of the polygon representing the shape as the user draws it on the monitor by dragging the mouse. The planar hand drawn shape can be described by an ordered set N vertices $v_i = (x_i, y_i)$; i = 1,2,…..,N where $v_{i+1}$ is a neighbor of $v_i$(modulo N) and $x_i = x(i)$, $y_i = y(i)$. With this notation, the boundary itself can be represented as the sequence of S(i) = (x(i), y(i)). Moreover, each coordinate pair can be treated as a complex number so that S(i) =x(i) + j y(i).

Zahn and Roskies [4] developed a method for the analysis and synthesis of closed curves in plane using the Fourier descriptors. A curve is represented parametrically as a function of arc length by the accumulated change in direction of the curve since the starting point. This function is expanded in a Fourier series and the coefficients are arranged in the amplitude/ phase-angle form. It is shown that the amplitudes are pure form invariants as well as are certain simple functions of phase angles. Zahn and Roskies define Fourier descriptors as-

$$\phi^*(t) = \mu_0 + \sum_{k=1}^{\infty}(a_k \cos kt + b_k \sin kt)$$

In polar form the expansion is-

$$\phi^*(t) = \mu_0 + \sum_{k=1}^{\infty} A_k \cos(kt - \alpha_k)$$

where $(A_k, \alpha_k)$ are polar coordinates of $(a_k, b_k)$. The set $\{A_k, \alpha_k\}$ are polar Fourier descriptors for the curve. Among the advantages are that no redundant information is present in the set $\{A_k, \alpha_k\}$ as will be the case for the Fourier Descriptors defined earlier. Therefore, every sequence $\{A_k, \alpha_k\}$ describes one curve and each curve has only one sequence $\{A_k, \alpha_k\}$. Among the disadvantages we have the following:

Some sequences $\{A_k, \alpha_k\}$ describe not closed curves. $\phi^*(t)$ for polygonal curves contains discontinuities, and therefore, the $A_k$ will decrease rather slowly as *k* increases.

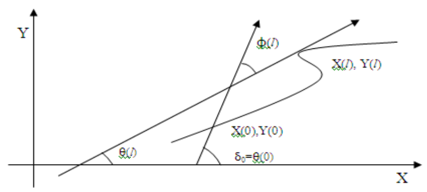


Fig 1. θ (*l*) and ϕ (*l*) for a planar curve

If curve $\gamma$, as shown in Figure1, is described by $\theta(l)$ and starting point Z(0) the position of point, Z(1) can be obtained from the expression

$$Z(1) = Z(0) + \int_0^1 \exp\{i\theta(\lambda)\}\, d\lambda$$

Using the reconstruction theorem and a truncated Fourier series expansion of $\phi^*$, we obtain a formula

$$Z(1) = Z(0) + \frac{L}{2\pi}\int_0^{\frac{2\pi l}{L}} \exp\{i[-t+\delta_0+\mu_0+\sum_{k=1}^{N} A_k \cos(kt-\alpha_k)]\}dt$$

Zahn and Roskies [4] use arbitrary Fourier Descriptor to reconstruct and plot curves.

## METHOD TO CALCULATE FOURIER DESCRIPTORS

### Fourier Descriptors for a Polygonal Curve

For a continuous curve, $U(1) = X(1) + jY(1)$ with period T, the Fourier Descriptors are the Fourier series coefficients-

$$a(k) = \frac{1}{T}\int_0^T u(t)\exp\left(\frac{-j2\pi kt}{T}\right)dt$$

Eric Persoon [5] give a formula for computing the Fourier Descriptors for a polygon, as shown in Figure 2, whose vertices are represented by $V_k$,

where k = 0,1, 2, ….., m-1.

$$a_k = \frac{T}{(2\pi k)^2}\sum_{i=1}^{m}(b_{i-1}-b_i)\exp\left(\frac{-j2\pi k t_i}{T}\right)$$

where,

$$b_i = \frac{V_{i+1}-V_i}{|V_{i+1}-V_i|}, t_k = \sum_{i=1}^{k}|V_i - V_{i-1}|, \text{ for } k>0 \text{ and } t_0 = 0$$

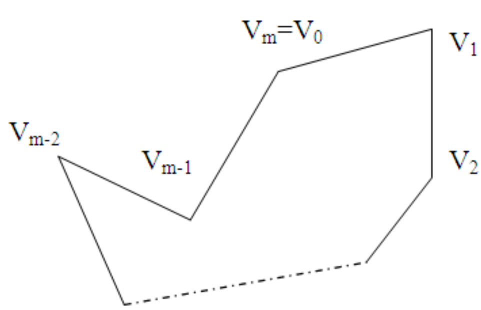


Fig 2. A closed polygon with vertices $V_k$

### Fourier Descriptors for line pattern

When a curve is not closed and non-overlapping: Typical hand drawn line patterns are given in Figure 3.

Fig 3.Examples of Hand Drawn Line patterns

In order to be able to use the Fourier descriptors, we shall trace the line pattern once and then retrace it so that a closed boundary curve $\gamma$ is obtained, as shown in Figure 4.

Fig 4. Tracing of a line pattern

To obtain skeletons of patterns (as given in Figure 5) Eric Persoon [5] defines an algorithm for finding skeleton using Fourier descriptors and proposed a distance measure that measures the difference between two boundary curves.

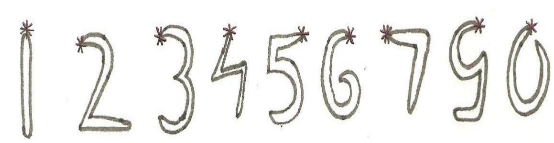

Fig 5. Patterns with thickness

The numerals in Figure 5 have a certain thickness. Since the starting point on $\gamma$ does not carry any information for classifying those numerals, it is useful to normalize the starting point, for example, at the end of point of the numerals (see the * in the Figure 5). This normalization is needed when we scan the field of the numeral from left to right and top to bottom and take the first black point as the starting point of the boundary $\gamma$, as shown in Figure 6.

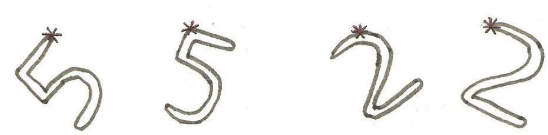

Fig 6. Normalized starting point

Sometimes it becomes more difficult to normalize with respect to starting point. This is, for example, the case with outer boundaries given in Figure 7.

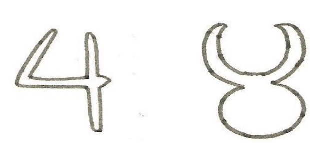

Fig 7. Some outer boundaries

## CONCLUSION

We have studied the behavior of normalized Fourier descriptors under Rotation, Scaling, Translation and various distortion for hand drawn planar curves. The proposed approach, which is stable and robust, preserved sufficient similarity under RST and distortion. We observed that Fourier descriptors and optimal curve matching criteria can be used to identification of noisy hand drawn planar curves.

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