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Spherical symmetric cosmological model for higher dimensional space–time

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Abstract
In this paper thenecessary condition for perfect fluid distribution and the value of pressuredensity and velocity are obtained in terms of components of curvature tensor for five dimensional sphericalSymmetric space – times.

Keywords: SphericalSymmetries, Cosmology Higher Dimension

INTRODUCTION

The spherical symmetry has been studied by Takeno (1966) [1]. His study depends on the group of motion and Killing vectors, while Burman (1969) [2] has given static symmetrically exterior solution. Which was detectable departure from the prediction of the Schwarzschild solution. The concept of spherical symmetry for higher dimensions can be obtained by generalizing that of the spherical symmetry in three dimensional Euclidean space, which has been done by Khadekar et al. (1987) [3]. Here we consider the spherical symmetric line element in five dimensional space – time in the form.

$$
\begin{aligned}
ds^2 &= h_1(dx^2+dy^2+dz^2) + h_2(xdx+ydy+zdz)^2 + h_3 dt_1{}^2 + \\
&2h_4(xdx+ydy+zdz)dt_1 + 2h_5(xdx+ydy+zdz)dt_2 + 2h_6 dt_1 dt_2 + h_7 dt_2{}^2
\end{aligned} \tag{1}
$$

where

$$h_1 = A - \frac{Bx}{y} = h_2, h_3 = C, h_4 = E/Z, h_5 = E/Z, h_6 = D, h_7 = G$$

In which $A, B, C--------G$ are function of $x, t_1 \& t_2$.
The line element (1) can be transformed to the orthogonal form

$$ds^2 = h_1(dx^2+dy^2+dz^2) + h_3 dt_1{}^2 + h_7 dt_2{}^2 \tag{2}$$

Where $h_1 = A - \frac{Bx}{y}, h_3 = C, h_7 = G$ $A, B, C \& G$ are functions of $x, t_1 \& t_2$.

For the metric (2), the necessary condition for the distribution, pressure and density have been calculated. Also metric reduces to $ds^2 = p(t)(dx^2+dy^2+dz^2+dt_1{}^2+dt_2{}^2)$ as a special case .

Components of curvature tensor

The fourteen algebraic independent and non-vanishing components of the curvature tensor R_{hijk} for the metric (2) are given by

$$f_1 = R_{1212} = -\frac{1}{2}\left[A'' - \frac{B''x}{y} - \frac{2B'}{y} - \frac{2Bx}{y^3}\right] + \frac{\left(A' - \frac{B'x}{y} - \frac{B}{y}\right)^2}{2(A-Bx|y)} - \frac{\left(\dot{A} - \frac{\dot{B}x}{y}\right)^2}{4C} - \frac{\left(\overset{*}{A} - \frac{\overset{*}{B}x}{y}\right)^2}{4G} + \frac{B^2x^2|y^4}{2(A-Bx|y)},$$

$$f_2 = R_{1313} = -\frac{1}{2}\left[A'' - \frac{B''x}{y} - \frac{2B'}{y}\right] + \frac{\left(A' - \frac{B'x}{y} - \frac{B}{y}\right)^2}{2(A-Bx|y)} + \frac{B^2x^2|y^4}{4(A-Bx|y)} - \frac{\left(\dot{A} - \dot{B}x/y\right)^2}{4C} - \frac{\left(\overset{*}{A} - \overset{*}{B}x/y\right)^2}{4G},$$

$$f_3 = R_{1224} = \frac{1}{2}\left(\dot{A}' - \frac{\dot{B}'x}{y} - \frac{\dot{B}}{y}\right) - \frac{\left(A' - \frac{B'x}{y} - \frac{B}{y}\right)\left(\dot{A} - \frac{\dot{B}x}{y}\right)}{2(A-Bx/y)} - \frac{\left(\dot{A} - \frac{\dot{B}x}{y}\right)C'}{4C} = R_{1334},$$

$$f_5 = R_{1225} = \frac{1}{2}\left(\overset{*}{A}' - \frac{\overset{*}{B}'x}{y} - \frac{\overset{*}{B}}{y}\right) - \frac{\left(A' - \frac{B'x}{y} - \frac{B}{y}\right)\left(\overset{*}{A} - \frac{\overset{*}{B}x}{y}\right)}{2\left(A - \frac{Bx}{y}\right)} - \frac{G'\left(\overset{*}{A} - \frac{\overset{*}{B}x}{y}\right)}{4G} = R_{1335},$$

$$f_4 = R_{1414} = \frac{-1}{2}\left(C'' + \ddot{A} - \frac{\ddot{B}x}{y}\right) + \frac{\left(\dot{A} - \frac{\dot{B}x}{y}\right)^2}{4\left(A - \frac{Bx}{y}\right)} + \frac{C'\left(A' - \frac{B'x}{y} - \frac{B}{y}\right)}{4\left(A - \frac{Bx}{y}\right)} + \frac{C'^2}{4C} - \frac{\overset{*}{C}\left(\overset{*}{A} - \frac{\overset{*}{B}x}{y}\right)}{4G} - \frac{\dot{C}\left(\dot{A} - \frac{\dot{B}x}{y}\right)}{4C},$$

$$f_6 = R_{1415} = -\frac{1}{2}\left(\overset{\bullet *}{A} - \frac{\overset{\bullet *}{B}x}{y}\right) + \frac{\left(\dot{A} - \frac{\dot{B}x}{y}\right)\left(\overset{*}{A} - \frac{\overset{*}{B}x}{y}\right)}{4\left(A - \frac{Bx}{y}\right)} + \frac{\overset{*}{C}\left(\dot{A} - \frac{\dot{B}x}{y}\right)}{4C} + \frac{\dot{G}\left(\overset{*}{A} - \frac{\overset{*}{B}x}{y}\right)}{4G},$$

$$f_7 = R_{1445} = \frac{1}{2}\overset{*}{C}' - \frac{C'\left(\overset{*}{A} - \frac{\overset{*}{B}x}{y}\right)}{4\left(A - \frac{Bx}{y}\right)} - \frac{C'\overset{*}{C}}{4C} - \frac{G'\overset{*}{C}}{4G},$$

$$f_8 = R_{1515} = -\frac{1}{2}\left(G'' + \overset{**}{A} - \frac{\overset{**}{B}x}{y}\right) + \frac{\left(\overset{*}{A} - \frac{\overset{*}{B}x}{y}\right)^2}{4\left(A - \frac{Bx}{y}\right)} + \frac{G'\left(A' - \frac{B'x}{y} - \frac{B}{y}\right)}{4\left(A - \frac{Bx}{y}\right)} - \frac{\dot{G}\left(\dot{A} - \frac{\dot{B}x}{y}\right)}{4C} + \frac{\overset{*}{G}\left(\overset{*}{A} - \frac{\overset{*}{B}x}{y}\right)}{4G} + \frac{G'^2}{4G}$$

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$$f_9 = R_{1545} = -\frac{\dot{G}'}{2} + \frac{G'\left(\dot{A}-\frac{\dot{B}x}{y}\right)}{4\left(A-\frac{Bx}{y}\right)} + \frac{C'\dot{G}}{4C} + \frac{\dot{G}G'}{4G}, \tag{3}$$

$$f_{10} = R_{2323} = \frac{Bx}{y^3} - \frac{\left(A'-\frac{B'x}{y}-\frac{B}{y}\right)^2}{4\left(A-\frac{Bx}{y}\right)} + \frac{\left(\frac{B^2x^2}{y^4}\right)}{4\left(A-\frac{Bx}{y}\right)} - \frac{\left(\dot{A}-\frac{\dot{B}x}{y}\right)^2}{4C} - \frac{\left(\overset{*}{A}-\frac{\overset{*}{B}x}{y}\right)^2}{4G}$$

,

$$f_{11} = R_{2424} = R_{3434} = -\frac{1}{2}\left(\ddot{A}-\frac{\ddot{B}x}{y}\right) - \frac{C'\left(A'-\frac{B'x}{y}-\frac{B}{y}\right)^2}{4\left(A-\frac{Bx}{y}\right)} + \frac{\left(\dot{A}-\frac{\dot{B}x}{y}\right)^2}{4\left(A-\frac{Bx}{y}\right)} + \frac{\dot{C}\left(\dot{A}-\frac{\dot{B}x}{y}\right)}{4C} - \frac{\overset{*}{C}\left(\overset{*}{A}-\frac{\overset{*}{B}x}{y}\right)}{4G},$$

$$f_{12} = R_{2525} = R_{3535} = -\frac{1}{2}\left(\overset{**}{A}-\frac{\overset{**}{B}x}{y}\right) - \frac{\left(A'-\frac{B'x}{y}-\frac{B}{y}\right)G'}{4\left(A-\frac{Bx}{y}\right)} + \frac{\left(\overset{*}{A}-\frac{\overset{*}{B}x}{y}\right)^2}{4\left(A-\frac{Bx}{y}\right)} - \frac{\dot{G}\left(\dot{A}-\frac{\dot{B}x}{y}\right)}{4C} + \frac{\overset{*}{G}\left(\overset{*}{A}-\frac{\overset{*}{B}x}{y}\right)}{4G}$$

, $$f_{13} = R_{4545} = -\frac{1}{2}(\ddot{G}+\overset{**}{C}) - \frac{G'C'}{4\left(A-\frac{Bx}{y}\right)} + \frac{\overset{*}{C}^2}{4C} + \frac{\dot{G}\dot{C}}{4C} + \frac{\dot{G}^2}{4G} + \frac{\overset{*}{G}\overset{*}{C}}{4G}$$

and

$$f_{14} = R_{2425} = R_{3435} = -\frac{1}{2}\left(\overset{\bullet *}{A}-\frac{\overset{\bullet *}{B}x}{y}\right) + \frac{\left(\dot{A}-\frac{\dot{B}x}{y}\right)\left(\overset{*}{A}-\frac{\overset{*}{B}x}{y}\right)}{4\left(A-\frac{Bx}{y}\right)} + \frac{\overset{*}{C}\left(\dot{A}-\frac{\dot{B}x}{y}\right)}{4C} + \frac{\dot{G}\left(\overset{*}{A}-\frac{\overset{*}{B}x}{y}\right)}{4G}$$

where $A' = \frac{\partial A}{\partial x}, \dot{A} = \frac{\partial A}{\partial t_1}, \overset{*}{A} = \frac{\partial A}{\partial t_2}$

Consider energy momentum tensor for perfect fluid distribution for five dimensions as

$$T_{ij} = (p+\rho)v_i v_j - p g_{ij} \quad ij = 1,2,-----5 \tag{4}$$

Together with $g_{ij}v^i v^j = 1$ where (5)

p is the pressure, ρ is the density and $v_i = (v_1, 0, 0, v_4, v_5)$ is the flow vector which represents the motion of the fluid in x direction.

Solution of field equations

The field equation $R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij}$ for the metric (6.1.2)becomes.

$$\frac{f_{10}}{A-\frac{Bx}{y}} + \frac{2f_{11}}{C} + \frac{2f_{12}}{G} + \frac{f_{13}\left(A-\frac{Bx}{y}\right)}{CG} = -8\pi(p+\rho)v_1^2 + 8\pi p\left(A-\frac{Bx}{y}\right) \tag{6}$$

$$\frac{f_2}{\left(A-\frac{Bx}{y}\right)} + \frac{f_4+f_{11}}{C} + \frac{f_8+f_{12}}{G} + \frac{f_{13}\left(A-\frac{Bx}{y}\right)}{CG} = 8\pi p\left(A-\frac{Bx}{y}\right) \tag{7}$$

$$\frac{f_1}{\left(A-\frac{Bx}{y}\right)} + \frac{f_4+f_{11}}{C} + \frac{f_8+f_{12}}{G} + \frac{f_{13}\left(A-\frac{Bx}{y}\right)}{CG} = 8\pi p\left(A-\frac{Bx}{y}\right) \tag{8}$$

$$\frac{(f_1+f_2)C}{\left(A-\frac{Bx}{y}\right)^2} + \frac{f_{10}C}{\left(A-\frac{Bx}{y}\right)^2} + \frac{(f_8+2f_{12})C}{G\left(A-\frac{Bx}{y}\right)} = -8\pi(p+\rho)v_4^2 + 8\pi pC \tag{9}$$

$$\frac{(f_1+f_2)G}{\left(A-\frac{Bx}{y}\right)^2} + \frac{f_{10}G}{\left(A-\frac{Bx}{y}\right)^2} + \frac{(f_4+2f_{11})G}{C\left(A-\frac{Bx}{y}\right)} = -8\pi(p+\rho)v_5^2 + 8\pi pG \tag{10}$$

$$\frac{2f_3}{\left(A-\frac{Bx}{y}\right)} - \frac{f_9}{G} = -8\pi(p+\rho)v_1 v_4 \tag{11}$$

$$\frac{2f_5}{\left(A-\frac{Bx}{y}\right)} + \frac{f_7}{C} = -8\pi(p+\rho)v_1 v_5 \text{ and} \tag{12}$$

$$\frac{-(f_6+2f_{14})}{\left(A-\frac{Bx}{y}\right)} = -8\pi(p+\rho)v_4 v_5. \tag{13}$$

Eliminating p, & ν_l form equations (6) to (13) , the necessary condition for the perfect fluid distribution comes out to be

$$\left[\frac{(f_2+f_{10})C}{\left(A-\frac{Bx}{y}\right)^2} - \frac{(f_4+f_{11})}{\left(A-\frac{Bx}{y}\right)} - \frac{f_{13}}{G}\right]\left[\frac{f_1-f_{10}}{\left(A-\frac{Bx}{y}\right)} + \frac{f_4-f_{11}}{C} + \frac{f_8-f_{12}}{G}\right] = \left[\frac{2f_3}{\left(A-\frac{Bx}{y}\right)} - \frac{f_9}{G}\right]^2 \tag{14}$$

From equations (7) and (8), we get $f_1 = f_2$.

In view of the above identity (14), the pressure and density are given by

$$8\pi p = \frac{f_1}{\left(A-\frac{Bx}{y}\right)^2} + \frac{f_4+f_{11}}{C\left(A-\frac{Bx}{y}\right)} + \frac{f_8+f_{12}}{G\left(A-\frac{Bx}{y}\right)} + \frac{f_{13}}{CG} \text{ and} \tag{15}$$

$$8\pi\rho = \frac{f_4-2f_{11}}{C\left(A-\frac{Bx}{y}\right)} + \frac{f_8-2f_{12}}{G\left(A-\frac{Bx}{y}\right)} - \frac{3f_{10}}{\left(A-\frac{Bx}{y}\right)^2} - \frac{2f_2}{\left(A-\frac{Bx}{y}\right)^2} + \frac{f_{13}}{CG} \tag{16}$$

The velocity vector distribution is obtained as

$$v_i = \left[\left(A-\frac{Bx}{y}\right)^{\frac{1}{2}}\frac{E^{\frac{1}{2}}}{F^{\frac{1}{2}}}, 0, 0, \frac{M^{\frac{1}{2}}}{F^{\frac{1}{2}}}, \frac{N^{\frac{1}{2}}}{F^{\frac{1}{2}}}\right], \tag{17}$$

where

$$E = (f_{10}-f_1)CG + (f_{11}-f_4)(A-\frac{Bx}{y})G + (f_{12}-f_8)(A-\frac{Bx}{y})C,$$

$$F = (f_1+3f_{10})CG + (f_{11}-2f_4)(A-\frac{Bx}{y})G + (f_{12}-2f_8)(A-\frac{Bx}{y})C - 2f_{13}(A-\frac{Bx}{y})^2,$$

$$M = -(f_4+f_{11})CG(A-\frac{Bx}{y}) + (f_1+f_{10})C^2G + f_{12}C^2(A-\frac{Bx}{y}) - f_{13}(A-\frac{Bx}{y})^2 C \text{ and}$$

$$N = f_{11}G^2(A-\frac{Bx}{y}) + (f_2+f_{10})CG^2 - (f_8+f_{12})CG(A-\frac{Bx}{y}) - f_{13}(A-\frac{Bx}{y})^2 G$$

Special cases

If we choose $A - \frac{Bx}{y} = C = G$.

Then metric (2) reduces to

$$ds^2 = C(dx^2 + dy^2 + dz^2 + dt_1^{\,2} + dt_2^{\,2}) . \qquad (18)$$

Further if we take C as a function of t_1 & t_2 the components of curvature tensor simplify to

$$f_1 = -\frac{\dot{C}^2}{4C} - \frac{\overset{*}{C}^2}{4C} = f_2,\ f_4 = -\frac{\ddot{C}}{2} + \frac{\dot{C}^2 - \overset{*}{C}^2}{4C},$$

$$f_6 = -\frac{\overset{\bullet *}{C}^2}{2} + \frac{3\overset{*}{C}\dot{C}}{4C},\ f_8 = -\frac{\overset{**}{C}}{2} + \frac{\overset{*}{C}^2}{2C} - \frac{\dot{C}^2}{4C},$$

$$f_{10} = -\frac{\dot{C}^2}{4C} - \frac{\overset{*}{C}^2}{4C},\quad f_{11} = -\frac{1}{2}\ddot{C} + \frac{\dot{C}^2}{2C} - \frac{\overset{*}{C}^2}{4C},$$

$$f_{11} = -\frac{1}{2}\ddot{C} + \frac{\dot{C}^2}{2C} - \frac{\overset{*}{C}^2}{4C},\quad f_{12} = -\frac{1}{2}\overset{**}{C} + \frac{\overset{*}{C}^2}{2C} - \frac{\dot{C}^2}{4C},$$

$$f_3 = \frac{\ddot{C}}{2},\quad f_5 = 0,\quad f_7 = 0,$$

$$f_9 = 0,\ f_{13} = -\frac{1}{2}\left(\ddot{C} + \overset{**}{C}\right) + \frac{\overset{*}{C}^2}{2C} + \frac{\dot{C}^2}{2C},\quad f_{14} = -\frac{\overset{\bullet *}{C}}{2} + \frac{3\dot{C}\overset{*}{C}}{4C},$$

Moreover if we assume that $f_4 = f_{11}$ then

$$-\frac{\ddot{C}}{2} + \frac{\dot{C}^2}{4C} - \frac{\overset{*}{C}^2}{4C} = -\frac{\ddot{C}}{2} - \frac{\overset{*}{C}^2}{4C} + \frac{\dot{C}^2}{2C}$$

i.e, $\frac{\dot{C}^2}{4C} = 0$ i.e, $\dot{C} = 0$

Integrating we get $C = p(t_2)$ where p is the function of t_2 only The Necessary Condition for the distribution assumes the form.

$$\left(\frac{2f_3}{C} - \frac{f_9}{C}\right)^2 = \left[\frac{Cf_2 + Cf_{10}}{C^2} - \frac{f_4 + f_{11}}{C} - \frac{f_{13}}{C}\right]\left[\frac{f_1 - f_{10}}{C^2} + \frac{f_4 - f_{11}}{C} + \frac{f_8 - f_{12}}{C}\right]$$

Moreover pressure and density becomes.

$$8\pi p = \frac{f_1 + f_4 + f_{11} + f_8 + f_{12} + f_{13}}{C^2}$$ and

$$8\pi\rho = \frac{f_4 - 2f_2 - 3f_{10} - 2f_{11} + f_8 - 2f_{12} + f_{13}}{C^2} .$$

Therefore metric (18) takes the form.

$$ds^2 = p(t)(dx^2 + dy^2 + dz^2 + dt_1^{\,2} + dt_2^{\,2}).$$

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