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Numerical accuracy of method of false position

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Abstract
A computer program in C++ language has been developed to calculate square roots of numbers from 1 to 25 in interval [0, 6] using method of false position. Accuracy of method of false position has been found out in each calculation. Lowest percentage error has been obtained in the calculation of square root of 24 in the interval [0, 6] using method of false position and is equal to 0.000006094208. Highest percentage error has been obtained in the calculation of square root of 1 in the interval [0, 6] and is equal to 0.000399351100. Average percentage error in method of false position in the calculation of square roots of natural numbers from 1 to 25 has been found to be 0.000051494950. Average percentage error of bisection method in the calculation of square roots of natural numbers from 1 to 25 has been found to be 0.000041549568 which indicates that the accuracy of bisection method is greater than that of the method of false position.

Keywords: Method of false position, numerical accuracy, percentage error, algorithm, approximation.

INTRODUCTION

The numerical analysis is a branch of Mathematics concerned with finding accurate approximations to the solutions of problems whose exact solution is either impossible or infeasible to determine [1-5]. In this field the analysis of error is of great importance. Numerical stability is an important notion in numerical analysis. An algorithm is called numerically able if an error, whatever its cause, does not grow to be much larger during the calculation. This happens if the problem is well-conditioned, meaning that the solution changes by only a small amount if the problem data are changed by a small amount [6-9].

In order to discuss the method of false position, let us choose c as the intercept of the secant line through *(a, f(a))* and *(b, f(b))*. Let us also assume that *f(x)* is continuous such that $f(a)\,f(b) < 0$ then formula for the secant line is given by-

$$\frac{y - f(b)}{x - b} = \frac{f(a) - f(b)}{a - b}$$

Let y = 0, the intercept then the next approximation is

$$x_1 = \frac{af(b) - bf(a)}{a - b}$$

x_1 is first approximation to x^* [10-15]

As in bisection, if $f(x_1) \neq 0 \implies f(a)\,f(x_1) < 0$ or $f(b)\,f(x_1) < 0$

$\implies$ there must be a root $x^* \in [x_1, b]$

Let us suppose $f(b)\,f(x_1) < 0$, then

$$x_2 = \frac{x_1 f(b) - bf(x_1)}{f(b) - f(x_1)} \quad \text{etc.}$$

Similar to the secant method, the false position method also uses a straight line to approximate the function in the local region of interest. The only difference between these two methods is that the secant method keeps the most recent two estimates, while the false position method retains the most recent estimate and the next recent one which has an opposite sign in the function value. [16-20]

The false position method, which sometimes keeps an older reference point to maintain an opposite sign bracket around the root, has a lower and uncertain convergence rate compared to the secant method. The emphasis on bracketing the root may sometimes restrict the false position method in difficult situations while solving highly nonlinear equations. [21-24]

MATERIAL AND METHOD

Algorithm of method of false position is given below-[25-28]

To find a root of $f(x) = 0$ in the interval $[a_0, b_0]$ with which $f(a_0)\,f(b_0) < 0$ with tolerance δ

$x_{n+1} = b_n - (b_n - a_n)\,f(b_n) / [f(b_n) - f(a_n)]$, n=0, 1, 2, ……
if ($|f(x_{n+1})| < \delta$) root found, stop iteration
else
if [$f(x_{n+1})\,f(b_n) < 0$] $a_{n+1} = x_{n+1}$; $b_{n+1} = b_n$
else
$a_{n+1} = a_n$; $b_{n+1} = x_{n+1}$

Computer program developed by us to calculate square roots of natural numbers from 1 to 25 is given below-

```
#include<conio.h>
#include<stdio.h>
#include<math.h>
// method of false position
```

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```
void main(void)
{
    FILE *fpt;
    int n;
    float a[1000],b[1000],c[1000],delta,rl,ru,d,aa;
    double f(float x);
    //avr is the variable whose square root is to be calculated
    double avr =1.0;
    clrscr();
    //Filename to store result
    fpt=fopen("nddf1.txt", "w");
    rl=0; ru=6.0; n=0; a[0]=rl; b[0]=ru; aa=fabs(rl-ru);
    //Value of function f(x)
    fprintf(fpt,"f(x)=x^2-25\n");
    fprintf(fpt,"rl= %6.2f\n",rl);
    fprintf(fpt,"ru= %6.2f\n",ru);
    //to check existence of root between the interval
    d=f(rl)*f(ru);
    delta=0.00001;
    fprintf(fpt,"    n     a[n]        b[n]        c[n]          f(c[n])\n");
    printf(" n            a[n]        b[n]        c[n]          f(c[n])\n");
    if (d<0)
    {
  while(aa > delta)
  {
    if (a[n]==b[n]) break;
    c[n+1]=b[n]-(b[n]-a[n])*f(b[n])/(f(b[n])-f(a[n]));
    if (f(c[n+1])*f(b[n])<0)
    {
        a[n+1]=c[n+1];
        b[n+1]=b[n];
    }
    else
    {
        b[n+1]=c[n+1];
        a[n+1]=a[n];
    }
    aa=fabs(f(c[n]));
fprintf(fpt,"%3d %15.12f %15.12f %15.12f %18.12f\n",n+1,a[n],b[n],c[n],
  f(c[n]));
printf("%3d %15.12f %15.12f %15.12f %18.12f\n",n+1,a[n],b[n],c[n],
  f(c[n]));
    if (aa > delta)      n=n+1;
  }
  printf("Root= %20.15f\n",c[n]);
  printf("Value of function=%20.15f\n", f(c[n]));
  printf("No. of iterations=%3d\n",n+1);
  printf("Actual value of root=%15.12f\n",sqrt(avr));
  fprintf(fpt,"Actual value of root=%15.12f\n",sqrt(avr));
  printf("\n");
  getch();
      }
      else
        {
          printf("There is no root in the given interval\n");
          getch();
        }
      fclose(fpt);
   }
//Function definition
double f(float x)
{
    double r;
    r=x*x-1;
    return(r);
}
```

With the help of above computer program, square roots of the number from 1 to 25 in the interval [0, 6] have been calculated. For this, the following functions have been taken

$f(x) = x^2 - n = 0$ where n = 1, 2, 3, ……. , 25

Numerical accuracy of method of false position has been has been measured by percentage error and defined as follows–

Percentage error = error in the value of square root * 100/actual value of square root

Numerical accuracy of method of false position is inversely proportional to percentage error.

RESULT AND DISCUSSION

Calculation of square root of 1 by method of false position

Method of false position has been applied to calculate the roots of equation

$f(x) = x^2 – 1 = 0$

in the interval [0, 6] using computer program developed by us. Initial value of interval, last value of interval, estimated value of root and value of function at estimated value of root in each iteration is included in Table-1. Estimated value of square root of 1 by method of false position after each iteration is shown in Graph-1.

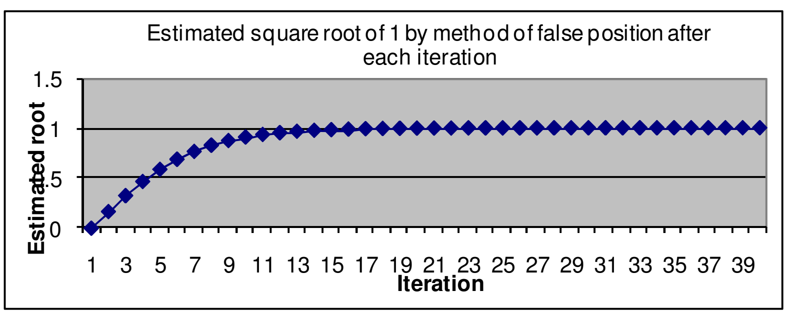
Table1. Initial value of interval, last value of interval, estimated root and value of function at estimated root in the calculation of square root of 1 by method of false position

Iteration	Initial value of interval	Last value of interval	Estimated root by method of false position	Value of function at estimated root
1	0.000000000000	6.000000000000	0.000000002401	-1.000000000000
2	0.166666671634	6.000000000000	0.166666671634	-0.972222220567
3	0.324324339628	6.000000000000	0.324324339628	-0.894813722725
4	0.465811967850	6.000000000000	0.465811967850	-0.783019210608
5	0.586913406849	6.000000000000	0.586913406849	-0.655532652861
6	0.686433851719	6.000000000000	0.686433851719	-0.528808567214
7	0.765520632267	6.000000000000	0.765520632267	-0.413978161574
8	0.826710045338	6.000000000000	0.826710045338	-0.316550500938
9	0.873079478741	6.000000000000	0.873079478741	-0.237732223802
10	0.907668352127	6.000000000000	0.907668352127	-0.176138162547
11	0.933167278767	6.000000000000	0.933167278767	-0.129198829839

Iteration	Initial value of interval	Last value of interval	Estimated root by method of false position	Value of function at estimated root
12	0.951802194118	6.000000000000	0.951802194118	-0.094072583271
13	0.965334296227	6.000000000000	0.965334296227	-0.068129696529
14	0.975115537643	6.000000000000	0.975115537643	-0.049149688246
15	0.982161998749	6.000000000000	0.982161998749	-0.035357808214
16	0.987226009369	6.000000000000	0.987226009369	-0.025384806426
17	0.990859031677	6.000000000000	0.990859031677	-0.018198379344
18	0.993462204933	6.000000000000	0.993462204933	-0.013032847369
19	0.995325803757	6.000000000000	0.995325803757	-0.009326544376
20	0.996659040451	6.000000000000	0.996659040451	-0.006670757087
21	0.997612476349	6.000000000000	0.997612476349	-0.004769347033
22	0.998294055462	6.000000000000	0.998294055462	-0.003408978829
23	0.998781144619	6.000000000000	0.998781144619	-0.002436225154
24	0.999129235744	6.000000000000	0.999129235744	-0.001740770281
25	0.999377965927	6.000000000000	0.999377965927	-0.001243681219
26	0.999555647373	6.000000000000	0.999555647373	-0.000888507804
27	0.999682605267	6.000000000000	0.999682605267	-0.000634688727
28	0.999773263931	6.000000000000	0.999773263931	-0.000453420728
29	0.999838054180	6.000000000000	0.999838054180	-0.000323865413
30	0.999884307384	6.000000000000	0.999884307384	-0.000231371846
31	0.999917387962	6.000000000000	0.999917387962	-0.000165217251
32	0.999940991402	6.000000000000	0.999940991402	-0.000118013715
33	0.999957859516	6.000000000000	0.999957859516	-0.000084279192
34	0.999969899654	6.000000000000	0.999969899654	-0.000060199785
35	0.999978482723	6.000000000000	0.999978482723	-0.000043034091
36	0.999984622002	6.000000000000	0.999984622002	-0.000030755760
37	0.999989032745	6.000000000000	0.999989032745	-0.000021934389
38	0.999992191792	6.000000000000	0.999992191792	-0.000015616356
39	0.999994397163	6.000000000000	0.999994397163	-0.000011205642
40	0.999996006489	6.000000000000	0.999996006489	-0.000007987006

Actual value of square root of 1	1.000000000000
Calculated value of square root of 1 by method of false position	0.999996006489
Difference between actual value and calculated value of square root of 1	0.000003993511
Percentage error in the value of square root 1calculated by method of false position	0.000399351100

Graph 1. Estimated value of square root of 1 by method of false position after each iteration



Similar calculations in the calculation of square root of natural numbers from 2 to 25 have been done by the method of false position.

CONCLUSION

Lowest percentage error has been obtained in the calculation of square root of 24 in the interval [0, 6] using method of false position and is equal to 0.000006094208. It means if roots lies in the middle of the interval then the error in the calculated value of root of equation by method of false position is least.

Highest percentage error has been obtained in the calculation of square root of 1 in the interval [0, 6] and is equal to 0.000399351100. It means if roots lies in the beginning of the interval then the error in the calculated value of root of equation by method of false position is greatest. Exact value of root, value of root calculated by method of false position and percentage error in the calculation of root by method of false position is shown in Table-26. Average percentage error in method of false position in the calculation of square roots of natural numbers from 1 to 25 has been found to be 0.000051494950. It is clear that percentage error decreases as the value of root shifts towards last value of interval.

Average percentage error of bisection method in the calculation of square roots of natural numbers from 1 to 25 has been found to be 0.000041549568 which indicates that the accuracy of bisection method is greater than that of the method of false position.

Table 2. Exact value of root, value of root calculated by method of false position and percentage error in the calculation of root by method of false position

S. No.	Function	Exact Value of root	Value of root obtained by method of false position	Percentage error in method of false position
1	$f(x)=x^2-1$	1.000000000000	0.999996006489	0.000399351100
2	$f(x)=x^2-2$	1.414213562373	1.414211034775	0.000178728169
3	$f(x)=x^2-3$	1.732050807569	1.732048511505	0.000132563317
4	$f(x)=x^2-4$	2.000000000000	1.999998092651	0.000095367450
5	$f(x)=x^2-5$	2.236067977500	2.236066579819	0.000062506195
6	$f(x)=x^2-6$	2.449489742783	2.449488878250	0.000035294412
7	$f(x)=x^2-7$	2.645751311065	2.645749807358	0.000056834782
8	$f(x)=x^2-8$	2.828427124746	2.828425884247	0.000043858263
9	$f(x)=x^2-9$	3.000000000000	2.999998807907	0.000039736433
10	$f(x)=x^2-10$	3.162277660168	3.162277221680	0.000013866208
11	$f(x)=x^2-11$	3.316624790355	3.316624164581	0.000018867796
12	$f(x)=x^2-12$	3.464101615138	3.464100599289	0.000029325035
13	$f(x)=x^2-13$	3.605551275464	3.605550765991	0.000014130239
14	$f(x)=x^2-14$	3.741657386774	3.741656541824	0.000022582239
15	$f(x)=x^2-15$	3.872983346207	3.872982978821	0.000009485866
16	$f(x)=x^2-16$	4.000000000000	3.999999284744	0.000017881400
17	$f(x)=x^2-17$	4.123105625618	4.123105049133	0.000013981815
18	$f(x)=x^2-18$	4.242640687119	4.242639541626	0.000026999529
19	$f(x)=x^2-19$	4.358898943541	4.358898162842	0.000017910463
20	$f(x)=x^2-20$	4.472135955000	4.472135543823	0.000009194197
21	$f(x)=x^2-21$	4.582575694956	4.582574844360	0.000018561526
22	$f(x)=x^2-22$	4.690415759823	4.690415382385	0.000008047005
23	$f(x)=x^2-23$	4.795831523313	4.795831203461	0.000006669375
24	$f(x)=x^2-24$	4.898979485566	4.898979187012	0.000006094208
25	$f(x)=x^2-25$	5.000000000000	4.999999523163	0.000009536740
Average percentage error in method of false position				0.000051494950

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