International Multidisciplinary Research Journal 2011, 1(10):12-15
ISSN: 2231-6302
Available Online: http://irjs.info/

Generalized fuzzy modified distribution method for generalized fuzzy transportation problem

S. Sagaya Roseline*, **E.C. Henry Amirtharaj**

Department of Mathematics, BishopHeberCollege, Tiruchirappalli-620017, Tamilnadu, India

Abstract
In this paper, a new approach is proposed to solve Generalized fuzzy transportation problem. Here for the initial solution,Generalized fuzzy Vogel's approximation method is preferred and to find the optimum solution a new approach, Generalized fuzzy modified distribution method is developed. This new approach is applied to a numerical example and it works well.

Keywords: Fuzzy numbers, Trapezoidal Fuzzy Numbers, Generalized Trapezoidal Fuzzy Numbers, Generalized Fuzzy Vogel's Approximation Method, Generalized Fuzzy Modified Distribution Method

INTRODUCTION

TheTransportation problem is the special type of linear programming problem where special mathematical structure of restrictions is used. Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. There are cases that the cost coefficients and the supply and demand quantities of a transportation problem may be uncertain due to some uncontrollable factors. To deal quantitatively with imprecise information in making decisions, Bellman and Zadeh [4] and Zadeh[3] introduced the notion of fuzziness. Lai and Hwang[7] others considered the situation where all parameters are fuzzy.

In 1979 Isermann [2] introduced algorithm for solving the transportation problem which provides effective solutions. The Rin guest and Rings [8] proposed two iterative algorithms for solving linear, multi-criteria transportation problem. Similar solution proposed in [2]. In works by S. Chanas and D. Kuchta[5] the approach based on interval and fuzzy coefficients had been elaborated. The further development of this approach introduced in work [8].

In this paper, a new approach is proposed to solve Generalized fuzzy transportation problem. Here for the initial solution, Generalized fuzzy Vogel's approximation method is preferred and to find the optimum solution a new approach, Generalized fuzzy modified distribution method is developed. This new approach is applied to an numerical example and it works well.

Preliminaries :(Kaufmann and Gupta [10])

Definition

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) ; x \in X \}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition

A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}} : R \to [0,1]$ is continuous.
(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty , a] \cup [d , \infty)$
(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on [a , b] and strictly decreasing on [c , d].
(iv) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b , c]$, where $a \le b \le c \le d$.

Definition

A fuzzy number $\tilde{A} = (a , b , c ,d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} , & a \le x \le b \\ 1 , & b \le x \le c \\ \dfrac{(x-d)}{(c-d)} , & c \le x \le d \end{cases}$$

Definition

A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R , is said to be generalized fuzzy number if its membership function has the following characteristics

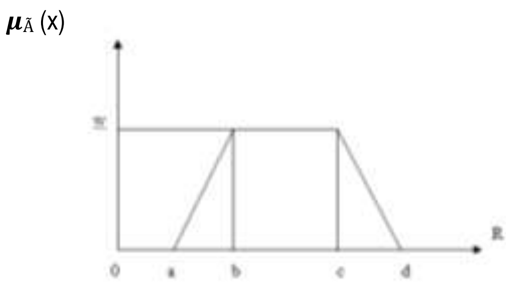
(i) $\mu_{\tilde{A}} : R \to [0,1]$ is continuous.
(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty , a] \cup [d , \infty)$
(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on [a , b] and strictly decreasing on [c , d].
(iv) $\mu_{\tilde{A}}(x) = w$, for all $x \in [b , c]$, where $0 \le w \le 1$.

Definition

A generalized fuzzy number $\tilde{A} = (a , b , c ,d; w)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

*Corresponding Author,
Email: sagaya_rose@yahoo.co.in, henry_23@rediffmail.com

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{w(x-a)}{(b-a)}, & a \le x \le b \\ w, & b \le x \le c \\ \dfrac{w(x-d)}{(c-d)}, & c \le x \le d \end{cases}$$



A Generalized trapezoidal fuzzy number

Let $\tilde{A} = (a, b, c, d; w)$ be a generalized trapezoidal fuzzy number then

(i) Rank $\mathrm{R}(\tilde{A}) = \dfrac{w(a+b+c+d)}{4}$, (ii) mode $(\tilde{A}) = \dfrac{w(b+c)}{2}$,
(iii) divergence$(\tilde{A}) = w(d-a)$, (iv)Left spread$(\tilde{A}) = w(b-a)$,
(v) Right spread $(\tilde{A}) = w(d-c)$

Two generalized trapezoidal fuzzy numbers $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ can be compared using the ranking functions given in [9].

ARITHMETIC OPERATIONS

In this section, arithmetic operations between two generalized trapezoidal fuzzy numbers, defined on universal set of real numbers R, are reviewed [1],[6].

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers then

(i) $\tilde{A} \oplus \tilde{B} = (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2; \min(w_1, w_2))$
(ii) $\tilde{A} \ominus \tilde{B} = (a_1-a_2, b_1-b_2, c_1-c_2, d_1-d_2; \min(w_1, w_2))$
(iii) $\tilde{A} \otimes \tilde{B} = (a_1 x a_2, b_1 x b_2, c_1 x c_2, d_1 x d_2; \min(w_1, w_2))$
(iv) $\tilde{A} \oslash \tilde{B} = (a_1 / a_2, b_1 / b_2, c_1 / c_2, d_1 / d_2; \min(w_1, w_2))$
where $a_1, b_1, c_1, d_1, a_2, b_2, c_2$ and d_2 are positive real numbers

Generalized Fuzzy Transportation Problem

Consider a transportation problem with m generalized fuzzy origins (rows) and n generalized fuzzy destinations (columns). Let $\tilde{C}_{ij} = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}, w_{cij}]$ be the cost of transporting one unit of the product from i[th] generalized fuzzy origin to j[th] generalized fuzzy destination. $\tilde{a}_i = [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, w_{ai}]$ be the quantity of commodity available at generalized fuzzy origin i. $\tilde{b}_j = [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, w_{bj}]$ be the quantity of commodity needed at generalized fuzzy destination j. $\tilde{X}_{ij} = [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}, w_{xij}]$ is the quantity transported from i[th] generalized fuzzy origin to j[th] generalized fuzzy destination.

The generalized fuzzy transportation problem can be stated in the tabular form as

	GFD_1	GFD_2		GFD_n	GFC
GFO_1	$\tilde{X}_{11}$ $\tilde{C}_{11}$	$\tilde{X}_{12}$ $\tilde{C}_{12}$	…	$\tilde{X}_{1n}$ $\tilde{C}_{1n}$	$\tilde{a}_1$
GFO_2	$\tilde{X}_{21}$ $\tilde{C}_{21}$	$\tilde{X}_{22}$ $\tilde{C}_{22}$	…	$\tilde{X}_{2n}$ $\tilde{C}_{2n}$	$\tilde{a}_2$
	…	…	…	…	
GFO_m	$\tilde{X}_{m1}$ $\tilde{C}_{m1}$	$\tilde{X}_{m2}$ $\tilde{C}_{m2}$	…	$\tilde{X}_{mn}$ $\tilde{C}_{mn}$	$\tilde{a}_m$
GFD	$\tilde{b}_1$	$\tilde{b}_2$		$\tilde{b}_n$	

GFO_i (i=1,2,…m) – Generalized Fuzzy Origin, GFD_j(j= 1,2,…n)– Generalized Fuzzy Destination ,GFC – Generalized Fuzzy Capacity,GFD – Generalized Fuzzy Demand.

Mathematically, the problem may be stated as a linear programming problem as follows

$$\text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}, w_{cij}]\,[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}, w_{xij}]$$

Subject to the Constraints

$$\sum_{j=1}^{n} [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}, w_{xij}] = [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, w_{ai}] \quad (i = 1,2,\ldots, m)$$

$$\sum_{i=1}^{m} [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}, w_{xij}] = [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, w_{bj}] \quad (j=1,2,\ldots,n)$$

$$[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}, w_{xij}] \ge 0 \quad \text{for all i and j}$$

The given generalized fuzzy transportation problem is said to be balanced if

$$\sum_{i=1}^{m} [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, w_{ai}] = \sum_{j=1}^{n} [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, w_{bj}]$$

(ie) if the total generalized fuzzy capacity is equal to the total generalized fuzzy demand.

The Generalized fuzzy transportation problem can be solved in two stages (i) initial solution and (ii) optimal solution. We discuss the Generalized Fuzzy Vogel's Approximation Method for finding the initial generalized fuzzy basic feasible solution and propose the Generalized Fuzzy Modified Distribution method for finding the optimal solution.

Generalized Fuzzy Vogel's Approximation Method

Step1 : Check whether the weight of Generalized Fuzzy Capacities and Generalized Fuzzy Demands are same . If not then consider the weight of the minimum among them as the weight of all capacities and demands.

Step2 : Check whether the given Generalized Fuzzy transportation problem is balanced or not. If

$$\sum_{i=1}^{m} [a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, w_{ai}] > \sum_{j=1}^{n} [b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, w_{bj}]$$

introduce a dummy destination in the generalized fuzzy transportation table. The cost of transporting to this destination are all set equal to generalized fuzzy zero and the requirement at this dummy destination is then assumed to be equal to

$$\sum_{i=1}^{m}[a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, w_{ai}] - \sum_{j=1}^{n}[b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, w_{bj}]$$

If $\sum_{i=1}^{m}[a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, w_{ai}] < \sum_{j=1}^{n}[b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, w_{bj}]$

introduce a dummy origin in the generalized fuzzy transportation table. The cost of transporting from this origin to any destination are all set equal to generalized fuzzy zero and the availability at this dummy origin is then assumed to be equal to

$$\sum_{j=1}^{n}[b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, w_{bj}] - \sum_{i=1}^{m}[a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, w_{ai}]$$

Step3 : Determine the generalized fuzzy penalty cost, namely the generalized fuzzy difference between the smallest and next smallest generalized fuzzy costs in each row and column.

Step4 : Choose the generalized fuzzy maximum penalty, by ranking method.

Step5 : In the selected row or column as by step 4, find out the cell having the least generalized fuzzy cost. Allocate to this cell the maximum feasible amount depending on the generalized fuzzy capacity and generalized fuzzy demands. Delete the row or column which is fully exhausted.

Step6 : Recompute the column and row generalized fuzzy penalties for the reduced generalized fuzzy transportation problem and then go to step 4, repeat the procedure until all the rim requirements are satisfied.

Generalized Fuzzy Modified Distribution Method

This proposed method is used for finding the optimal generalized fuzzy basic feasible solution. Generalized Fuzzy optimality test is applied to Generalized fuzzy initial basic feasible solution having (m+n-1) occupied cells where m is the number of generalized fuzzy origins and n is the number of generalized fuzzy destinations. The following iterative procedure is used to find out the same.

Step1 : For each occupied cell , find out a set of numbers, $[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}, w_{ui}]$ and $[v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}, w_{vj}]$ for each row and column satisfying $[u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}, w_{ui}] + [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}, w_{vj}] = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}, w_{cij}]$ for each occupied cell. To start with assign a generalized fuzzy zero to any row or column having maximum number of allocations. If this maximum number of allocation is more than one, choose any one arbitrary.

Step2 : Compute the net evaluations$[z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}, w_{zij}] = [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}, w_{cij}] - ([u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}, w_{ui}] + [v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}, w_{vj}])$ for all unoccupied basic cells.

If all $[z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}, w_{zij}] > [-1\lambda, -0.5\lambda, 0.5\lambda, 1\lambda]$, the current basic feasible solution is optimal and a unique solution exists.

If all $[z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}, w_{zij}] \geq [-1\lambda, -0.5\lambda, 0.5\lambda, 1\lambda]$, the current basic feasible solution is optimal but an alternate solution exists.

If atleast one $[z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}, w_{zij}] < [-1\lambda, -0.5\lambda, 0.5\lambda, 1\lambda]$, select the unoccupied cells having most negative net evaluation to enter the basis and then go to next step.

Step3 : From that unoccupied cell (having most negative net evaluation) draw a loop starting and ending at that unoccupied cell with corner cell occupied cell. Assign sign + and – alternatively starting from that unoccupied cell and find the generalized fuzzy minimum allocation from the cells having negative sign. This allocation should be added to the allocation of the cell with + sign and subtracted from the allocation of the cell with - sign.

Step4 : Step3 gives a better solution by making one or more occupied cell as unoccupied and one unoccupied cell as occupied. For this new set of generalized fuzzy basic feasible allocation repeat from the step1 till a generalized fuzzy optimal feasible solution is obtained.

Numerical Example

Consider a generalized fuzzy transportation problem with rows representing three generalized fuzzy origins GFO_1 , GFO_2 , GFO_3 and columns representing four generalized fuzzy destinations GFD_1 ,GFD_2,GFD_3, GFD_4

	GFD_1	GFD_2	GFD_3	GFD_4	GFC
GFO_1	[-1,1,3,7,0.2]	[1,4,5,7,0.5]	[-1,0,1,2,0.1]	[2,4,6,8,0.6]	[1,4,6,10,0.2]
GFO_2	[0,3,6,13,0.3]	[1,2,3,6,0.1]	[0,3,6,14,0.1]	[2,6,11,15,0.3]	[2,4,6,8,0.2]
GFO_3	[2,6,10,17,0.2]	[2,6,8,12,0.2]	[0,2,4,6,0.1]	[0,3,7,10,0.1]	[5,10,14,20,0.2]
GFD	[2,6,9,14,0.2]	[3,6,8,10,0.2]	[2,3,4,7,0.2]	[1,3,5,7,0.2]	

Since $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j = [8,18,26,38,0.2]$, the problem is balanced generalized fuzzy transportation problem.Using generalized fuzzy Vogel's approximation method the initial generalized fuzzy basic feasible solution is obtained as follows

	GFD_1	GFD_2	GFD_3	GFD_4	GFC
GFO_1	[1,4,6,10,0.2] [-1,1,3,7,0.2]	[1,4,5,7,0.5]	[-1,0,1,2,0.1]	[2,4,6,8,0.6]	[1,4,6,10,0.2]
GFO_2	[1,2,3,4,0.2] [0,3,6,13,0.3]	[1,2,3,4,0.2] [1,2,3,6,0.1]	[0,3,6,14,0.1]	[2,6,11,15,0.3]	[2,4,6,8,0.2]
GFO_3	[2,6,10,17,0.2]	[2,4,5,6,0.2] [2,6,8,12,0.2]	[2,3,4,7,0.2] [0,2,4,6,0.1]	[1,3,5,7,0.2] [0,3,7,10,0.1]	[5,10,14,20,0.2]
GFD	[2,6,9,14,0.2]	[3,6,8,10,0.2]	[2,3,4,7,0.2]	[1,3,5,7,0.2]	

Since the number of occupied cells is 6, there exists a non degenerate generalized fuzzy basic feasible solution.

Therefore, the initial generalized fuzzy transportation minimum cost is $[Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)} w_Z] = [4,53,136,330,0.1]$

Using generalized fuzzy modified distribution method the optimal generalized fuzzy basic feasible solution is obtained as follows

	GFD_1	GFD_2	GFD_3	GFD_4	GFC
GFO_1	[1,4,6,10,0.2] [-1,1,3,7,0.2]	[1,4,5,7,0.5]	[-1,0,1,2,0.1]	[2,4,6,8,0.6]	[1,4,6,10,0.2]
GFO_2	[0,3,6,13,0.3]	[2,4,6,8,0.2] [1,2,3,6,0.1]	[0,3,6,14,0.1]	[2,6,11,15,0.3]	[2,4,6,8,0.2]
GFO_3	[1,2,3,4,0.2] [2,6,10,17,0.2]	[1,2,2,2,0.2] [2,6,8,12,0.2]	[2,3,4,7,0.2] [0,2,4,6,0.1]	[1,3,5,7,0.2] [0,3,7,10,0.1]	[5,10,14,20,0.2]
GFD	[2,6,9,14,0.2]	[3,6,8,10,0.2]	[2,3,4,7,0.2]	[1,3,5,7,0.2]	

The optimal generalized fuzzy transportation minimum cost is[$Z^{(1)}$, $Z^{(2)}$, $Z^{(3)}$, $Z^{(4)}$ w_Z] = [5,51,133,322,0.1].

Conclusion

Generalized Fuzzy Modified Distribution Method provides the optimal value of the objective function and the transported quantity as generalized trapezoidal fuzzy numbers for the generalized fuzzy transportation problem with transported costs, the supply quantities and the demand quantities as generalized trapezoidal fuzzy numbers. This method is a systematic procedure, both easy to understand and to apply. This approach can be extended to solve other fuzzy optimization problem.

REFERENCES

[1] Chen S. H., 1985. Operations on Fuzzy Numbers with function principal, Tamkang. J. Manag. Sci.6, 13-25.

[2] Isermann H., 1979. The enumeration of all efficient solution for a linear multiple objective transportation problem, Naval Research Logistics Quarterly 26, pp.123-139.

[3] Zadeh L.A., 1978. Fuzzy sets as a basis for a theory of possibility, Fuzzy sets and systems, 1, 3-28

[4] Bellman R.E., Zadeh L.A.,1970. Decision making in a fuzzy environment, Management Science.17,pp.141-164

[5] Chanas S., Kuchta D., 1998. Fuzzy Integer Transporting problem, Fuzzy Sets and Systems 98, pp.291-298.

[6] Hsieh C.H., Chen S.H., 1999 . Similarity of generalized Fuzzy numbers with graded mean integration representation In : Proceedings of the eighth International Fuzzy Systems Association world Congress, TaipeiTaiwan, Republic of China,pp.551-555.

[7] Lai Y.J., Hwang C.L., 1992. Fuzzy Mathematical Programming Methods and Application, Springer, Berlin .

[8] Ringuset J.L., Rinks D.B., 1987. Interactive Solutions for the Linear Multi Objective Transportation Problem, European Journal of Operational Research 32,pp.96-106.

[9] Amit Kumar, Pushpinder Singh, Amarpreet Kaur, Parmpreet Kaur, 2010. Ranking of Generalized Trapezoidal Fuzzy Numbers Based on Rank, Mode, Divergence and Spread,Turkish Journal of Fuzzy Systems, Vol, No.1.

[10] Kaufmann A., Gupta, M.M., 1988. Fuzzy mathematical models in engineering and management science, Elsevier Science Publishers, Amsterdam, Netherland.