

Regular Article

# Applications of topology in automobile engineering

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## Abstract

Topology is a branch of Geometry concerned with the properties remain unchanged when the figure is deformed. Topology is used in automobile engineering, computer science engineering and other fields. The significance of topology is given in the following pages which will help to develop light weight components which are used in manufacturing two wheeler carts etc, the result is we get better automobile fuel efficiency. This in turn results to energy conservation and global environmental preservation. In industries several attempts are made to produce light weight designs. In order to manufacture light weight component our first step is to decrease the weights of hub bearing. Many companies have succeeded to reduce the weights of various products. By applying shape optimization techniques, many have dramatically succeeded in reducing weights.

Our main objective is to reduce the weight of a hub-bearing of a car. To achieve this target we make use of topological optimization technique as well as shape optimization technique.

**Keywords:** Shape optimization, contractible, decomposition, quotient topology

## Preliminaries

If  $X$  and  $Y$  are topological spaces and  $f: X \rightarrow Y$  is continuous and either open or closed, then the topology  $\tau$  on  $Y$  is the quotient topology  $\tau_f$ .

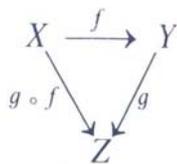
Let  $X = [0, 2\pi]$  with the usual topology,  $Y = \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 = 1\}$  with its usual topology and define  $f: X \rightarrow Y$  by  $f(x) = (\cos x, \sin x)$ . Then  $f$  is continuous and closed, so the unit circle with its usual topology is a quotient space of  $[0, 2\pi]$ .

The following are central useful facts for weak topology

If  $X$  has the weak topology induced by a collection  $\{f_\alpha : \alpha \in A\}$  of functions  $f_\alpha : X \rightarrow X_\alpha$ , then  $f: X \rightarrow Y$  is continuous iff  $f_\alpha \circ f$  is continuous for each  $\alpha \in A$ .

The following is the fundamental result about quotient topologies.

Let  $Y$  have the quotient topology induced by a map  $f$  of  $X$  onto  $Y$ . Then an arbitrary map  $g: Y \rightarrow Z$  is continuous iff  $g \circ f: X \rightarrow Z$  is continuous [18].



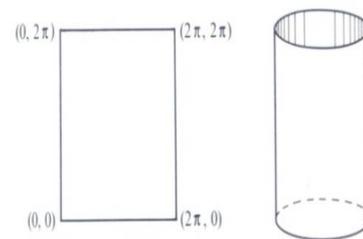
Let  $X$  be a topological space. A decomposition  $D$  of  $X$  is a collection of disjoint subsets of  $X$  whose union is  $X$ . If a decomposition  $D$  is endowed with the topology in which  $S \subseteq D$  is open iff  $\bigcup \{F / F \in S\}$  is open in  $X$ , then  $D$  is referred to as a decomposition space of  $X$  [12].

Define a map  $\pi$  of  $X$  onto  $D$  by letting  $\pi(x)$ , for each  $x \in X$ , be the element of  $D$  containing  $x$ .  $\pi$  is called the natural map (or decomposition map) of  $X$  onto  $D$ .

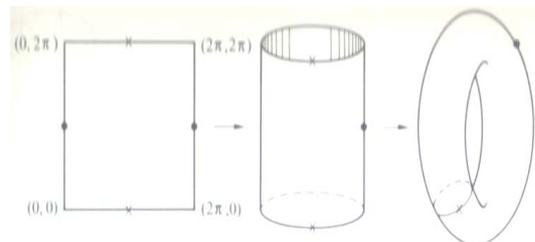
If  $\sim$  is an equivalence relation on the topological space  $X$ , then the identification space  $X/\sim$  is defined to be the decomposition space  $D$ . Whose elements are the equivalence classed for  $\sim$ .

The unit circle is a quotient space of  $[0, 2\pi]$  viewed as a decomposition space, the appropriate elements of the decomposition are the one point set  $\{x\}$  for which  $0 < x < 2\pi$  together with the set  $[0, 2\pi]$ .

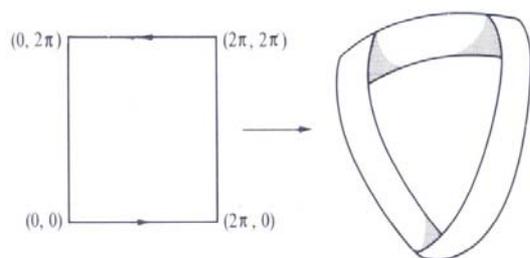
Consider the square  $[0, 2\pi] \times [0, 2\pi]$ . If we identify each point  $(0, x)$  with the point  $(2\pi, x)$  the resulting identification space is homeomorphic to the cylinder  $S^1 \times [0, 2\pi]$



Consider the square  $[0, 2\pi] \times [0, 2\pi]$ . Identify each point  $(0, y)$  with the point  $(2\pi, y)$  and also identify each point  $(x, 0)$  with the point  $(x, 2\pi)$ . Intuitively, it is clear that the resulting identification space is what one obtains by first rolling the square to obtain a cylinder, as we did before then matching the ends of the cylinder to obtain torus [7,10].



Again consider  $[0, 2\pi] \times [0, 2\pi]$ . Now identify point  $(x, 0)$  with  $(2\pi - x, 2\pi)$  the result is a twisted strip, called the Moebius strip [10,13].



Given any topological space  $X$ , we can describe two constructions. We obtain the cone  $\Delta X$  over  $X$  by identifying all the points  $(x, 1)$  in  $X \times I$  with a single point [3].

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3. Nano car is the best example and etc.

### Topology optimization

In the topology optimization step, an optimal structure to achieve the functions necessary for the part in question is obtained. In some cases, dramatic changes in the structure are possible. We used a homogenization method as a topology optimization technique.

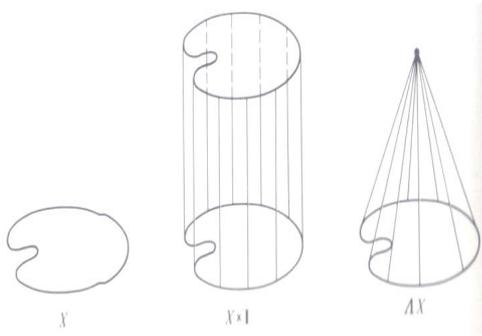
#### Analysis model

The analysis model we used is illustrated in fig 2. The hub ring was integrated with the inner ring, mesh was plotted and the 3 translational degrees of freedom for the bolt hole nodes on the flange were constrained. For the external load working on the hub ring, load concentration was assumed to occur on the rolling -16- fig 3. Result of topology optimization surface and a rolling element load value obtained from a bearing internal force analysis program was utilized. As shown in fig 2, regions where the shape can be altered (design regions) were set larger, and regions where the shape cannot be changed (such as the wheel spigot joint outside diameter and bearing outside diameter) due to interactions with counterpart members were defined as unalterable regions (non - design regions). Furthermore, analysis was executed considering the forging draft, cyclic symmetry conditions of bolt holes and symmetrical ness relative to the straight lines connecting the bolt holes and the flange center. The object function and restricting conditions were as follows.

Objective function: Distortion energy minimization  
Restricting conditions: 22.5%, 27.5% and 30% relative to the volume (or mass) of initial shape

The analysis was performed with two hub bolt patterns - 0° (cross-pattern) and 45° (X-pattern) relative to the vertical direction. Since the stress was greater in the 0° direction, further analysis was limited to this direction.

30% the initial volume 27.5% the initial volume 22.5% the initial volume



A space X is contractible iff, the identify map  $I: X \rightarrow X$  is homotopic to some constant map  $c(x) = x_0$  from X to a point  $x_0 \in X$ .

A convex subset of the Euclidean space is contractible.

X is contractible iff for any space T, any two continuous maps  $f, g: T \rightarrow X$  are homotopic.

### Applications of topology in optimization

Topology is a branch of Geometry concerned with the properties which remain unchanged even if the figure is deformed. Also, topology is the study of continuous functions. In some cases some changes in the structure are possible by using topology optimization step. We use a homogenization method as a topology optimization technique [19,20].

1. During the durability test it was found the weight of 1.4 kg of hub-bearing has succeeded but 1.3 kg weight failed in the test. Slowly it was succeeded in producing 1 kg low weight hub-bearing for a car. This result marked the beginning of our analysis for optimal bearing shape.
2. Ten years back our cell phone weights are more than 600 grams but nowadays we have a cell phone and also volume of the cell phone are very very small.

Fig. 2 Topology optimization model

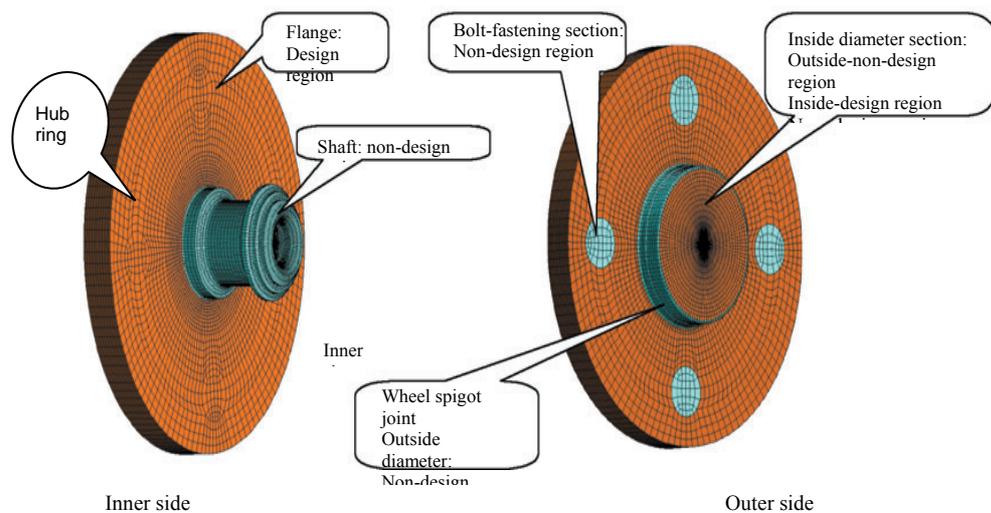
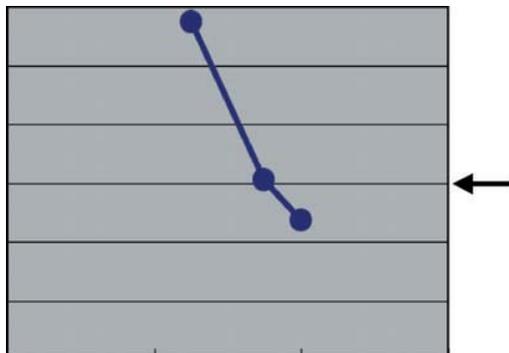


Fig2 .Topology optimization model

**Analysis results**

The shapes and principal stresses shown in fig 3 and 4 were attained. As shown in fig 4, with greater weight reduction, stress rapidly increased. We adopted a shape that was 27.5% the initial volume and near the stress limit as the basis for the basic structure of the hub bearing [5,9].

**Fig. 4 Proportion of initial volume**



Principal stress of each design 30% of the initial volume [17].

**Fig.3 27.5% of the initial volume 22.5% of the initial volume**



**Shape optimization**

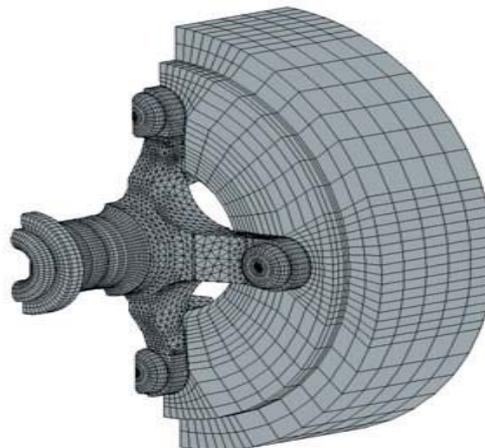
**Analysis model**

Using the shape obtained from the previously described topology analysis as a basis, the basic shape was set as in fig 5 and shape optimization analysis was executed by varying the dimensions. The fully automatic analysis system summarized in fig 7 was developed and the analysis was executed based on the experiment plan method.

Objective function: Volume (mass) minimization Restricting conditions: Principle stress at three designated nodes

The software of the shape optimization analysis system drives the nonlinear analysis solver to fig 5 Basic design of shape optimization calculate stress values while altering the shape according to the mesh data. In altering the shape, the shape base vectors\* were set using the morphing software so that alteration of the mesh could be synchronized with alteration of the shape in order to maintain the cyclic symmetry of the bolt holes as well as the symmetrical ness relative to the straight lines connecting the bolt holes and the flange center [6].

The calculation was executed on a 2.8 GHz Pentium 4 computer and took 2.5 days to complete. From this result, a response curved surface approximation model was developed to achieve an optimal solution. \*Vectors that defines how each node shifts when the nodes used as parameters are shifted.



**Analysis results**

(a) summarizes volumes, while (b) shows stress values. "L27 best solution" represents the combination that performed most effectively in the calculations performed according to the experimental method while Optimization" represents the optimal solution obtained by calculation in accordance with the response curved surface approximation. As a result of this series of calculation operations, many of the optimal values obtained coincided with the upper or lower limits (limits in terms of avoidance of interaction with counterpart parts). At every evaluation point, the stress value is lower than the current level. From these findings, we determined the shape of our final design.

**Final design**

In the development work, we also applied the analysis technique described above to the outer ring. In addition to shape optimization, we attempted to achieve lightening in the finalized bearing specification through alteration of bearing internal design and development of new materials and grease. The newly developed hub bearing shape, which is illustrated in fig 9, has achieved the target mass of 1.0 kg and cleared the target values for strength, durability and rigidity. In addition, the hub ring and outer ring have also achieved lightening targets as illustrated in fig 10 [15].

**Fig. 9 Shape of final design**

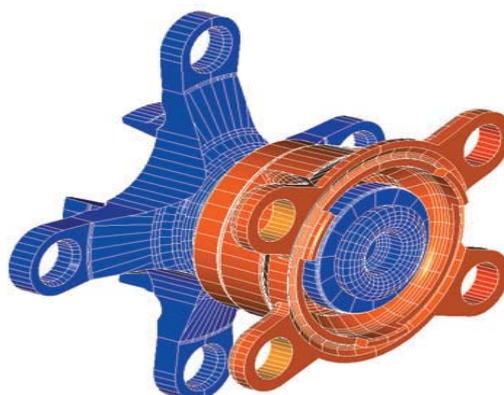
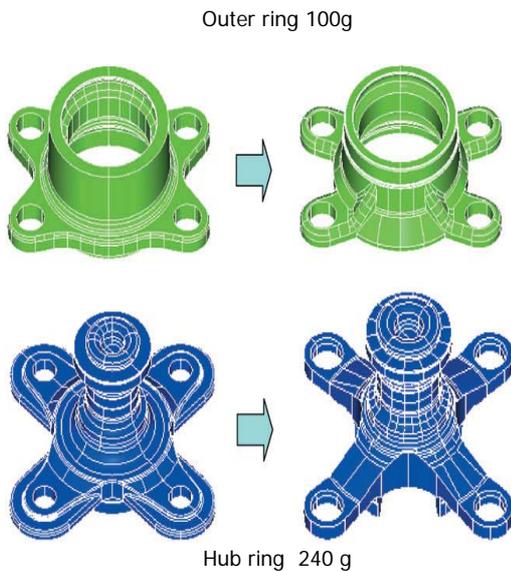


Fig. 10 Amount of each component lightening [18]



## Conclusion

This report has introduced the shape optimization analysis executed for lightening hub bearings for reduced weight cars. With this technique, we have succeeded in achieving a target mass of 1.0 kg, attaining dramatic lightening that was previously considered impossible. Note, however, that this newly developed hub bearing is not suitable for drum brake structures in which the hub bearing also functions as a brake seal.\* For such applications, another hub bearing type developed through use of our shape optimization technique should be used (at a penalty of additional 50 g weight). Our analysis in this report was centered on Mechanical strength. However, we also need to set targets for hub bearing rigidity. Therefore, we will aim for multi-faceted optimization in which we attempt to promote further lightening while maintaining sufficient bearing rigidity so that we can establish hub bearing technology to cope with the needs of various car manufacturers [1,2].

\*As shown in fig 9, the newly developed hub bearing has four pawls that radially guide the members installed to the hub ring. If the hub bearing is applied to a drum brake, the pawls need to be replaced with a circumferential rim to provide sealing function.

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