Journal of Experimental Sciences 2012, 3(1): 01-04
ISSN: 2218-1768
Available Online: http://jexpsciences.com/

Comparison of Lagrange's and Newton's interpolating polynomials

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Abstract
A set of fourteen functions have been considered in various intervals. Lagrange's and Newton's interpolating polynomials have been obtained for each function using a computer program developed in C++ programming language. Average of the maximum percentage error for the functions in Newton's interpolating polynomial and Lagrange's interpolating polynomial are 765.3107 and 898.9139 respectively. This indicates that the Newton's interpolating polynomial is approximately 1.174574 times better than the Lagrange's interpolating polynomial.

Keywords: Newton's divided difference formula, Lagrange's interpolation formula, interpolating polynomial, difference triangle.

INTRODUCTION

Numerical analysis [1-4] is the area of mathematics and computer science that creates, analyzes and implements numerical method for solving numerically the problems of continuous mathematics. Such problems originates from real-world applications of algebra, geometry and calculus and they involve variables that vary continuously, such problems occur throughout the natural sciences, social sciences, engineering, medicine and business. During the second half of the twentieth century and continuing up to the present day, digital computers have grown in power and availability. This has led to the use of increasingly realistic mathematical models in science & engineering and numerical analysis of increasing sophistication has been needed to solve the more sophisticated mathematical models of the world. The formal academic area of numerical analysis varies from quite foundational mathematical studies to the computer science issues involved in the creation and implementation of several algorithms.

In almost any discussion of interpolation formula is a certain collection, the interpolation formulas are derived which find the interpolated value of a function in terms of certain of its values. These standard formulas are all expressions for the polynomial, which gives the function at certain values. Polynomial interpolation methods include Newton's divided difference and Lagrange's interpolation formulas. These formulas involve finding a polynomial of order n-1 that passes through the n data points. We published the paper for interpolation of $\tan^{-1}x$ and found that Newton divided difference formula is two times better than Lagrange's interpolation formula [5-6]. Here, we have extended our study for a set of functions defined in definite intervals.

Received: Jan 02, 2012; Revised: Jan 15, 2012; Accepted: Jan 28, 2012.

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MATERIAL AND METHOD

The work this paper is mainly based on Newton divided formula[7-10] and Lagrange's interpolation formula[11] described below

Lagrange's formula for equal interval

We have divided the interval [a, b] into 10 equal parts with the help of the points $a=x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}=b$ and calculated interpolating polynomial using Lagrange's formula

$$f(x)=\frac{(x-x_1)(x-x_2)....(x-x_{10})}{(x_0-x_1)(x_0-x_2)....(x_0-x_{10})}f(x_0)+\frac{(x-x_0)(x-x_2)(x-x_3)....(x-x_{10})}{(x_1-x_0)(x_1-x_2)(x_1-x_3)....(x_1-x_{10})}f(x_1)$$
$$+....+\frac{(x-x_0)(x-x_1)....(x-x_9)}{(x_{10}-x_0)(x_{10}-x_1)....(x_{10}-x_9)}f(x_{10})$$

It can be solved to give $f(x) = c_0 + c_1 x + c_2 x^2 + ... + c_{10} x^{10}$, with

$$c_i=\sum_{j=0}^{j=10} b_j\, y_j\, p(j,10,10-i)$$

where $b_i = (-1)^i/\{i!\,(10-i)!\,h^{10}\}$ and p(i,10,j) = sum of product of j terms in all combinations among $x_0,x_1,\ldots,x_{i-1},x_{i+1},\ldots, x_{10}$

Newton's divided difference formula

The Lagrange interpolation formula involves very considerable computation and its use can be quite risky. It is much more efficient to use the divided differences method for interpolation [12-14].

We have divided the interval [a, b] into 10 equal parts with the help of the points $a=x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}=b$. Let the differences be defined as

$$d^n y_i = d^{n-1}y_i - d^{n-1}y_{i-1} \quad \text{where } n, i = 0, 1, 2, ... , 10$$

Now, the Newton's divided difference formula

$$f(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + ... + a_{10}(x-x_0)(x-x_1)....(x-x_9)$$

where $a_n = (d^n y_0)/(n! h^n)$; n = 0,1, 2, …. , 10.
becomes
$f(x) = c_0 + c_1 x + c_2 x^2 + \ldots + c_{10} x^{10}$
where
$C_0=a_0-a_1*p(1,1)+a_2*p(2,2)-a_3*p(3,3)+a_4*p(4,4)-a_5*p(5,5)+a_6*p(6,6)-a_7*p(7,7)+a_8*p(8,8)-a_9*p(9,9)+a_{10}*p(10,10)$;
$C_1=a_1-a_2*p(2,1)+a_3*p(3,2)-a_4*p(4,3)+a_5*p(5,4)-a_6*p(6,5)+a_7*p(7,6)-a_8*p(8,7)+a_9*p(9,8)-a_{10}*p(10,9)$;
$C_2=a_2-a_3*p(3,1)+a_4*p(4,2)-a_5*p(5,3)+a_6*p(6,4)-a_7*p(7,5)+a_8*p(8,6)-a_9*p(9,7)+a_{10}*p(10,8)$;
$C_3=a_3-a_4*p(4,1)+a_5*p(5,2)-a_6*p(6,3)+a_7*p(7,4)-a_8*p(8,5)+a_9*p(9,6)-a_{10}*p(10,7)$;
$C_4=a_4-a_5*p(5,1)+a_6*p(6,2)-a_7*p(7,3)+a_8*p(8,4)-a_9*p(9,5)+a_{10}*p(10,6)$;
$C_5=a_5-a_6*p(6,1)+a_7*p(7,2)-a_8*p(8,3)+a_9*p(9,4)-a_{10}*p(10,5)$;
$C_6=a_6-a_7*p(7,1)+a_8*p(8,2)-a_9*p(9,3)+a_{10}*p(10,4)$;
$C_7=a_7-a_8*p(8,1)+a_9*p(9,2)-a_{10}*p(10,3)$;
$C_8=a_8-a_9*p(9,1)+a_{10}*p(10,2)$;
$C_9=a_9-a_{10}*p(10,1)$;
$C_{10}=a_{10}$;
p(i, j)=summation of the product of j elements in all combinations among $x_0, x_1, .., x_{i-1}, x_{i+1}, \ldots, x_{10}$

We have developed a computer program in C++ for obtaining the polynomial interpolation of the functions using Newton's divided difference and Lagrange's interpolation formulas. Fourteen functions given in Table-1 have been considered for polynomial interpolation using Newton's divided difference and Lagrange's interpolation formulas

Table 1. Function for polynomial interpolation

S. No.	Function f(x)	Interval
1	cos x	[-3,3]
2	cos x	[-1,1]
3	cos x	[-2,2]
4	sin x	[-1,1]
5	sin x	[-2,2]
6	sec x	[-1,1]
7	$\cos^{-1}x$	[-1,1]
8	$\sin^{-1}x$	[-1,1]
9	e^x	[0,2]
10	$\sqrt{x}$	[0, 2]
11	x	[0,2]
12	log x	[0.02,2]
13	$(1-x)^{-1/2}$	[-1.5,.95]
14	$(1+10x^2)^{-1}$	[-1,1]

RESULT AND DISCUSSION

We have divided the interval into 10 equal parts with the help of the points $x_0, x_1, \ldots x_{10}$ such that $x_i = x_0 + ih$, i=0,1,2,…………10, h=(b-a)/10 for the calculation of interpolating polynomials. Interpolating polynomials for the function obtained by Newton's divided difference formula and Lagrange's interpolation formula are shown in Tables-2-3. For the adjudgement of errors, the interval [a, b] in which the function is defined, has been divided into 200 equal points and the errors in each interpolating polynomial have been calculated. Maximum errors in these interpolating polynomials have been calculated and shown in Table-4.

Table 2. Polynomial obtained by Newton's divided difference formula

S. No	Function	Interval	Newton's Interpolating polynomial
1	cos x	[-1, 1]	$1.00000000 + 0.00000001 x - 0.49999908 x^2 + 0.00000001 x^3 + 0.04165433x^4 - 0.00134065 x^6 - 0.00004504 x^8 + 0.00003272 x^{10}$
2	cos x	[-2, 2]	$0.99999994 - 0.00000003 x - 0.50000018 x^2 - 0.00000055 x^3 + 0.04166721 x^4 + 0.00000063 x^5 - 0.00138944 x^6 - 0.00000025 x^7 + 0.00002498 x^8 + 0.00000003 x^9 - 0.00000029 x^{10}$
3	cos x	[-3, 3]	$1.00000012 + 0.00000013 x - 0.49999970 x^2 - 0.00000004 x^3 + 0.04166586 x^4 + 0.00000001 x^5 - 0.00138816 x^6 + 0.00002454 x^8 - 0.00000024 x^{10}$
4	sin x	[-1, 1]	$1.00000000 + 0.00000005 x - 0.49999985 x^2 - 0.00000066 x^3 + 0.04166621 x^4 - 0.00000067 x^5 - 0.00138884 x^6 - 0.00000005 x^7 + 0.00002454 x^8 - 0.00000024 x^{10}$
5	sin x	[-2, 2]	$0.00000006 + 0.99999994 x - 0.00000015 x^2 - 0.16666619 x^3 + 0.00000051 x^4 + 0.00833239 x^5 - 0.00000050 x^6 - 0.00019771 x^7 + 0.00000018 x^8 + 0.00000253 x^9 - 0.00000002 x^{10}$
6	sec x	[-1, 1]	$0.99999988 - 0.00000062 x + 0.50002587 x^2 + 0.00000039 x^3 + 0.20744291 x^4 + 0.00000010 x^5 + 0.09241693 x^6 + 0.00000005 x^7 + 0.01065832 x^8 + 0.04027120 x^{10}$
7	$\cos^{-1}x$	[-1, 1]	$1.57079613 - 1.00169110 x - 0.00000135 x^2 - 0.10639933 x^3 + 0.00001642 x^4 - 0.57960111 x^5 - 0.00005271 x^6 + 1.36122882 x^7 + 0.00005947 x^8 - 1.24432909 x^9 - 0.00002406 x^{10}$
8	$\sin^{-1}x$	[-1, 1]	$- 0.00000002 + 1.00169086 x - 0.00000120 x^2 + 0.10639693 x^3 + 0.00000495 x^4 + 0.57960939 x^5 - 0.00001412 x^6 - 1.36123753 x^7 + 0.00002500 x^8 + 1.24433196 x^9 - 0.00001235 x^{10}$
9	e^x	[0, 2]	$1.00000000 + 1.00000167 x + 0.49998301 x^2 + 0.16671911 x^3 + 0.04165243 x^4 + 0.00812289 x^5 + 0.00183556 x^6 - 0.00022259 x^7 + 0.00023522 x^8 - 0.00005165 x^9 + 0.00000610 x^{10}$
10	$\sqrt{x}$	[0, 2]	$4.22807598 x - 17.03731537 x^2 + 50.84386826 x^3 - 98.82740021 x^4 + 127.30678558 x^5 - 109.65661621 x^6 + 62.43516922 x^7 - 22.54080772 x^8 + 4.67118597 x^9 - 0.42295042 x^{10}$
11	x	[0, 2]	$0.99999046 x + 0.00013426 x^2 - 0.00075699 x^3 + 0.00228581 x^4 - 0.00411959 x^5 + 0.00463710 x^6 - 0.00328498 x^7 + 0.00142151 x^8 - 0.00034287 x^9 + 0.00003529x^{10}$

S. No	Function	Interval	Newton's Interpolating polynomial
12	log x	[0.02,2]	$-4.46995497 + 30.49288559x - 137.54426575x^2 + 401.33236694x^3 - 759.88439941x^4 + 957.13061523x^5 - 809.16748047x^6 + 453.56219482x^7 - 161.59304810x^8 + 33.10969162x^9 - 2.96867371x^{10}$
13	$(1-x)^{-1/2}$	[-1.5,.95]	$0.99981147 + 0.49352676x + 0.37325808x^2 + 0.47205245x^3 + 0.52293235x^4 - 0.43204543x^5 - 1.37324429x^6 + 0.20573542x^7 + 2.35216498x^8 + 1.89666903x^9 + 0.46798301x^{10}$
14	$(1+10x^2)^{-1}$	[-1, 1]	$1.00000000 + 0.00000024x - 8.91812038x^2 + 0.00000118x^3 + 49.59495163x^4 + 0.00002386x^5 - 136.96646118x^6 - 0.00008102x^7 + 168.75018311x^8 + 0.00000763x^9 - 73.36964417x^{10}$

Table 3. Polynomial obtained by Lagrange's interpolation formula

S. No.	Function	Interval	Lagrange's Interpolating polynomial
1	cos x	[-1, 1]	$0.99999994 - 0.00000013x - 0.49999520x^2 - 0.00005539x^3 + 0.04162361x^4 + 0.00062712x^5 - 0.00122348x^6 - 0.00028937x^7 - 0.00024315x^8 - 0.00007797x^9 + 0.00007581x^{10}$
2	cos x	[-2, 2]	$0.99999994 + 0.00000006x - 0.49999893x^2 - 0.00000724x^3 + 0.04166568x^4 + 0.00001896x^5 - 0.00138716x^6 - 0.00000233x^7 + 0.00002419x^8 - 0.00000006x^9 - 0.00000025x^{10}$
3	cos x	[-3, 3]	$1.00000000 + 0.00000005x - 0.49999985x^2 - 0.00000066x^3 + 0.04166621x^4 - 0.00000067x^5 - 0.00138884x^6 - 0.00000005x^7 + 0.00002454x^8 - 0.00000024x^{10}$
4	sin x	[-1, 1]	$1.00000012x - 0.00000031x^2 - 0.16666347x^3 - 0.00004135x^4 + 0.00831725x^5 - 0.00002395x^6 - 0.00034726x^7 + 0.00003643x^8 - 0.00000853x^9 - 0.00000209x^{10}$
5	sin x	[-2, 2]	$1.00000000x - 0.00000026x^2 - 0.16666538x^3 - 0.00000415x^4 + 0.00833171x^5 - 0.00000126x^6 - 0.00019984x^7 + 0.00000047x^8 + 0.00000248x^9 - 0.00000003x^{10}$
6	sec x	[-1, 1]	$0.99999994 + 0.00000005x + 0.50003064x^2 - 0.00005681x^3 + 0.20743188x^4 + 0.00066193x^5 + 0.09250873x^6 - 0.00030908x^7 + 0.01043347x^8 - 0.00009522x^9 + 0.04033199x^{10}$
7	$\cos^{-1}x$	[-1, 1]	$1.57079625 - 1.00169015x + 0.00000608x^2 - 0.10648999x^3 + 0.00001894x^4 - 0.57858944x^5 + 0.00015964x^6 + 1.36094809x^7 - 0.00033046x^8 - 1.24444938x^9 + 0.00004564x^{10}$
8	$\sin^{-1}x$	[-1, 1]	$1.00169039x - 0.00000051x^2 + 0.10640211x^3 - 0.00003981x^4 + 0.57959783x^5 - 0.00002853x^6 - 1.36139894x^7 + 0.00004951x^8 + 1.24432528x^9 - 0.00001136x^{10}$
9	e^x	[0, 2]	$1.00000000 + 0.99997425x + 0.49996591x^2 + 0.16670969x^3 + 0.03763726x^4 + 0.02828430x^5 - 0.02202225x^6 + 0.01616962x^7 - 0.00474117x^8 - 0.00121481x^9 + 0.00015702x^{10}$
10	$\sqrt{x}$	[0, 2]	$4.22805595x - 17.03724098x^2 + 50.84366226x^3 - 98.83029938x^4 + 127.31250763x^5 - 109.66347504x^6 + 62.44139481x^7 - 22.54339027x^8 + 4.67076111x^9 - 0.42287174x^{10}$
11	x	[0, 2]	$0.99997783x + 0.00003184x^2 - 0.00035747x^3 - 0.00163670x^4 + 0.00017150x^5 - 0.00742745x^6 + 0.00390315x^7 - 0.00106466x^8 - 0.00073051x^9 + 0.00008202x^{10}$
12	log x	[0.02,2]	$-4.46995592 + 30.49288177x - 137.54432678x^2 + 401.33270264x^3 - 759.88085938x^4 + 957.12689209x^5 - 809.16607666x^6 + 453.56692505x^7 - 161.59251404x^8 + 33.10970306x^9 - 2.96867681x^{10}$
13	$(1-x)^{-1/2}$	[-1.5,.95]	$0.99981141 + 0.49352601x + 0.37326023x^2 + 0.47204998x^3 + 0.52291656x^4 - 0.43194848x^5 - 1.37347054x^6 + 0.20551522x^7 + 2.35218930x^8 + 1.89668214x^9 + 0.46798608x^{10}$
14	$(1+10x^2)^{-1}$	[-1, 1]	$0.99999994 - 0.00000003x - 8.91811752x^2 - 0.00004816x^3 + 49.59489441x^4 + 0.00044524x^5 - 136.96626282x^6 - 0.00027129x^7 + 168.74998474x^8 - 0.00001552x^9 - 73.36961365x^{10}$

Table 4. Maximum percentage error in Newton's and Lagrange's interpolating polynomials

S. No.	Function	Interval	Maximum percentage error in Newton's interpolating polynomial	Maximum percentage error in Lagrange's interpolating polynomial
1	cos x	[-3, 3]	0.064992690	2.294741840
2	cos x	[-1, 1]	0.000059420	0.049786950
3	cos x	[-2, 2]	0.041294390	12.05671096
4	sin x	[-1, 1]	0.000200020	0.024267050
5	sin x	[-2, 2]	0.000600110	0.040910650
6	sec x	[-1, 1]	0.011954750	0.015129500
7	$\cos^{-1}x$	[-1, 1]	99.39362100	73.26376200
8	$\sin^{-1}x$	[-1, 1]	6.639571000	6.626192000
9	e^x	[0, 2]	0.000022710	7.618216000
10	$\sqrt{x}$	[0, 2]	59.37311000	59.37330000
11	x	[0, 2]	0.000800000	28.09092000
12	log x	[0.02, 2]	1.547646000	111.1734000
13	$(1-x)^{-1/2}$	[-1.5, 0.95]	9.423458530	9.418445080
14	$(1+10x^2)^{-1}$	[-1, 1]	588.8133427	588.8680960
Average			765.3107	898.9139

CONCLUSION

Average of the maximum percentage error for the function in Newton's interpolating polynomial is 765.3107 where as it is 898.9139 in Lagrange's interpolating polynomial. It is clear that Newton's interpolating polynomial is approximately 1.174574 times better than Lagrange's interpolating polynomial.

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